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Partial cooperation and convex sets

J. Enrique Romero García*, Jorge J. López Vázquez

Universidad de Sevilla, Spain

Abstract

We consider games of transferable utility, those that deal with partial cooperation situations, made up of coalition systems, in which every unit coalition is feasible and every coalition of players can be expressed as a disjoint union of maximal feasible coalitions. These systems are named *partition systems* and cause restricted games. To sum up, we study feasible coalition systems defined by a partial order designed for a set of players and we analyze the characteristics of a feasible coalition system developed from a family of convex sets.

MSC: 90D12

Keywords: cooperative games, partial cooperation, convex sets

1 Partial cooperation

A system of feasible cooperations is defined by $(N, \mathcal{F})$, $\mathcal{F} \subseteq 2^N$, *that proves the following axiom:*
$(P1)$ $\emptyset \in \mathcal{F}$, *and the group* $\{i\} \in \mathcal{F} \;\; \forall\, i \in N$.

Considering the given explanation, it results that any coalition $S \subseteq N$ can be expressed by a disjoint union of feasible coalitions, as

$$S = \bigcup_{a \in S} \{a\}.$$

However, this partition of S for *feasible coalitions* should not be unique. In general, we will denote $\mathcal{P}_{\mathcal{F}}(S)$ the set made up of partitions of $S \subseteq N$ in nonempty feasible

* *Address for correspondence:* José Enrique Romero García. romerogje@us.es. Facultad de Ciencias Económicas y Empresariales. Universidad de Sevilla. Avda. Ramón y Cajal, nº 1 41018-Sevilla. Spain.

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coalitions. Obviously $\mathcal{P}_{\mathcal{F}}(\emptyset) = \{\emptyset\}$. The previous reasoning gives sense to and makes consistent the idea of a *restricted cooperation game*: *Define the triple* $(N, \mathcal{F}, v)$*, in which* $(N, \mathcal{F})$ *is a feasible coalition system and* (N, v) *a transferable utility game. Then the couple* $(N, v^{\mathcal{F}})$ *in which*

$$v^{\mathcal{F}} : 2^N \longrightarrow \mathbb{R}, \quad v^{\mathcal{F}}(S) = \max \left\{ \sum_i v(T_i) \mid \{T_i\} \in \mathcal{P}_{\mathcal{F}}(S) \right\}.$$

is termed a game with restricted cooperation by the feasible coalition system $(N, \mathcal{F})$*.*

The supplied explanation for *game of restricted cooperation by a system of feasible coalitions* is for every coalition of players, an extension of the one by Faigle (1989) concerning games with restricted cooperations and by Bergantiños, Carreras and García–Jurado (1993) when using communication graphs to show incompatibility among some of the players. Indeed, it can be shown that $v^{\mathcal{F}}(S) \geq \sum_{i \in S} v(\{i\})$. Defined this way, the game is always superadditive.

Let $(N, \mathcal{F})$ *be a system of feasible coalitions. Let* $S \subseteq N$*. It is said that* T *is* $\mathcal{F}$*–component of* S *if it is proved that* $T \in \mathcal{F}$ *and* $T' \in \mathcal{F}$ *does not exist, as* $T \subset T' \subseteq S$*.* That is to say, the $S \subseteq N$ $\mathcal{F}$–components are the *maximal feasible coalitions* included in S and, for any $S \subseteq N$, the $\mathcal{F}$–components *of* S are a collection $\{T_k\}_k \subset 2^S$ such that

$$S = \bigcup_k T_k$$

But, the $\mathcal{F}$–components of $S \subseteq N$ are not necessarily a partition of S as its intersection can be nonempty.

It can be proved that if we consider $(N, \mathcal{F}, v)$*, where* $(N, \mathcal{F})$ *is a feasible coalition system,* (N, v) *a superadditive game and, for each coalition* $S \subseteq N$*, the* $\mathcal{F}$*–components of* S *are a partition of itself, then the restricted cooperation game* $(N, v^{\mathcal{F}})$ *verifies*

$$v^{\mathcal{F}}(S) = \sum_k v(T_k),$$

where $\{T_k\}_k \in \mathcal{P}_{\mathcal{F}}$*, the* S *partition for its maximal feasible coalitions (*$\mathcal{F}$*–components of* S*).*

Therefore, if the $\mathcal{F}$–components of any coalition are a partition of itself, and the game (N, v) is superadditive, then the restricted game by the system of feasible coalitions is determined by

$$v^{\mathcal{F}}(S) = \sum_k v(T_k),$$

in which $\{T_k\}_k$ is the S partition for maximal feasible coalitions. As the previous expression requires that maximal feasible coalitions must be disjointed, a new definition for a concrete feasible coalitions system has to be looked for. It will be named a *partition system* .

