

Universal GeoGebra Astrolabe

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It is shown how to operate, design and create an astrolabe using GeoGebra, available on the GeoGebraTube platform. This material was the first of its kind with anverse and reverse fully operational. It is formed an instrument to which, throughout history, complete treatises on celestial mechanics and mathematics have been dedicated. The branches of knowledge involved have been reunited and the basics have been collected.

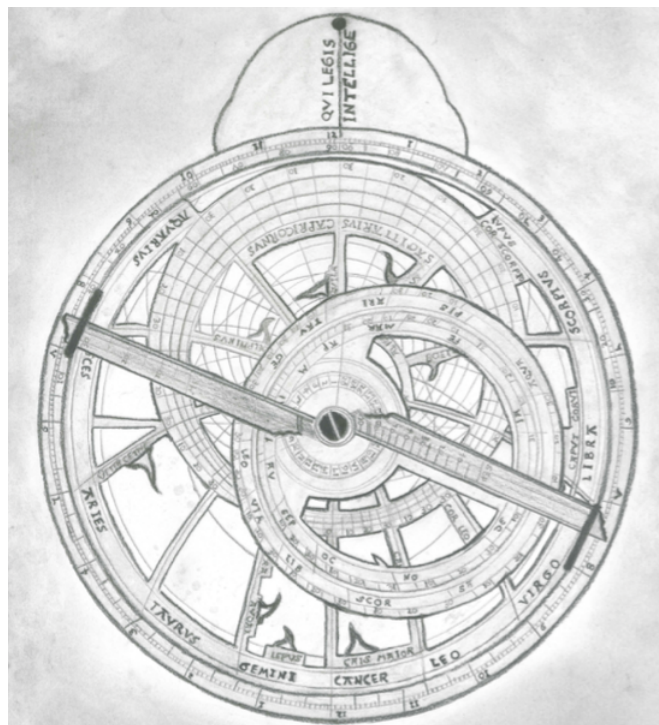


Figure 1: Ancient astrolabe.

1. Case study: how to calculate the time with the Universal GeoGebra Astrolabe

An astrolabe can perform hundreds of astronomical operations and, to illustrate them all, would require several treaties. Our objective, in this section, is to show its basic operation. The instrument can be adapted to any latitude and is made in its entirety through GeoGebra, hence its name: Universal GeoGebra Astrolabe. We will calculate the civil hour from the observation of the height of a star, information that we will process through the archives available in [1], fifth section (figures 2, 3 and 4).

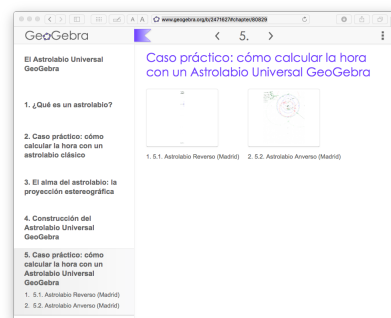


Figure 2: GeoGebra Book *El Astrolabio Universal GeoGebra*.

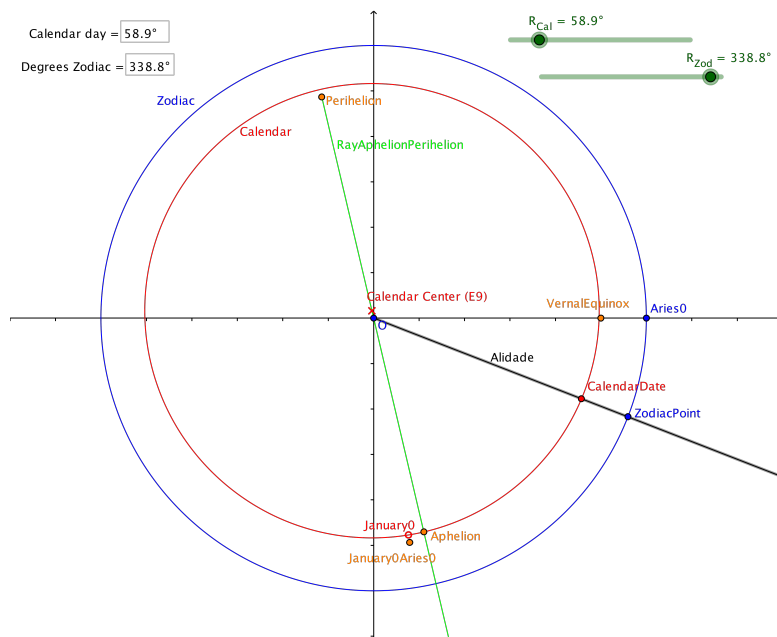


Figure 3: Astrolabe Back (Madrid).

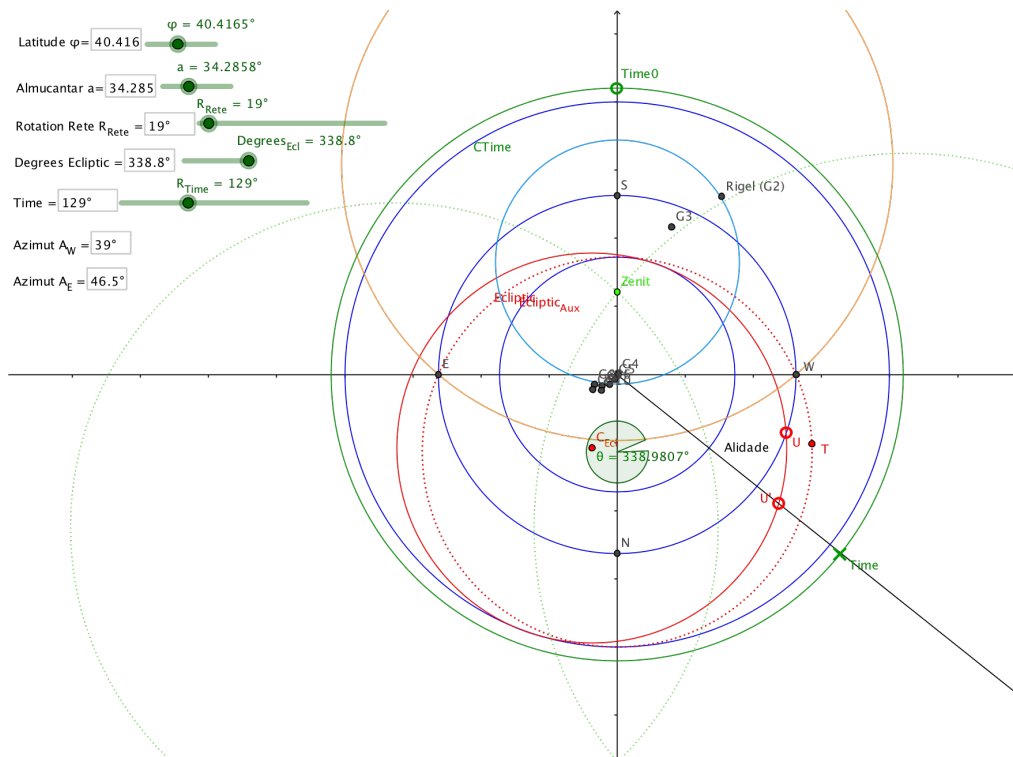


Figure 4: Astrolabe Front (Madrid).

Let's calculate the time in Madrid, with latitude 40.4165° North and longitude $3^\circ 42' 9''$ West, let's see:

On the night of February 27, 2016, Rigel is placed in the Southwest sky, leaving Sirio to his left, with an apparent altitude of about 34.28575° .

We point an *almucantar* $a = 34.2858^\circ$, that is, the parallel on the celestial sphere of height a .

On the reverse of the astrolabe we move the alidade so that the Day of the Calendar is 58 (days elapsed), that is, February 27; as we are very close to day 28 we chose to choose 58.9° . This marks, by extension, an angle of 338.8° in the Zodiac (Pisces 8.8°).

