Top (2013) 21:268–274
DOI 10.1007/s11750-013-0277-9

DISCUSSION

Comments on: Multicriteria decision systems for financial problems

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Published online: 4 May 2013

Zopounidis and Doumpos consider a very interesting topic, namely Multiple Criteria Decision Aiding (MCDA) (for a collection of state-of-the-art surveys on MCDA, see (Figueira et al. 2005a)) for financial problems. The main perspective adopted in classical finance theory is normative and descriptive. As pointed out in the paper, real world financial problems require, however, to adopt a constructive perspective instead of the normative and descriptive one. We believe that this perspective is worth some more words in order to get a more complete image of the application of MCDA to decisions in finance.

According to Bell et al. (1988) there are three main approaches to decision making:

This comment refers to the invited paper available at doi:10.1007/s11750-013-0279-7.

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- the *normative approach*, which aims at defining how ideally decisions should be rationally taken,
- the *descriptive approach*, which aims at describing how decisions are taken in real life,
- the *prescriptive approach*, which aims at "the application of normative theories, mindful of the descriptive realities, to guide real decision making" (French and Rios Insua 2000) and to avoid "decision traps", i.e. systematic deviations from the rationality (Russo and Schoemaker 1989; Hammond et al. 1998).

Observe that, in fact, the above three approaches are related to an ideal of rationality: the normative approach defines the basic principles of rationality and deduces its consequences, the descriptive approach verifies if these principles of rationality are respected in real life decisions, and the prescriptive approach suggests how to avoid the violations of the same principles of rationality.

As observed by Roy (1993, 2010), MCDA represents a different fourth approach, called the *constructive approach*. The basic assumption of this constructive approach is that, in a given multiple criteria decision problem, the preferences of the Decision Maker (DM) with respect to the considered alternatives do not pre-exist in the DM's mind. Instead, the decision aiding process involves the DM in co-construction of his/her preferences by exploring, interpreting, debating and arguing. Thus, in general, MCDA assumes an active participation of DM in an interactive decision process, in which phases of preference elicitation are interleaved with phases of computation of a recommended decision.

Since uncertainty is an inherent component of decisions in finance, the constructive approach seems to be best suited. This opinion is strongly supported by arguments provided by Maurice Allais in one of his famous articles (Allais 1953), where he argues against the axiomatic approach to decision under risk of the so-called "American School". This great French economist writes that rationality has not to be understood as adherence to one of the systems of axioms which lead "a priori" to the decision model considered as rational as the Bernoulli expected value formulation. Instead, according to Allais, rationality has to be defined by referring to:

(a) a general criterion "implying the coherence of desired ends and the use of appropriate means for attaining them", and
(b) observation of the actions of people who can be regarded as acting rationally.

Observe that in point (a), the MCDA approach is implicitly imposed, because one can see the ends as the criteria used for evaluation of the actions. Point (b) is more related to the descriptive approach, however, in the article of Allais there is a strong emphasis on considering not decisions of an "average individual", as usually done in experiments of psychology of the decision, but rather decisions of individuals that are supposed to act with a certain rationality as, to cite some examples given by Allais, very prudent people gambling small sums or entrepreneurs when great losses are possible. One can observe that Allais proposes to pass from a quite abstract understanding of rationality, to an effort of giving it a more practical and concrete sense, which, in our opinion, is perfectly concordant with the MCDA constructive approach.

In order to exemplify the constructive approach of MCDA on a financial decision problem, we shall shortly recall and comment one possible application of MCDA to the most typical financial decision problem, i.e. the Markowitz portfolio selection problem.