A partition system is the couple $(N, \mathcal{F})$, $\mathcal{F} \subseteq 2^N$ *that verifies the following axioms:*

$(P1)$ $\emptyset \in \mathcal{F}$, $\{i\} \in \mathcal{F}$ $\forall$ $i \in N$.

$(P2)$ $\forall$ $S \subseteq N$, *the* S *maximal subsets in* $\mathcal{F}$ *(*$\mathcal{F}$*–components of* S*) are a partition of* S*, denoted by*

$$C_{\mathcal{F}}(S) = \{S_1, \dots, S_k\}.$$

Evidently, a *partition system* is a *feasible coalitions system*, so, the $\mathcal{F}$ elements will not change their name.

Example 1 *Let* $N = \{1, 2, \dots, n\}$*, a natural number n, and considering the collection* $\mathcal{L}_n$ *made of all the sets such as* $[i, j] = \{i, i+1, \dots, j-1, j\}$ *for* $1 \leq i \leq j \leq n$*. This model represents a one-dimensional political election situation and the couple* $(N, \mathcal{L}_n)$ *is a partition system.*

Example 2 *A communication situation is the triple* (N, G, v)*, in which* (N, v) *is a game and* $G = (N, E(N))$ *is a graph. This idea was first developed by Myerson (1977), and researched by Owen (1986) and Borm, Nouweland and Tijs (1992, 1993). It is easy to see that the couple* $(N, \mathcal{F})$*, in which*

$$\mathcal{F} = \{S \subseteq N \mid (S, E(S)) \text{ is a connected subgraph of } G\},$$

is a partition system. We must point out that the opposite is not always true, because every G graph is a collection of pairs $\{i, j\}$*, and as a result, there must be feasible collections made up of two elements, but this might not happen.*

The previous definitions come from an extension of communication situation and communication graph-restricted game, developed by Myerson (1977) and Owen (1986).

The following theorem shows a characterization of the concept of partition systems.

Theorem 1 *A feasible coalitions system* $(N, \mathcal{F})$*,* $\mathcal{F} \subseteq 2^N$ *is a partition system if and only if*

$$\forall A \in \mathcal{F}, B \in \mathcal{F}, \text{ con } A \cap B \neq \emptyset \implies A \cup B \in \mathcal{F}.$$

Proof. $(\Leftarrow)$ Considering that the $\mathcal{F}$-components of $A \subseteq N$ form a recover, it is only necessary to prove that every pair of $\mathcal{F}$*–components* of A are disjointed. Let T_i, T_j $(i \neq j)$ maximal feasible coalitions of A. If $T_i \cap T_j \neq \emptyset$,it would mean, hypothesizing, $T_i \cup T_j \in \mathcal{F}$ being $T_i \cup T_j \subset A$. This contradicts that T_i and T_j are maximal feasible coalitions of A.

$(\Rightarrow)$ Let $A \in \mathcal{F}$, $B \in \mathcal{F}$ with $A \cap B \neq \emptyset$. If $A \cup B \notin \mathcal{F}$, then

$$A \cup B = \bigcup_k T_k,$$

where $\{T_k\}$ is the partition of $A \cup B$ for maximal sets. As A and B are feasible coalitions contained in $A \cup B$, thus $A \subseteq T_j$, $B \subseteq T_p$ for every j and p. If $j \neq p$, then $T_j \cap T_p = \emptyset$

and, so, $A \cap B = \emptyset$ against the hypothesis; then $A \cup B \in \mathcal{F}$. If $j = p$ then $A \subseteq T_j \subseteq A \cup B$ and $B \subseteq T_j \subseteq A \cup B$, implies $A \cup B = T_j \in \mathcal{F}$. □

2 Partially ordered set restricted games

The aim of this section is to study a feasible coalition system defined by a partial order for all players. From this moment only posets $P = (N, \leq)$ will be considered and the feasible coalition system characteristics developed from the family of convex sets will be analyzed.