We go to the obverse of the astrolabe, we locate Rigel and we are moving the spider until Rigel is placed in the almucantar $a = 34.2858^\circ$ (falls in the Southwest sky). The spider rotates about 19° .

Once fixed the spider we marked the 338.80 in the Ecliptic and aligned the Hour, that gives about 129° . If we consider the equivalence $1 \text{ hour} = 15^\circ$, we have $129^\circ : 15^\circ = 8.6$, that is, 8 h. 36' in the evening *local time*.

To adjust it with the civil hour in which we need the latitude of our locality:

$3^{\circ}42'9.22'' = 3.7025^{\circ}$ West (Madrid).

The equivalence $60' = 15^{\circ}$ gives us a Time Correction:

$$CH = 3.7025^{\circ} \times 60' / 15^{\circ} = 14.81' = 14'48''$$

Therefore, the set time is $8 \text{ h. } 36' - (-14' 48'') = 8 \text{ h. } 50' 48''$. and, as our time zone is +01:00 UTC hours, it would be 9:50''.

Taking data from the *Stellarium* program it would be 22 h 00' 00'', ie 9' 12'' below, which is an order error:

$$552/60^3 \cdot 100 \% = 0.002555 \dots \cdot 100 \% = 0.2555 \dots \% < 0.26 \%$$

If we do not need to put any satellite or other space craft into orbit, it seems more than assumeable.

A simple look at our clock would be enough to know the time, but the obverse of the astrolabe gives us a snapshot of the universe as well. Thus we can know when certain stars will be placed or others will come out... In short, a classic astrolabe allows to hold the universe in the palm of the hand. Today we would need special software to achieve the same effect.

2. The soul of the astrolabe: the stereographic projection

Once we have shown, very above, the basic management of the instrument, we will try to explain its construction and, to a lesser extent, the reason for its design.

The celestial bodies are so far apart that we agree to place them all at the same distance. The celestial sphere contains these objects and we occupy them right at its center ([1]). If we project the content of that sphere over the celestial South Pole (projection pole) and imagine that the celestial equator is contained in a plane (projection plane), the marks that leave the rays of projection on this plane will be the image of the firmament that we are looking for (Fig. 5).

We can project stars, parallel and meridian celestial, planets... any object of the firmament. An astrolabe concentrates on the same plane the whole known universe.

2.1. Properties

At the theoretical level, to project the basic elements of the celestial sphere (meridians, parallels...) would require the following properties.

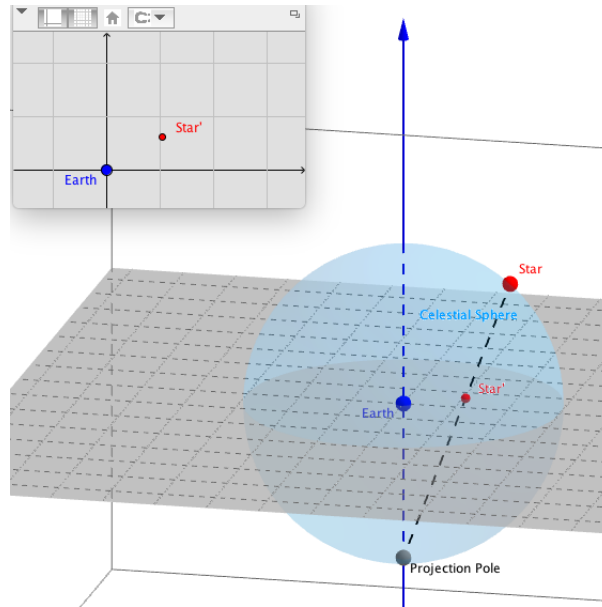


Figure 5: Stereographic projection of the celestial sphere.

2.1.1. Geometric results to consider

Property 1. *The sum of the interior angles of a triangle is 180° .*

Property 2. *Two similar rectangular triangles always have the same ratio between the opposite leg and the adjacent leg. This relationship depends exclusively on the acute angle that relates them, and can name that ratio based solely on that acute angle.*

Thus it is possible to give the following:

Definition 1. *It is called tangent of the angle a , $tg(a)$, to the ratio of the opposite leg relative to the adjacent leg of a right triangle.*

Property 3. *Every angle inscribed in a semicircle is a right angle.*

Property 4. *Any circumference on the sphere passing through the projection pole is projected as a segment.*

Theorem 1. (Fundamental equation of the astrolabe)

If it is a point of a circle parallel to the equator, then all points of the circle are projected at the same distance:

$$r = R \cdot tg\left(\frac{90^\circ - d}{2}\right) = R \cdot \left(\frac{\cos \delta}{1 + \sin \delta}\right)$$

where δ is the declination of X and R (Fig. 6).

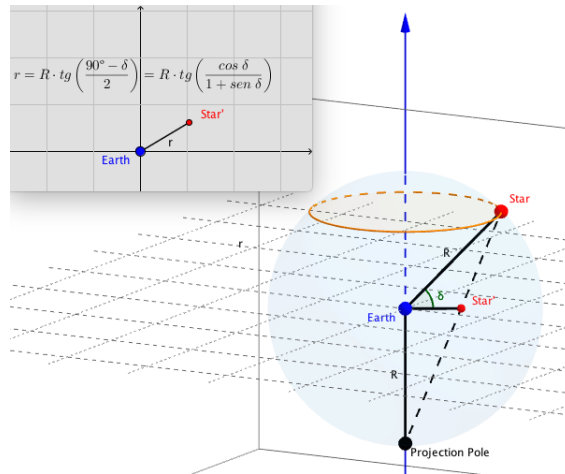


Figure 6: Fundamental equation of the astrolabe.

Note: Given a point on the celestial sphere, if the working context is the space we will denote by the stereographic projection of that point.

It is convenient to know the main characteristics of the stereographic projection presented below and whose demonstration can find the interested reader in [3].

2.1.2. Positive aspects: preservation of circles and angles

Property 5. *The stereographic projection transforms circumferences that do not pass through the projection pole in circumferences (Fig. 7).*

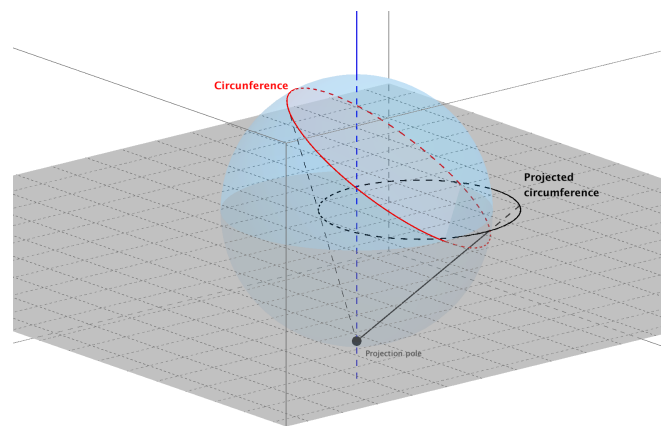


Figure 7: Preservation of circumferences.

Property 6. (Conformality of the stereographic projection)
The stereographic projection preserves angles (Fig. 8).

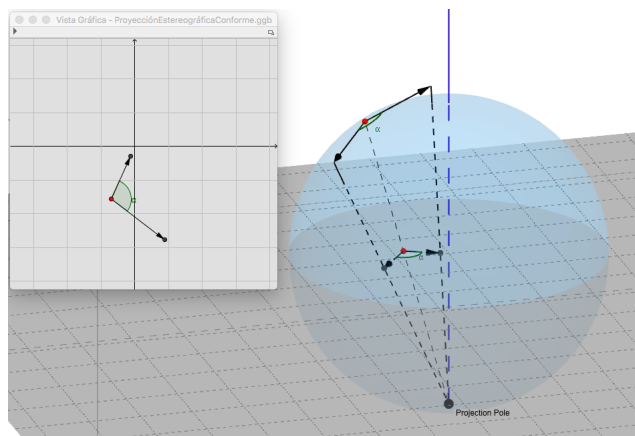


Figure 8: Conformality of the Stereographic Projection.