Assuming the existence of a set $J = \{1, \ldots, j, \ldots, z\}$ of risk securities (assets) in the capital market, representing a single portfolio by its vector allocation $\mathbf{x} = [x_1, \ldots, x_z]$, with $\sum_{j=1}^{z} x_j = 1$, and denoting by $R(\mathbf{x})$ the return of portfolio $\mathbf{x}$, in the Markowitz model (Markowitz 1952, 1959, 1970, 1991), the problem of portfolio selection is a bi-objective optimization problem, where the two objectives are: maximization of the expected return $E(R(\mathbf{x}))$, and minimization of the variance $\sigma^2(R(\mathbf{x}))$ which represents the risk of the portfolio. In such a context, the efficient frontier is composed of portfolios $\mathbf{x}$ for which it is not possible to increase the expected return $E(R(\mathbf{x}))$ without increasing also the variance $\sigma^2(R(\mathbf{x}))$, or, equivalently, it is not possible to decrease the variance $\sigma^2(R(\mathbf{x}))$ without decreasing also the expected return $E(R(\mathbf{x}))$. To select the best portfolio, that is to find the optimal allocation vector $\mathbf{x}$, a utility function $U(E(R(\mathbf{x})), \sigma^2(R(\mathbf{x})))$ representing the preferences of the investor with respect to expected return $E(R(\mathbf{x}))$ and variance $\sigma^2(R(\mathbf{x}))$ has to be defined. The utility function $U(E(R(\mathbf{x})), \sigma^2(R(\mathbf{x})))$ has to be nondecreasing with respect to $E(R(\mathbf{x}))$ and nonincreasing with respect to $\sigma^2(R(\mathbf{x}))$. Once setting the utility function, the selection of optimum portfolio is obtained by maximizing the utility function on the set of efficient portfolios. Of course, the elicitation of such a function is a very difficult task and its interpretation can be non-straightforward for investors without a solid background in finance, statistics and economics.

For this reason, Greco et al. (2013), following Greco et al. (2010), Matos (2007), have recently proposed an alternative approach to the Markowitz portfolio selection, based on consideration of some meaningful quantiles of the probability distribution of returns corresponding to each portfolio as a set of criteria to be maximized. These quantiles are easily understandable even for an investor without a sophisticated financial preparation, because each quantile simply says the minimal return that a portfolio $\mathbf{x}$ gives with a corresponding and meaningful value of probability. The resulting formulation of the multiple criteria decision problem can be dealt with any methodology already existing in MCDA, such as value function methods (Dyer 2005; Keeney and Raiffa 1976), outranking relation methods (Figueira et al. 2005b; Roy 1991), decision rule methods based on Dominance-based Rough Set Approach (Greco et al. 2001, 2005; Słowiński et al. 2009, 2012), interactive multiobjective optimization and evolutionary multiobjective optimization (Branke et al. 2008). In simple words, while in a normative approach the Markowitz portfolio selection problem is solved by transforming a bi-objective problem in terms of $E(R(\mathbf{x}))$ and $\sigma^2(R(\mathbf{x}))$ to a single objective problem based on the utility function $U(E(R(\mathbf{x})), \sigma^2(R(\mathbf{x})))$, in a constructive approach the same problem is transformed to a multiple criteria decision problem in which a set of quantiles relative to some meaningful values of probability have to be maximized. Observe that in the constructive approach increasing the number of objectives to be considered (in general, the quantiles are more than two) makes easier and more understandable the portfolio selection problem from the point of view of its interpretation by the DM.

Note that the approach of Greco, Matarazzo and Słowiński aims at modeling preferences strictly related to the portfolio return distribution and thus it differs from the many methods that have already been proposed to deal with the portfolio selection problem using MCDA (some comprehensive state-of-the-art surveys on this subject are given in Spronk et al. (2005), Steuer and Na (2003), and Steuer et al. (2005)). These methods extend the basic Markowitz model by considering different aspects of the portfolio selection problem, such as maximum investment proportion weight, social responsibility, number of securities or economic sectors in the portfolio, short selling, and so on. The approach of Greco, Matarazzo and Słowiński proposes an alternative methodology as it abandons the bi-objective logic of the Markowitz model, however, it can also include criteria which are not directly related to the portfolio return distribution. Here, for the sake of simplicity, we consider preferences related to the portfolio return distribution only.