Let $P = (N, \leq)$ a poset. It is said that $A \subseteq N$ is convex in P if it is proved that

$$a \in A,\ b \in A \quad \text{and} \quad a \leq b \implies [a, b] \subseteq A.$$

If $P = (N, \leq)$ is a poset, we are interested in obtaining $P^* = (N, \leq)$, the dual of P, with

$$x \leq y \ \text{ en } P^* \iff y \leq x \ \text{ en } P.$$

It can be proved that $Co(P) \simeq Co(P^*)$, $\forall\ P$ (Birkoff and Bennett, 1985).The family of convex sets in P will be denoted

$$Co(P) = \{S \subseteq N \mid S \text{ is convex in } P\}.$$

This characterization implies, $\forall\ i \in N$, $\{i\} \in Co(P)$ so the couple $(N, Co(P))$ *is a feasible coalitions system*. Then, given a game (N, v), if there is an order relation among the players, it makes sense to take into consideration the triple $(N, Co(P), v)$ and the appropriate partial cooperation game,

$$v^{Co(P)} \mid 2^N \longrightarrow \mathbb{R},\ v^{Co(P)}(S) = \max \left\{ \sum_i v(T_i) \mid \{T_i\} \in \mathcal{P}_{Co(P)}(S) \right\},$$

where $\mathcal{P}_{Co(P)}(S)$ is the family of partitions from the coalition S in convex sets in P.

It is easy to prove that $A, B \in Co(P)$, that $A \cap B \in Co(P)$, impliying $(N, Co(P))$ a closure space. Also, Edelman and Jamison (1985), Birkoff and Bennett (1985) think that $(N, Co(P))$ proves the Minkowski–Krein–Milman condition, and, therefore an atomic convex geometry, named order convex in N.

As $(N, Co(P))$ is a feasible coalition system, every subset in N can be expressed as a union of it maximal convex sets. In this particular case, the maximal convex definition of $S \subseteq N$ in P is equivalent to the one by Tijs (1993), which is due to the two $(N, Co(P))$ being a convex geometry: *Let* $(N, Co(P))$ *be a feasible coalition system and let* $S \subseteq N$. *If* $T \in Co(P)$ *and* $T \subseteq N$, *then* T *is maximal convex* S *in* P *if and only if,* $\forall i \in S \setminus T, \quad T \cup \{i\} \notin Co(P)$.

Notice that this characterization for maximal convex is certain in all convex geometry, and, in general, the feasible coalition system $(N, Co(P)$ is not a partition system.

Example 2 *Let $(N, \leq)$ be a poset, whose Hasse diagram is shown in Figure 1,*