2.1.3. Negative aspects: Objects distorted away from the projection axis

The stereographic projection is one of so many incomplete solutions to an unsolvable problem, as it is to represent the sphere in a plane (always a deformation will be committed).

When we project non-maximum circles of the same radius by means of the stereographic projection they are distorted, because the nearer the smaller projection axis the circle becomes.

3. Construction of the Universal GeoGebra Astrolabe

We proceed to give the necessary guidelines for the drawing of the basic elements of an astrolabe (obverse and reverse).

3.1. Equator and Tropics

The ecliptic is the path that travels the Sun over the celestial vault each solar year ([4]). This trajectory has the celestial tropics of Cancer and Capricorn as upper and lower limits respectively (Fig. 9). The plane containing the ecliptic forms an angle called the obliquity of the ecliptic with the plane containing the celestial equator. The obliquity changes very slowly with the passage of time. We will set

with a checkbox the value for 2016 of 23.4372° ($23^\circ 26' 14''$). From Figure 9 it is possible to deduce the formulas (1) and (2):

$$R_{Equator} = R_{Capricorn} \cdot \operatorname{tg} \left(\frac{90^\circ - \varepsilon}{2} \right) \quad (1)$$

$$R_{Cancer} = R_{Equator} \cdot \operatorname{tg} \left(\frac{90^\circ - \varepsilon}{2} \right) \quad (2)$$

where $R_{Equator}$, $R_{Capricorn}$ and R_{Cancer} are the radius of the Equator, Tropic of Capricorn and Cancer respectively.

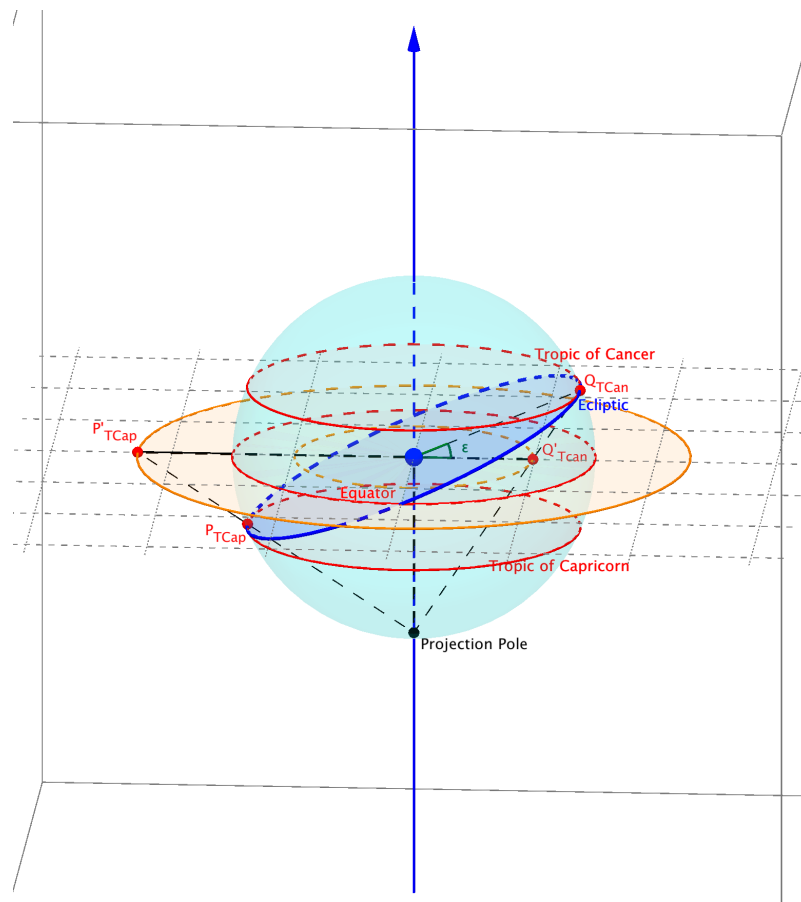


Figure 9: Ecuator and celestial tropics.

Figure 10 shows that the stereographic projection on the celestial sphere of Figure 9 is applied. In order to do this, we will have to reproduce the geometric conditions involved, that is, the equations (1) and (2) mentioned. We will follow the following process:

Note: It is convenient to set the overall program accuracy to 4 decimal places, for example (Options-Rounding) menu.

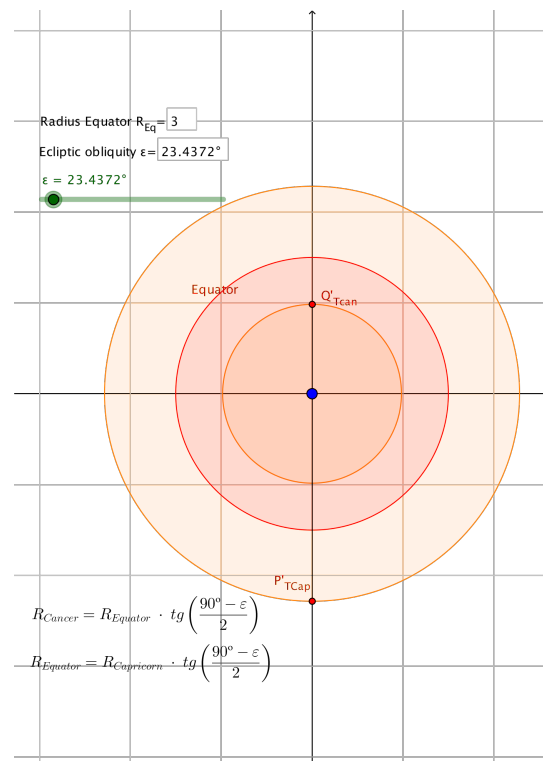


Figure 10: Equator and celestial Tropics projection.

- 1) Let O be the Cartesian coordinate origin.
- 2) With center in O and radio R_{Cap} a circumference is traced (Tropic of Capricorn) that gives us a point A on the positive axis OY (Fig. 11). We take a numeric slider between 1 and 20 for R_{Cap} .
- 3) Draw an angle $\varepsilon = 23.4372^\circ$ from the ordinate axis clockwise to mark point B on the Tropic of Capricorn. We associate angle ε with a slider and an input box.
- 4) We join point B with the intersection of the Tropic of Capricorn and the negative OX axis; We obtain point C (Fig. 11).
- 5) We trace the segment BC , obtaining in intersection with the axis OY the point P . The length of the segment OP gives us the radius of the Equator.
- 6) If we proceed with the equator instead of the Tropic of Capricorn as we have done in the previous steps, we will get the Tropic of Cancer.

Figure 9 justifies the construction made from the fundamental equation of the astrolabe. This fact is very useful for finding the radius of the projected circles.

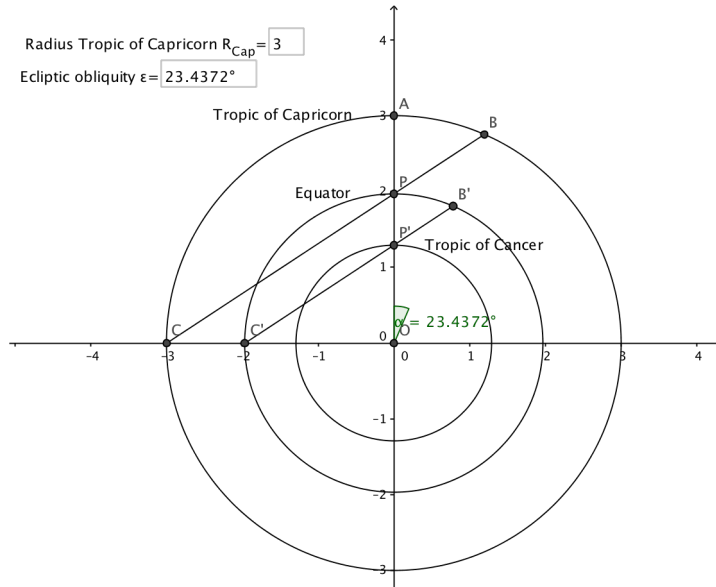


Figure 11: Construction of the Equator and the celestial Tropics.