Among the many MCDA methods, Greco et al. (2013) propose using the Interactive Multiobjective Optimization (IMO) method based on so-called Dominance-based Rough Set Approach (DRSA) (Greco et al. 2001) as the primary method for the portfolio selection problem (Greco et al. 2008b). IMO-DRSA is composed of two alternating phases: computation phase, and dialogue phase. In the computation phase, a sample of feasible portfolios is calculated and presented to the (DM). In the dialogue phase, the DM indicates portfolio solutions which are relatively attractive in a given sample; this binary classification of sample portfolios into 'good' and 'others' is an input preference information to be analyzed using DRSA. DRSA is producing decision rules relating conditions on particular quantiles with the qualification of supporting portfolios as 'good'. For example a rule can say

r1: if there is a probability of 25 % of obtaining a return of at least 18.74 %, and a probability of 99 % that the possible loss is not greater than −5.24 %, then the portfolio is 'good'.

A rule that best fits the current DM's preferences is chosen to constrain the previous multiobjective optimization in the next computation phase: for example, if rule r1 was chosen, then the search would be restricted to portfolios characterized by probability of 25 % of obtaining a return of at least 18.74 %, and a probability of 99 % that the possible loss is not greater than −5.24 %. In this way, the computation phase yields a new sample including better portfolios, and the procedure loops until the most preferred portfolio is found, or until the DM concludes that there is no satisfactory solution for the current problem setting. The advantage of this procedure is not only the interaction between the DM and the optimization procedure, but the transparent link between the input preference information and the output of the decision aiding process. Indeed, taking into consideration the decision rules chosen during the dialogue phase, the process concludes with recommendation of a portfolio to be selected and with some arguments in favor of the portfolio, such as, for example:

- in 75 % of cases the recommended portfolio gives a return not smaller than 8.03 %,
- in 99 % of cases the recommended portfolio gives a return not smaller than −5.24 %,
- in 25 % of cases the recommended portfolio gives a return not smaller than 18.93 %.

Another interesting remark about the methods to be used in such a type of problems is related to the case in which the portfolio selection problem formulated in terms of quantiles should be dealt with a value function. In this case, it is not reasonable to ask the DM to define directly his/her own value function, because this requires a technical background that many DMs do not have and, in any case, it is very demanding from a cognitive point of view. Thus, very often, the value function is defined using *ordinal regression* (Jacquet-Lagrèze and Siskos 1982) to induce one value function compatible with some preference information expressed by the DM, such as: portfolio **x** is preferred to portfolio **y** or portfolio **w** is preferred to portfolio **z** more than portfolio **u** is preferred to portfolio **t**. Ordinal regression induces one compatible value function which, in a sense, is the most representative for the preference information given by the DM. In general, however, there is more than one instance of the compatible value function, and to select one of them is always arbitrarily to some extent. Therefore, it is advisable to consider the whole set of value functions compatible with the preferences expressed by the DM. This is possible due to a recent extension of the ordinal regression called Robust Ordinal Regression (ROR) (Greco et al. 2008a; Figueira et al. 2009) which considers two preference relations, i.e.

- the necessary preference relation, when **x** is preferred to **y** for all compatible instances of the value function,
- the possible preference relation, in case **x** is preferred to **y** for at least one compatible instance of the value function.

During the decision aiding process the DM can add new preference information and then the preference model becomes more and more precise in the sense that necessary preference relation is enriched and the possible preference relation is impoverished. Therefore, in the perspective of the constructive approach, ROR seems much more appropriate than the classical ordinal regression, not only because considering the necessary and the possible preference relations it is more prudent in suggesting or rejecting preferences between alternative solutions, but also because it properly deals with the incompleteness of the DM's preferences, helping the DM in making these preferences more precise in the course of an interactive procedure. Observe, moreover, that ROR applied to outranking methods (Greco et al. 2011) can also be used to deal with the portfolio selection problem formulated in terms of quantiles.

Concluding this short comment, we would like to summarize in few points the main characteristics that a MCDA methodology should have to guide towards reasonable decisions in finance:

- the information that the DM gives and receives within a decision aiding process has to be simple and understandable;
- the recommendation has to be well argumented, because the strength of the recommendation is in the good explanation of its reasons;
- the conclusions communicated to the DM at successive stages of a decision aiding process have to be prudent and cautious, in the sense that only conclusions firmly established on the base of information received from the DM should be given;
- the DM should not be substituted by some "supposed rational" method considered as an oracle: the final decision must be taken solely by the DM after a sufficient maturation of his/her knowledge about the problem and about his/her own preferences.

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