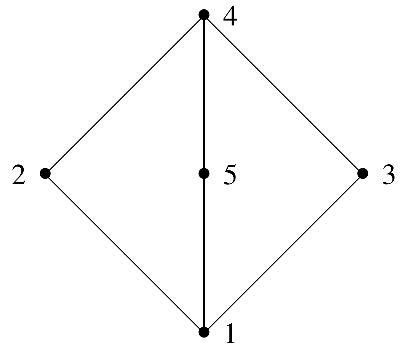


Figure 1

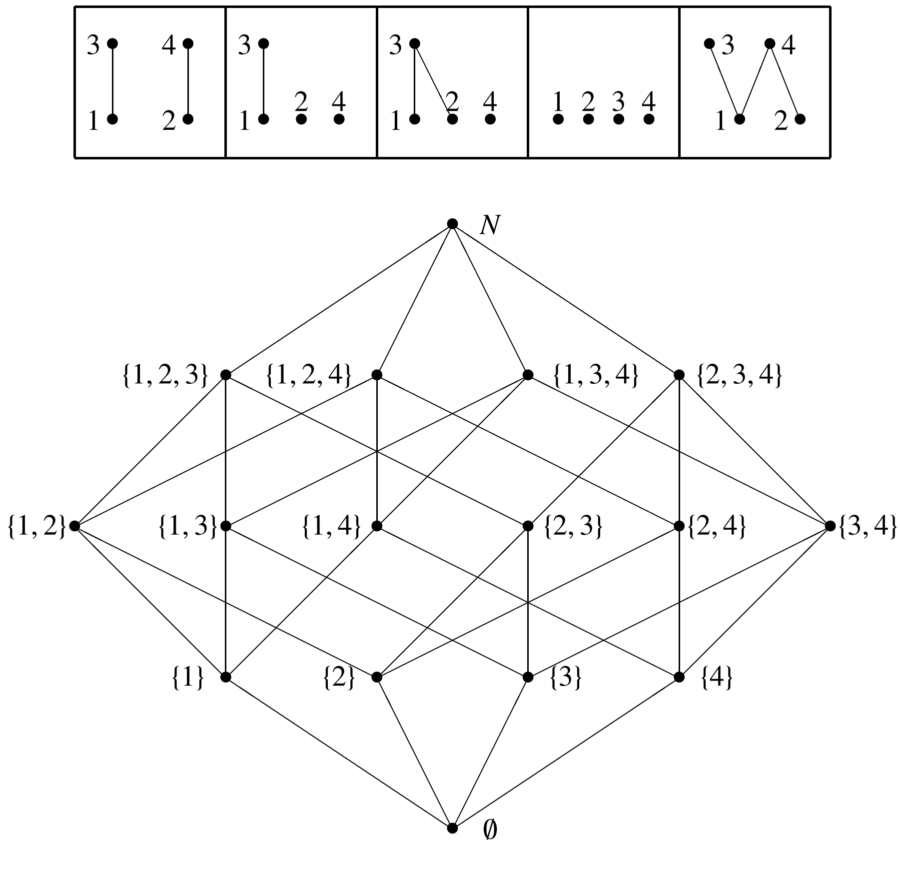


Figure 2: $(Co(P), \subseteq) \simeq (2^4, \subseteq)$

The couple $(N, Co(P))$ is not a partition system, applying Theorem 1, because $\{1,3\} \in Co(P)$, $\{3,4\} \in Co(P)$, the intesection is not empty, however, $\{1,3\} \cup \{3,4\} \notin Co(P)$ due to $1 \leq 4$ y $[1,4] \nsubseteq \{1,3,4\}$.

Let $P = (N, \leq)$ be a poset whose range or length $l(P)$ might equal 1 or be less than 1. That is to say:

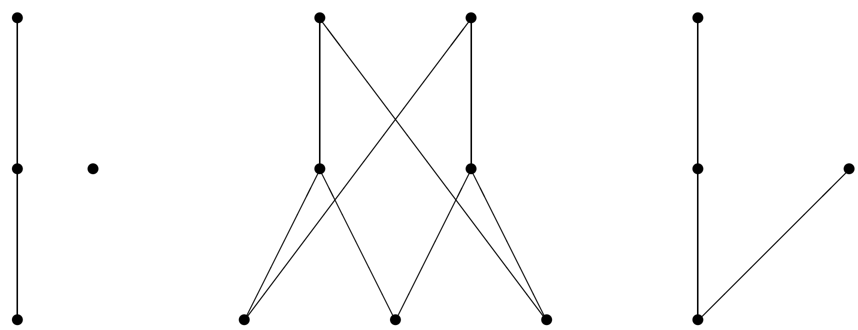
$$l(P) = \max\{l(C) \mid C \text{ is a chain in } P \text{ and } l(C) = |C| - 1\} \leq 1.$$

Then $(N, Co(P))$ is a partition convex geometry. As every subset in N is convex, either due to being an atom or a chain of two elements from N, it implies that $Co(P) \simeq 2^N$. For example, in Figure 2, $Co(P) \simeq 2^4$. If $l(P) \leq 1$ and if it is considered a partition system (or partition convex geometry) restricted $Co(P)$–game linked to the three $(N, Co(P), v)$, it verifies that $v^{Co(P)}(S) = v(S)$, $\forall S \in 2^N$ and, therefore restricted game and original game are the same.

It has been proved that if $l(P) \geq 2$, the atomic convex geometry $(N, Co(P))$ is not necessarily a partition system . This is the reason why only partially ordered sets with $l(P) \geq 2$ are taken into consideration, and we search for conditions to set $(N, Co(P))$ as a partition system. We will introduce the concept of completely coherent ordered sets as given by Birkoff and Bennett (1985).

A poset $P = (N, \leq)$ is *coherent* if it is connected and no maximal element from P covers any minimal element from P.