3.2. Horizon

As in the previous section, we compare figures 12 (3D) and 13 (2D), that is, the celestial sphere and its projection. From both figures the projections of the boundaries of the horizon are obtained by relations (3) and (4):

$$LH'_1 = -R_{Equator} \cdot tg\left(\frac{\varphi}{2}\right) \quad (3)$$

$$Zenit' = R_{Equator} \cdot tg\left(\frac{90^\circ - \varphi}{2}\right) \quad (4)$$

The projection of the Zenith, the point on the head of the observer on the celestial sphere, would be established by (5):

$$Zenit' = R_{Equator} \cdot tg\left(\frac{90^\circ - \varphi}{2}\right) \quad (5)$$

We impose the above relations to reproduce the geometric conditions that characterize the projection of the Horizon of the observer.

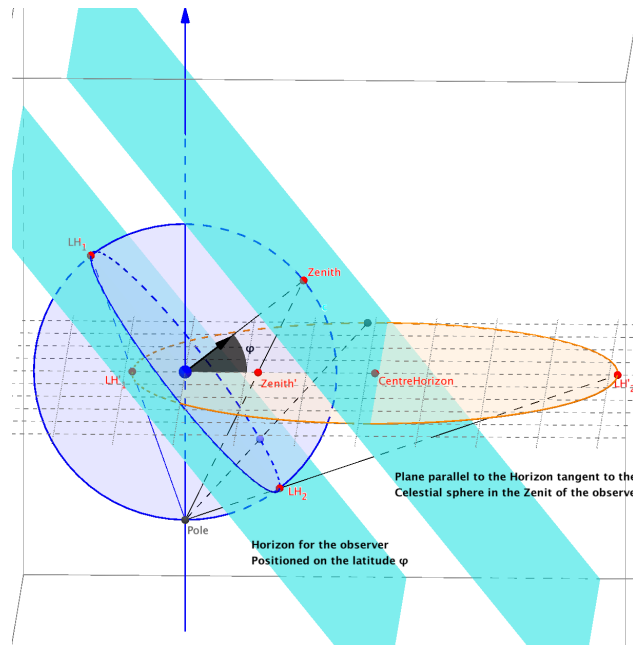


Figure 12: Horizon.

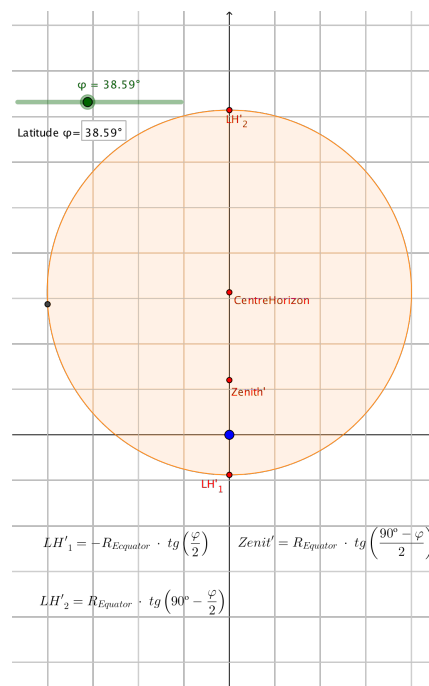


Figure 13: Projection of the Horizon.

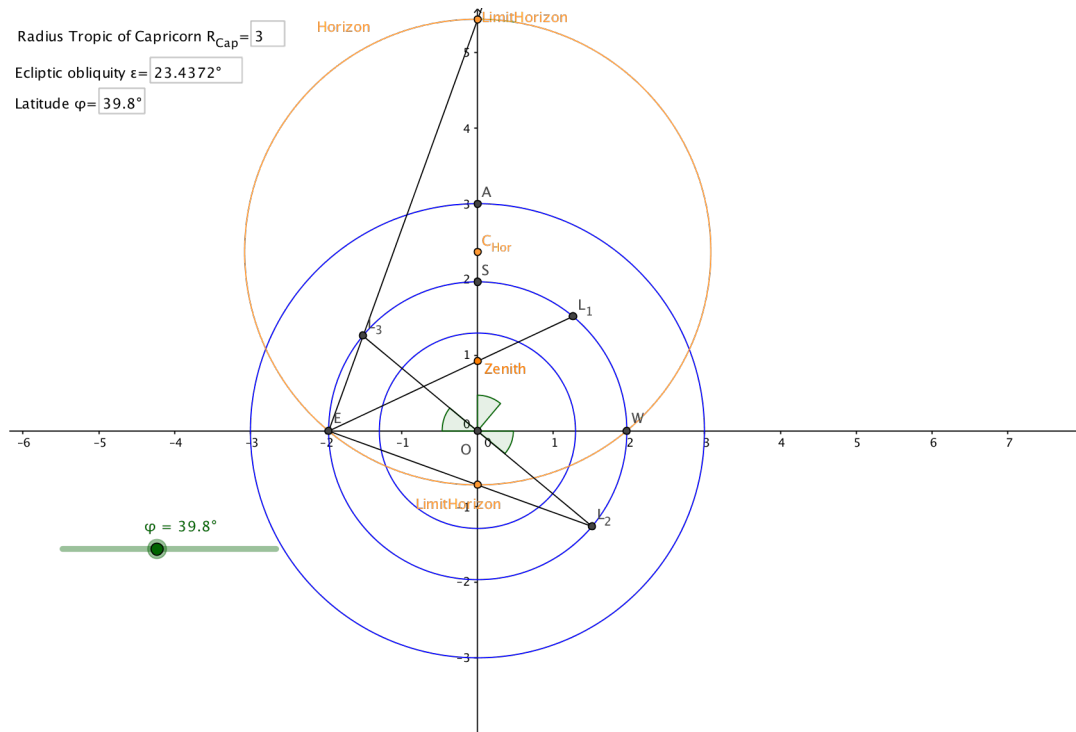


Figure 14: Construction of the Horizon.

Figure 14 if the final result of the following steps:

- 1) On the already constructed equator an arc of latitude φ is drawn from the meridian OY , so that the angle $OSL_1 = \varphi$ (see Fig. 14). We create a slider and an input box for φ .
- 2) Draw the line connecting the E -end of the Equator to the end of the arc, L_1 , newly obtained. The Z point where this line intersects the meridian is Zenith.
- 3) In an analogous way, an arc φ is measured on the equator under the east-west line, obtaining the point L_2 (Fig. 14). We draw the EL_2 segment. The point where this segment intersects the meridian is the *LimitHorizon₁* point.
- 4) In the same way, an arc φ is again measured on the equator, although this time above the east-west line, to obtain point L_3 .
- 5) If we trace the line from the point E , which passes through the point L_3 , it will cut the extension of the meridian at the point *LimitHorizon₂*.
- 6) The midpoint of *LimitHorizon₁* and *LimitHorizon₂*, which we will call C_{Hor} , will be the center of the circumference that defines the horizon, so that its radius will be given by $dist(LimitHorizon_1, C_{Hor})$.

Note: In an astrolabe only the part of the inner horizon to the Tropic of Capricorn would be seen.

3.3. Almucantar

An almucantar is the locus of celestial sphere's points which are at the same height to the plane of the horizon; in fact, the horizon is an almucantar of height 0° , that's why the following process is similar to the previous one.