For example, the poset in example 3 (Figure 1) is *coherent*. Other possible situations are considered below:

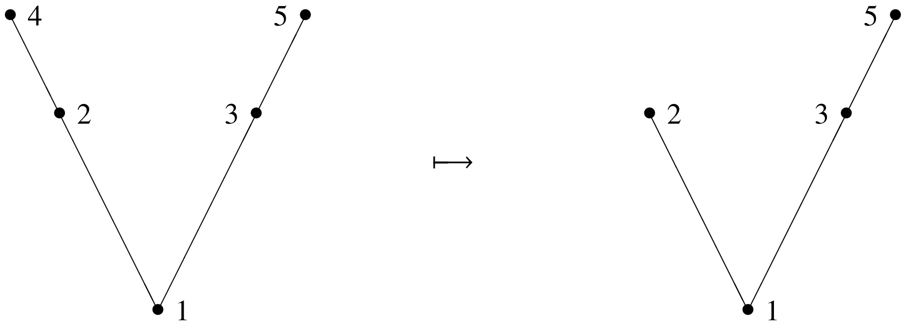


NON COHERENT NON COHERENT NON COHERENT

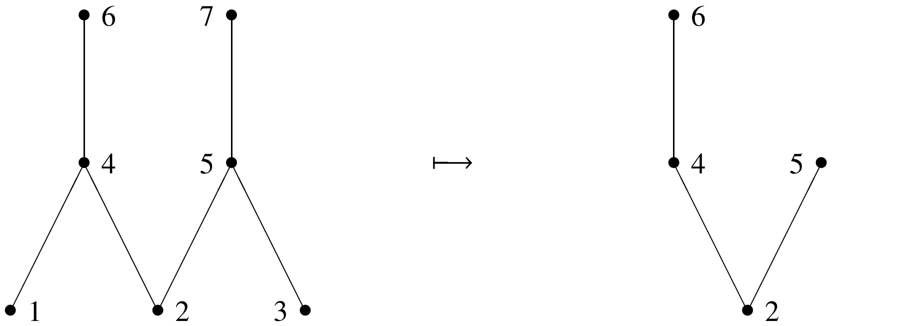
(non connected) (maximals are about minimals)

Figure 3

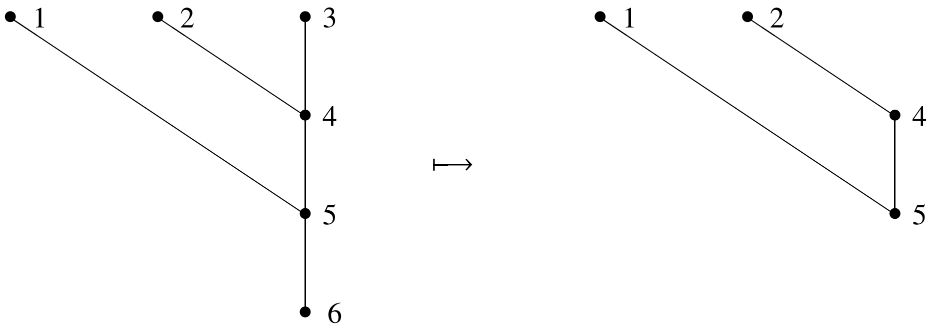
A poset P, with $l(P) \geq 2$, is completely coherent if any subposet infered by P, P' with $l(P') \geq 2$, is coherent. The following figures illustrate this concept. Figure 4 shows diagrams of *coherent* posets that are not *completely coherent*. On the other hand, Figure 5, shows examples of *completely coherent* posets.



P COHERENT $\qquad$ P', with $l(P') \geq 2$, NON COHERENT

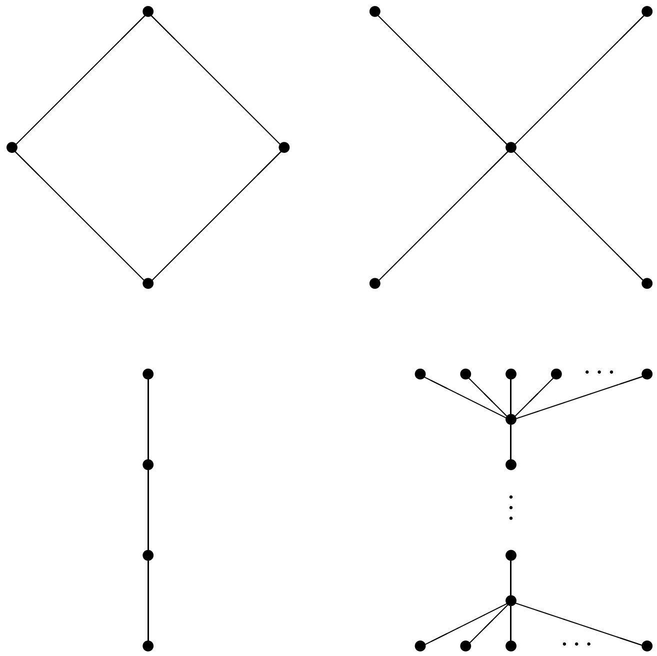


Q COHERENT $\qquad$ Q', with $l(Q') \geq 2$, NON COHERENT



H COHERENT $\qquad$ H', with $l(H') \geq 2$, NON COHERENT

Figure 4



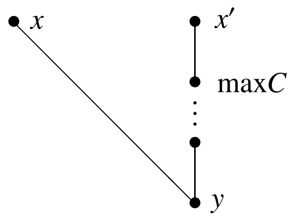
COMPLETELY COHERENT ORDERED SETS

Figure 5

Notice that completely coherent posets in Figure 5, except the first of them, verify that $P \setminus ex(P)$ is a chain. This property will be very important to prove that the couple $(N, Co(P))$ is a partition system.

Theorem 2 *Let $P = (N, \leq)$ be a completely coherent finite poset, as $P \backslash ex(P)$ is a chain C. Then, every maximal element from P covers the maximum in chain C and the minimal element from C covers every minimal element from P.*

Proof. If P is *coherent*, it is connected and its maximal elements do not cover any minimal. Therefore, if x is maximal, it follows that $y \in P$ is such that $x \succ y$ in which $y \notin ex(P)$ because set $ex(P)$ is the union of maximal and minimal elements. Then, $y \in C$ / $y \leq \max C$ exists.



If $y \neq \max C$, as $\max C$ is not maximal in P, there is $x' \succ \max C$. The induced subposet P', made up of the elements $\{y, \max C, x, x'\}$ verifies that $l(P') = 2$ and is not coherent, in opposition to the hypothesis. Consequently, $y = \max C$.

The reasoning for minimal elements is equivalent to the one above. □

The following theorem is the main result from this research. It establishes alternative characterization for the two $(N, Co(P))$ to be a partition system.

Theorem 3 *Let $P = (N, \leq)$ be a finite poset. The couple $(N, Co(P))$ is a partition system if and only if P is completely coherent and $P \setminus ex(P) = C$ is a chain.*

Proof. ($\Rightarrow$) Consider that $(N, Co(P))$ is a partition system. We must prove that P is completely coherent and $P \setminus ex(P) = C$.

If $P \setminus ex(P) \neq C$, there are $a, b \in P \setminus ex(P)$ so that $\{a, b\}$ is an antichain. As $\{a, b\} \nsubseteq ex(P)$, consider the sets

$$m(a) = \{m \in P \mid m \prec a\}, \quad M(a) = \{m' \in P \mid a \prec m'\},$$

and, analagously, $m(b)$ and $M(b)$. Obviously, these are not empty sets, and it is easy to notice that $m(a) \cap M(b) = m(b) \cap M(a) = \emptyset$. However, $m(a) \cap m(b)$ and $M(a) \cap M(b)$, these intersections cannot be empty. So, these are the alternatives:

(1) $m(a) \cap m(b) \neq \emptyset$
(2) $M(a) \cap M(b) \neq \emptyset$
(3) $m(a) \cap m(b) = M(a) \cap M(b) = \emptyset$

Using the duality $Co(P) \simeq Co(P^*)$, we only need to pay attention to (1) and (3).
(1) Let $m \in m(a) \cap m(b)$, $m' \in M(a)$. If $b \nleq m'$ (Figure 6), the set $\{m, b, m'\} \notin Co(P)$ and their maximal convexes $\{\{b, m'\}, \{m, b\}\}$ are not its partition. If $b \leq m'$ (Figure 7), $\{m, a, m'\} \notin Co(P)$ and their maximal convexes $\{\{a, m'\}, \{m, a\}\}$ are also not its partition.

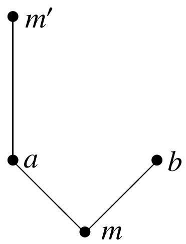


Figure 6

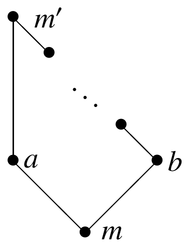
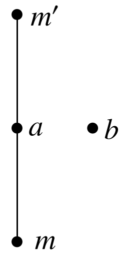


Figure 7

(3) Suppose that $m(a) \cap m(b) = M(a) \cap M(b) = \emptyset$ and let $m \in m(a)$ and $m' \in M(a)$. If there is no connection between b and elements m, m', then $\{m, b, m'\} \notin Co(P)$ and their maximal convexes$\{\{b, m'\}, \{m, b\}\}$ are not its partition (Figure 8). If there was connection it would be because, $m \leq b, b \leq m'$, one or both of them. In every situation, $m \notin m(b)$ and $m' \notin M(b)$ such that $m(a) \cap m(b) = M(a) \cap M(b) = \emptyset$. In all situations, we cannot find convex sets in which their maximal convexes are not a partition. Indeed, if $m \leq b$ there is a b_1 such that $m \leq b_1 \leq b$ (Figure 9) and, for $\{m, a, b\} \notin Co(P)$ their maximal convexes $\{\{m, a\}, \{a, b\}\}$ are not its partition. If $b \leq m'$ the reasoning is equivalent.