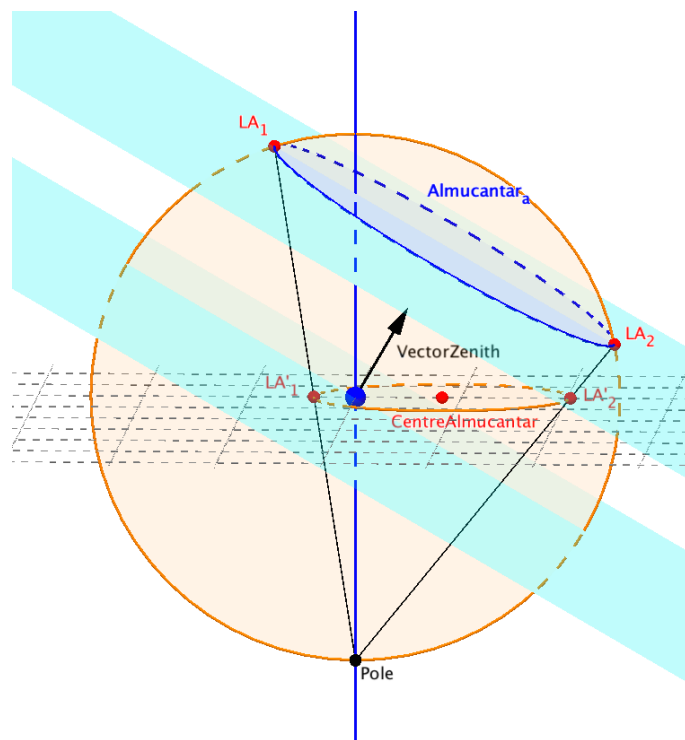


Figure 15: Almucantar.

Figures 15 and 16 establish the following ratios (6) and (7):

$$LA'_1 = -R_{Equator} \cdot tg\left(\frac{\varphi - a}{2}\right) \quad (6)$$

$$LA'_2 = R_{Equator} \cdot tg\left(90^\circ - \left(\frac{\varphi - a}{2}\right)\right) \quad (7)$$

Next, we construct the almucantar by imposing the geometric conditions identified in the relations (6) and (7).

Note: Construction can be done on either side of the meridian.

The east-west straight line is normally called *right horizon*.

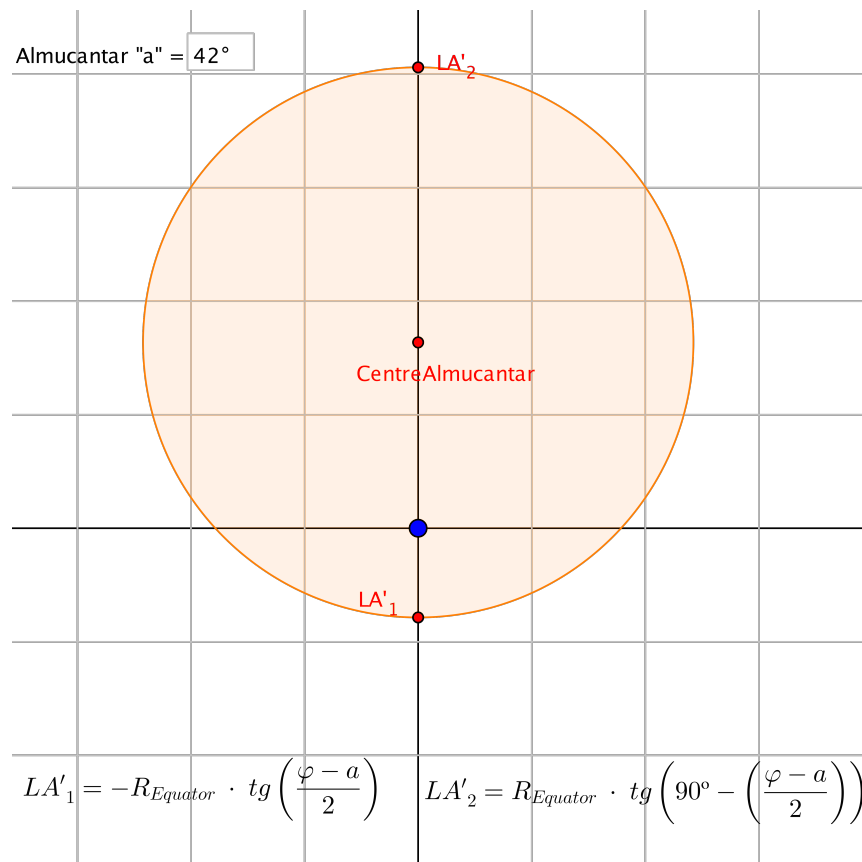


Figure 16: Projection of the Almucantar.

1) To find out the intersection of the almucantar a with the meridian, we should measure an angle $(\varphi - a)$ under the right horizon on the equator. There will be obtained a point that we will call $W_{\varphi-a}$ (Fig. 17).

Note: the resulting point will be under the right horizon if $a < \varphi$, otherwise it will be on it.

2) Draw the segment by joining the points $W_{\varphi-a}$ and E . The intersection of this segment with the meridian gives us the lower limit of almucantar a (Fig. 17).

3) In order to find out the upper limit we should measure $\varphi + a$ upwards; this must be done from the right horizon (Fig. 17). Once the angle measure has been taken into account, we should draw the segment $EE_{\varphi+a}$ whose extension cuts the meridian at the limit superior to the almucantar a (Fig. 17).

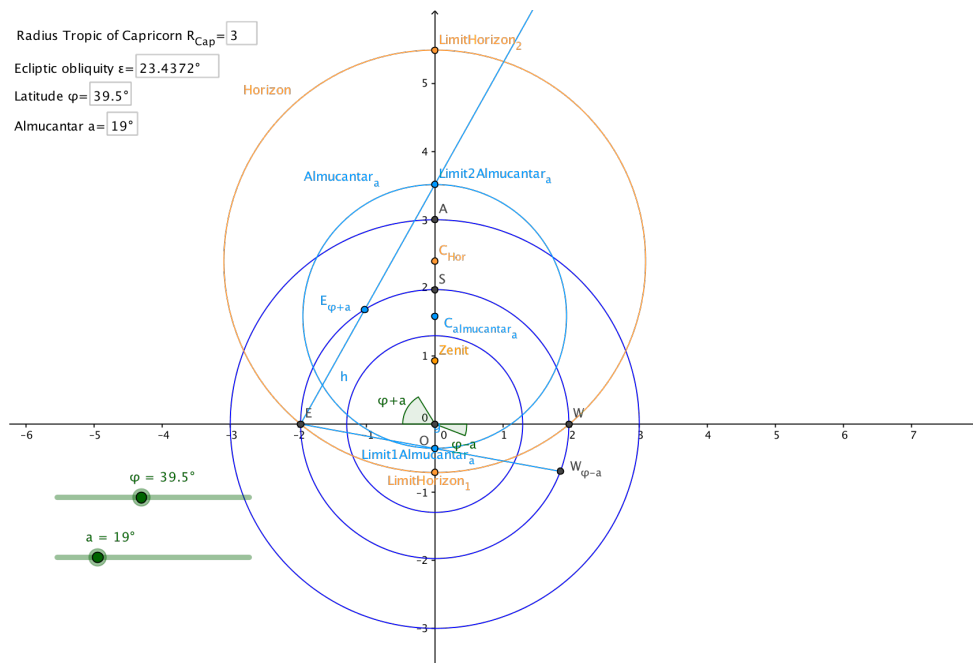


Figure 17: Construction of the Almucantar.

3.4. Nadir

The point of the celestial sphere opposite the zenith with respect to the center is nadir. While the zenith is above our heads, the nadir is below us (that's why both constructions are so similar).