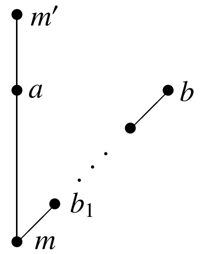


Figure 8 *Figure 9*

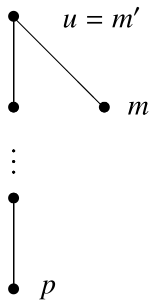
Thus, we have proved that if $P \setminus ex(P) \neq C$ then the hypothesis is not satisfied. Suppose that P is not completely coherent. Then, there is an inducted subset P', with $l(P') \geq 2$ that is not coherent, therefore P' is not connected nor does any maximal element from P' cover any minimal element from P'.

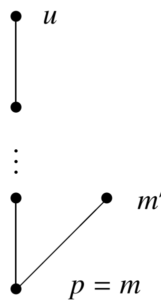
If P' is not connected, there are at least two connected components C_1, C_2 and all of them have to include a chain with length equal or bigger than 2. Suppose $l(C_1) \geq 2$. When we consider the first and last maximal chain C_1 element, indicated by $\{p, u\}$, together with any $a \in C_2$, there is for set $\{p, u, a\}$ the situation is analogue to the subposet in Figure 8, so there is a contradiction.

If P' is connected but any maximal element covers any minimal element, there are m and m' (minimal and maximal from P') such that $m \prec m'$. Nevertheless, that m' covers m in the subposet P' does not imply the same in P. So, there are two possibilities:

(1) $m \prec m'$ in P ($\{m, m'\} \in Co(P)$).
(2) $m \not\prec m'$ in P ($\{m, m'\} \notin Co(P)$).

(1), we consider the set $\{p, u, m, m'\}$ in which p and u are the first and the last elements included in a subposet maximal chain P' ($l(P') \geq 2$). As p and u are extreme elements in P', the three situations shown in Figures 10, 11 and 12 arise. There, $\{p, u, m, m'\} \notin Co(P)$ and its maximal convexes are not its partition. (Notice there is an unknown connection drawn between p and m', as well as between m and u).





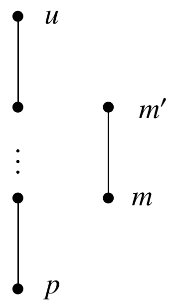


Figure 10 *Figure 11* *Figure 12*

In (2), if $m \nless m'$ there is $p_1 \in P \setminus P'$ such as $m < p_1 < m'$. Let p and u be the first and the last elements from a subposet maximal chain P'. Then, there is $u_1 \in P'$ such as $p < u_1 < u$ ($l(P') \geq 2$). Evidently it cannot be $u = m'$ and $p = m$, because then $m \nless m'$ in P'. Therefore, we must take into consideration the situations in which $u \neq m'$ and $p \neq m$. Because of the duality, it is enough to study one of them. If $u \neq m'$, the situations where it originates (drawn in Figures 13, 14 and 15) are due to $\{u_1, p_1\}$ being an antichain or not.

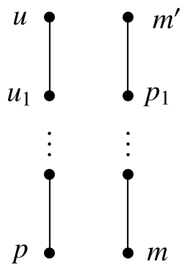


Figure 13

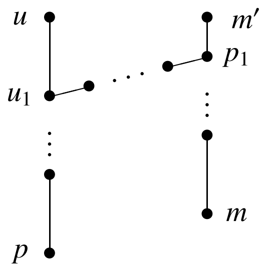


Figure 14

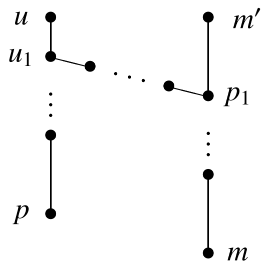


Figure 15

If $\{u_1, p_1\}$ is an antichain, there is a contradicion due to $P \setminus ex(P) \neq C$. If $u_1 < p_1$ or $p_1 < u_1$, we consider the sets $\{u, u_1, m'\} \notin Co(P)$, $\{u, p_1, m'\} \notin Co(P)$. In both cases, their maximal convexes are not their partition.