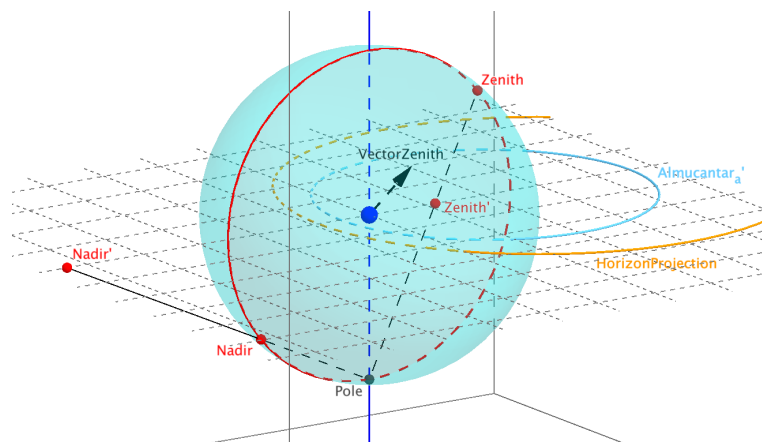


Figure 18: Zenith and Nadir.

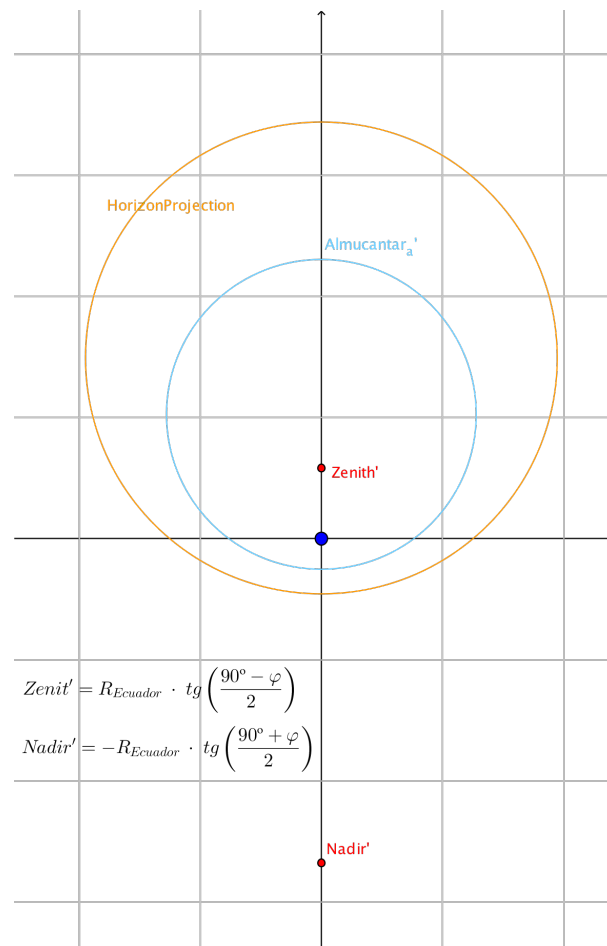


Figure 19: Projection of the Zenith and Nadir.

From the figures 18 and 19 we can deduce relations (8) and (9):

$$Zenith' = R_{Ecuador} \cdot tg\left(\frac{90^\circ - \varphi}{2}\right) \quad (8)$$

$$Nadir' = -R_{Ecuador} \cdot tg\left(\frac{90^\circ + \varphi}{2}\right) \quad (9)$$

The following construction is adjusted to the conditions established by equations (8) and (9):

1) We draw a ray whose origin is O from the negative axis OY with an angle φ in the clockwise direction. The isosceles triangle OEN' is obtained (Fig. 20) whose angle at the unequal vertex is $(90^\circ - \varphi)$ and the equal angles are about $\left(\frac{90^\circ + \varphi}{2}\right)$.

2) Draw a ray from the point E passing through N' . The extension of this line cuts the meridian in the nadir.

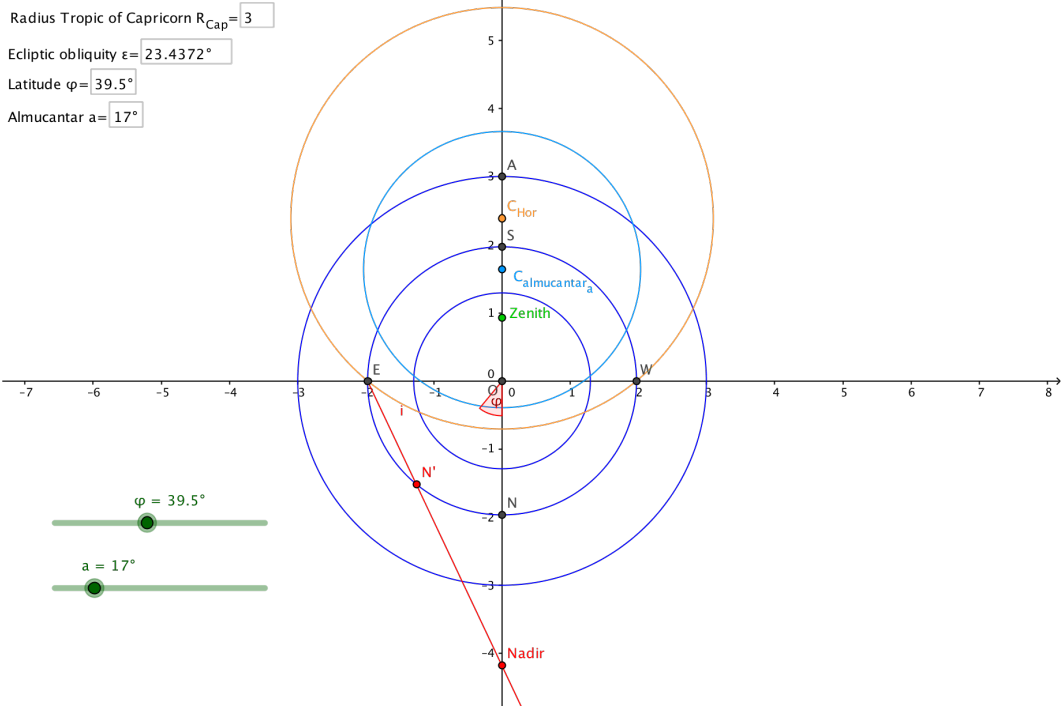


Figure 20: Construction of the Nadir.

3.5. Azimuth

By definition, all azimuths are circles on the celestial sphere that pass through zenith and nadir and are the equivalent of terrestrial meridians ([4]). The North-South azimuth on the celestial sphere is the equivalent of the Greenwich meridian on Earth, that is, the one we refer to. The stereographic projection of this azimuth is a segment, as it passes through the projection pole (Property 4), which is contained in the OY axis. Since the stereographic projection is conformal (Property 6), the projected azimuths will be identified by their angular relation to that OY axis.

Azimuths that do not pass through the projection pole are projected as circumferences. By symmetry, all projected azimuth centers must equidist from zenith' and nadir', and must be located in the so called *Line of the Azimuth Centers*, which is the line perpendicular to OY axis and equidistant to zenith' and nadir'.

On the other hand, the angle $\angle AE$ that forms the line tangent to the $azimuth_{AE}$ in the zenith with respect to the North-South azimuth, will be preserved when the projection is performed (Property 6). In addition, the radius of the circumference projected by the $azimuth_{AE}$ that joins its center with the zenith', will form a right angle with the projection of the tangent line to the $azimuth_{AE}$ in the zenith (Fig. 22). Thus, to find the center of the projected $azimuth_{AE}$, a ray is drawn from zenith' forming an angle of $(90^\circ - \angle AE)$ respect to the OY axis (Fig. 21 and 22).

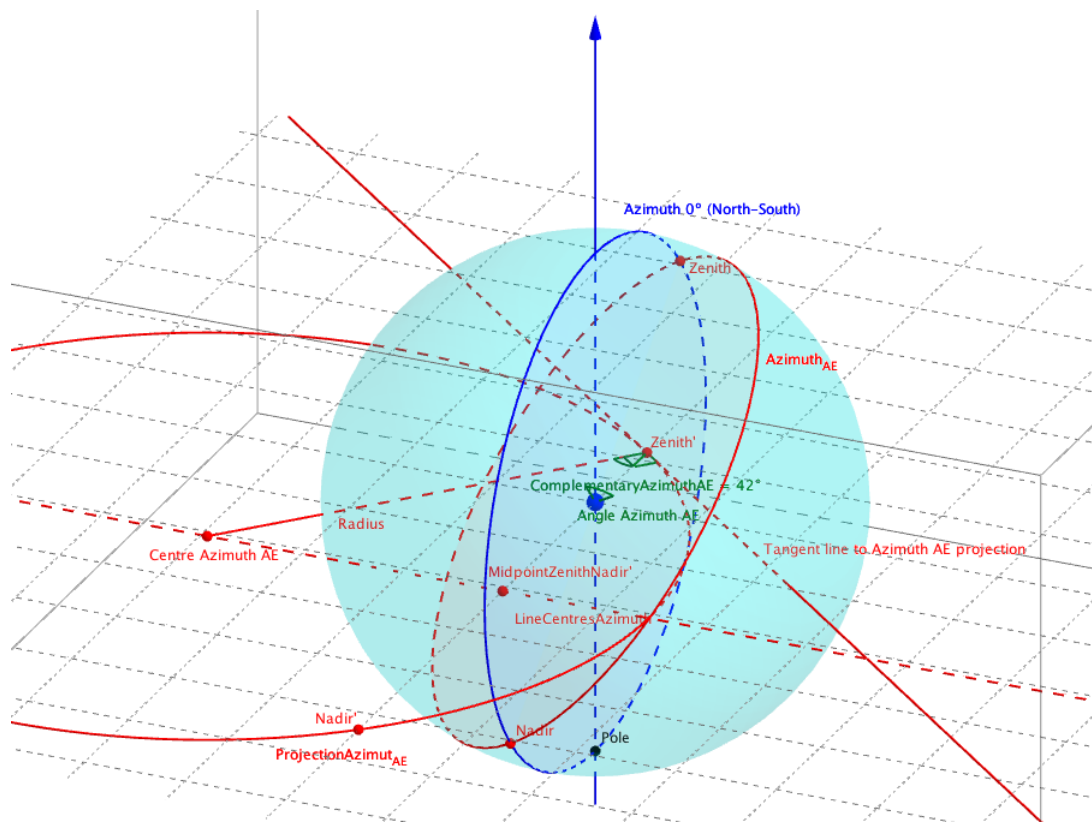


Figure 21: Azimuth.

From the above we obtain the process to construct the azimuth projection. Let's see:

- 1) Find the midpoint of zenith and nadir, then draw a perpendicular to the meridian by that point. This is the so called Line of Azimuth Centers (Fig. 23).
- 2) We turn the meridian an angle $(90^\circ - \angle AE)$ and the intersection with the line of centers gives us the center of the projection of the $azimuth_{(90^\circ - \angle AE)}$.

Note: The azimuth construction is erroneous in several manuals, since they assign the angle $\angle AE$ (instead of the correct one $(90^\circ - \angle AE)$). The 3D construction in GeoGebra allows to verify this fact.

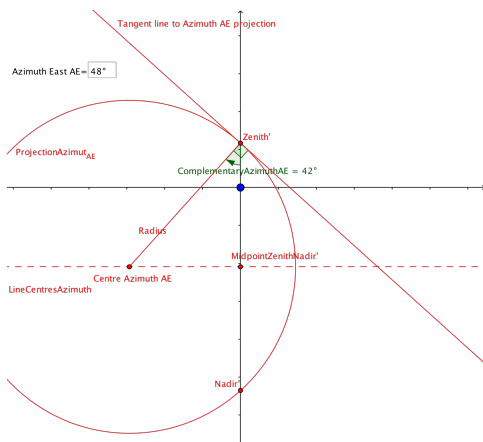


Figure 22: Projection of the azimuth.

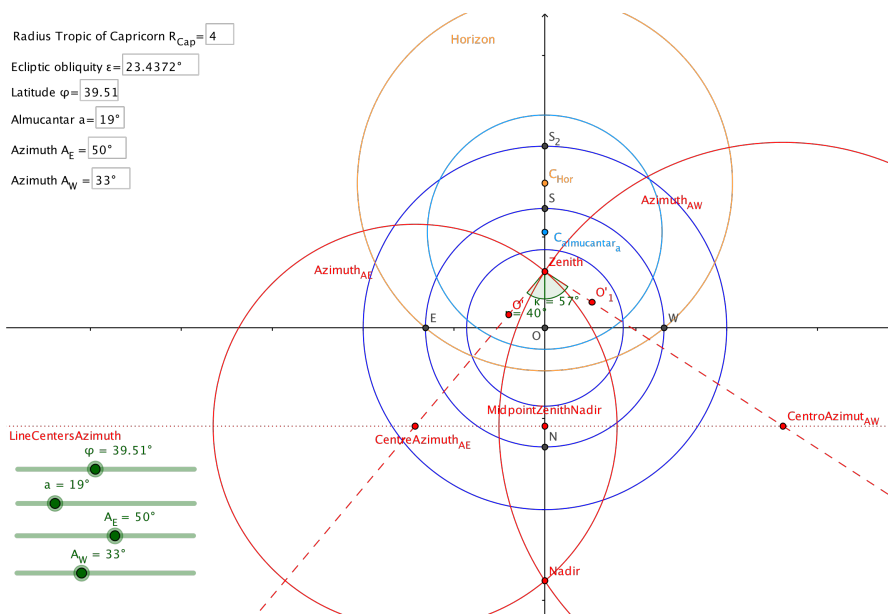


Figure 23: Azimuth construction.

3.6. Ecliptic

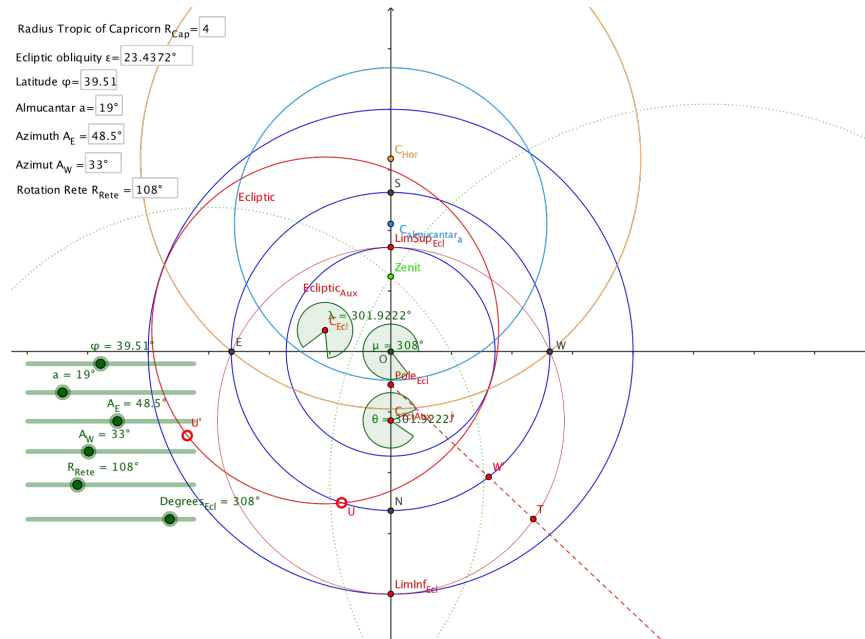


Figure 24: Ecliptic.

The ecliptic has as upper and lower limit the tropics of Capricorn and Cancer, respectively; It is true that the radius of the ecliptic, R_{Ecl} is the semisume of the radius of the tropics. As for the center of the ecliptic, C_{Ecl} , it suffices to perform the semidifference of the radius of the tropics.

We will need to find the projection of the *ecliptic pole*, that is, the point of the axis perpendicular to the ecliptic on the celestial sphere.