($\Leftarrow$) Notice that if $P = (N, \leq)$ is a completely coherent finite poset, such that $P \setminus ex(P)$ is a chain C, then $A \in Co(P)$, $B \in Co(P)$ and $A \cap B \neq \emptyset$ imply that $A \cup B \in Co(P)$. The set $A \cup B$ is convex if given $a \in A \cup B$, $b \in A \cup B$ with $a \leq b$, then $[a, b] \subseteq A \cup B$. The set $A \cup B$ is a disjoint union of $A \setminus B$, $A \cap B$ and $B \setminus A$; so, among the different possible alternatives for a and b, we only need to analyze a couple of them: $a \in A \setminus B$ and $b \in B \setminus A$, or $a \in B \setminus A$ and $b \in A \setminus B$. Furthermore, using the duality ($Co(P) \simeq Co(P^*)$), it is enough to analyze only one of the possibilities. Consequently, let $a \in A \setminus B$, $b \in B \setminus A$ with $a < b$.

It must be proved that $[a, b] \subseteq A \cup B$ and, by hypothesis, $A \cap B \neq \emptyset$. If there is an element $d \in A \cap B$ such that $d \in [a, b]$, then:

$$[a, b] = [a, d] \cup [d, b] \subseteq A \cup B,$$

as the intervals of P are always chains ($P \setminus ex(P) = C$), $\{a, d\} \subseteq A$, $\{d, b\} \subseteq B$ and $A, B \in Co(P)$.

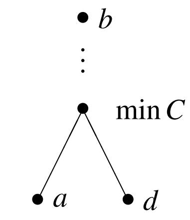
In the case that any $d \in A \cap B$ is not included in the interval $[a, b]$, the are four possible alternatives: 1) $d < a$, 2) $b < d$, 3) $\{a, d\}$ is an antichain and 4) $\{b, d\}$ is an antichain.

We are going to analyze:

(1) If $d < a < b$, then $[d, b] \subseteq B$. Therefore $a \in B$, instead of being $a \in A \setminus B$.

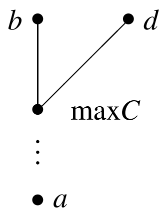
(2) If $a < b < d$, then $[a, d] \subseteq A$. Therefore, $b \in A$ which contradicts $b \in B \setminus A$.

(3) If $\{a, d\}$ is an antichain, then a and d are minimal elements (a is not maximal due to $a < b$ and the only possible antichains in P are made of maximal elements from P or of minimal elements).



Theorem 2 implies that minimal element from chain C, $\min C$ covers a and d. Then $d \prec \min C \leq b$ and $[d, b] \subseteq B$ as B is convex and $\{d, b\} \subseteq B$. Therefore,

$$[a, b] = \{a\} \cup [\min C, b] \subseteq \{a\} \cup [d, b] \subseteq A \cup B.$$



(4) Using an analogous reasoning, if $\{b, d\}$ is an antichain, both are maximals and it is deduced that $d \succ \max C \geq a$. Then, $[a, d] \subseteq A$ y

$$[a, b] = [a, \max C] \cup \{b\} \subseteq A \cup B.$$

□

Obviously, the results above have a theoretical interest. The knowledge of convex sets, and particulary those structures that lead to partition systems, have a practical interest, among other possibilities, in order to estimate power indexes —both Banzhaf's and Shapley's— in simple weighted voting games and in double-triple majority games, in which cooperation is restricted to a feasible coalition set. This application is discussed in more detail by Bilbao, Jiménez, López and Fernández (2000).

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Resum

Considerem jocs d'utilitat transferible que tracten amb situacions de cooperació parcial constituïdes per sistemes de coalicions, en els que tota coalició unitària és factible i tota coalició de jugadors es pot expressar com una unió disjunta de coalicions factibles maximals. Aquests sistemes reben el nom de sistemes de partició i donen lloc a jocs restringits. En particular, estudiem sistemes de coalició definits per un ordre parcial establert en el conjunt dels jugadors i analitzem les característiques de coalicions factibles construït a partir de la classe de conjunts convexos.

MSC: 90D12

Paraules clau: Jocs cooperatius, cooperació parcial, conjunts convexos