From Figure 25 it follows that

$$dist(O, PoleEcl'_1) = R_{Equator} \cdot tg\left(\frac{\varepsilon}{2}\right)$$

On the other hand, as the ecliptic rotates around the axis of rotation of the Earth, the construction will have to allow to turn it the desired angle. We will follow the following steps:

1) To mark the lower and upper limits of the ecliptic, which are the tropics, we make the lower point of the ecliptic coincide with the lower point of the tropic of Capricorn and the upper one coincides with the upper one of the Cancer (Fig. 24). The midpoint of both will be the center of the ecliptic.

2) We define the center of the ecliptic in polar coordinates as follows:

$$C_{Ecl} = (Distance[Midpoint[LimSup_{Ecl}, LimInf_{Ecl}], O]; -R_{Rete} + 90^\circ),$$

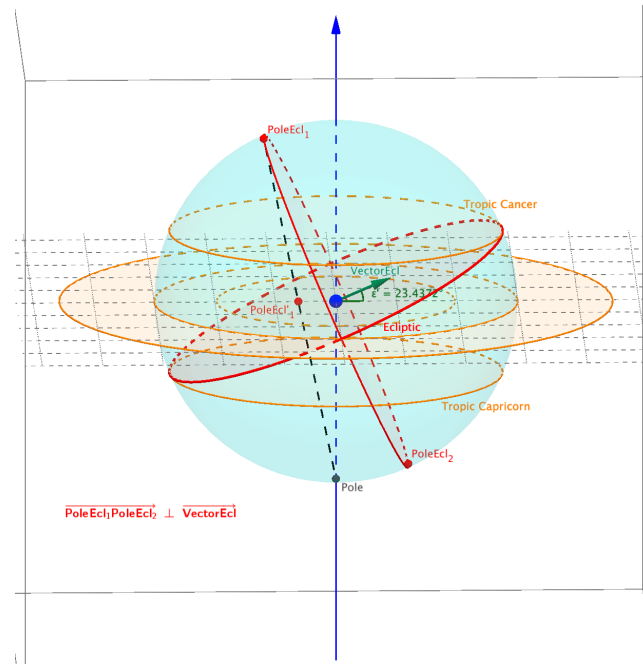


Figure 25: Ecliptic poles.

where R_{Rete} is an angle slider between 0° and 360° to rotate the ecliptic clockwise. We will consider as a starting point for the ecliptic the place where its center is right in the meridian that is under the straight horizon (axis OX).

3) We define the ecliptic as the circumference with center in C_{Ecl} and as radius the half of the segment that links $LimInf_{Ecl}$ and $LimSup_{Ecl}$, that is to say,

$$Ecliptic = Circunference \left[C_{Ecl}, \frac{Distance[LimInf_{Ecl}, LimSup_{Ecl}]}{2} \right]$$

4) We define the pole of the Ecliptic as:

$$Pole_{Ecl} = (0, -Distance[O, S] \cdot tg\left(\frac{\varepsilon}{2}\right)),$$

This point will be one of the two fixed reference points on the ecliptic to measure angles on it.

5) To graduate the ecliptic we will make an *auxiliary ecliptic*. We take a circumference whose center is the center of the ecliptic on the meridian, that is, the midpoint of the upper and lower limits of the ecliptic.

With center at the midpoint

$$C_{EclAux} = Midpoint [LimSup_{Ecl}, LimSup_{Ecl}],$$

we draw a circumference that passes said upper and lower limits of the ecliptic

$$Ecliptic_{Aux} = Circumference [C_{EclAux}, LimSup_{Ecl}],$$

6) We create the angle slider $Degrees_{Ecl}$ between 0° and 360° . On the equator we measure, from the point W (lateral point) and centered on O , using the tool *Angle given its amplitude*, a counterclockwise angle according to the $Degrees_{Ecl}$ value; it will give us the point W' on the equator (Fig. 24).

7) Draw a ray from the ecliptic pole and pass through W' , this will cut the auxiliary ecliptic at a point T (we will hide the rotating ecliptic so as not to confuse us).

8) Now we mark the angle that forms, on the auxiliary ecliptic, W with its center C_{EclAux} and the point we have just discovered T ; we will call θ at that angle.

9) It is now when we mark on the rotating ecliptic the angle θ (it is convenient to hide the auxiliary ecliptic, that is, the fixed ecliptic).

We place a point U on the rotating ecliptic, which occupies the same position as the point W when both ecliptic, rotary and rigid match. To do this last, place the Ecliptic in its lowest position and put a point U on it. We set coordinate $y(U) = 0$ (this must be done because of the precision).

Finally, using the tool *Angle given its amplitude* we measure an angle θ in anti-clockwise on the rotating ecliptic with U as lateral point and C_{Ecl} as center; these last are the two points that we will have as fixed reference in the ecliptic. This angle θ will give the point U' on the movable ecliptic that will correspond with the desired graduation $Degrees_{Ecl}$.

Note: The point U marks 0° Aries, ie the start of the Zodiac. Every 30° a new sign of the Zodiac appears: Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces.

As a curiosity, it is possible to say that all constellations that crossed the ecliptic, although they forgot to include Ofiuco, so that the horoscopes always have been designed obviate this constellation.

3.7. Stars and constellations

The rotation of the rete simulates the rotation of the celestial sphere and allows to position the positions of the stars at any instant. The set of stars rotates completely in a sidereal day, about 23 h. 56' 4". This period is about 3' 56" less than a *average solar day*, because the Earth has to rotate a little further until the Sun

reaches the desired meridian by having also moved in its orbit ([3]).

The position of a star on the arches of azimuth and almucantar shows its coordinates in the celestial sphere. When the spider is placed at a certain sidereal hour it shows us a snapshot of the sky at that moment.

The stars of the spider are usually grouped in *constellations* superimposed on some figures to identify them better. These groupings (*asterisms*) are preserved when projected, however, the sizes of the constellations near the South Pole and the Tropic of Capricorn are smaller than those located near the equator.

Star charts are made to hold them over our heads, imitating the image we see from the sky. However, the astrolabe is a representation of the celestial sphere seen from the outside, not from within. This orientation alters, for example, the image of the Ursa Major, whose shape is reversed.

Choosing the stars to include in the spider is a trivial exercise. Bright stars like Regulus, Sirius, Rigel..., are included almost always.

A list of stars, with their coordinates of the time J2000.0, is available in [6] or in Stellarium.

Note: The precession of the equinoxes causes the stars to change their position, although it is perceptible only with the passage of several decades.

Example: from [6] we are going to place Rigel on the rete; write down the name of the star in the search engine and press *Submit*. The fields that are going to interest us are the last two columns, that is, *-RA.icrs* and *-DE.icrs*.

-RA.icrs: value of right ascension in time J2000.0 given in hours: minutes: seconds (h: m: s).

Note: 1 hour is equivalent to 15° .

-DE.icrs: value of the decline of the period J2000.0 given in degrees: arcminutes: arcseconds (d: m: s), that is, d: arcmin: arcsec.

Note: 1 arcmin = $1/60$ degree.

In our case, Rigel's properties are:

Right ascension $\alpha = 5 \text{ h. } 14' 32.2'' \approx 5.2423 \text{ h. } \xrightarrow{\cdot(15)} 78.6345^\circ$
 Declination $\delta = -8^\circ 12 \text{ arcmin. } 6.08 \text{ arcsec. } \approx -8.2017^\circ$

A star is represented in polar coordinates as r_α , where α is the right ascension of the star counterclockwise from the vernal equinox W (intersection of the positive axis OX with the equator), while $r = R \cdot \operatorname{tg} \left(\frac{90^\circ - \delta}{2} \right)$

In our particular case, $r = R \cdot \operatorname{tg}(49.10085^\circ)$, so that if $R_{Eq} = 1$ then:

$$r = 1 \cdot \operatorname{tg}(49.10085^\circ) = 1.154466$$

In summary, Rigel would be represented in polar coordinates, according to GeoGebra notation, as follows:

$$(1.154466; 78.6345^\circ)$$

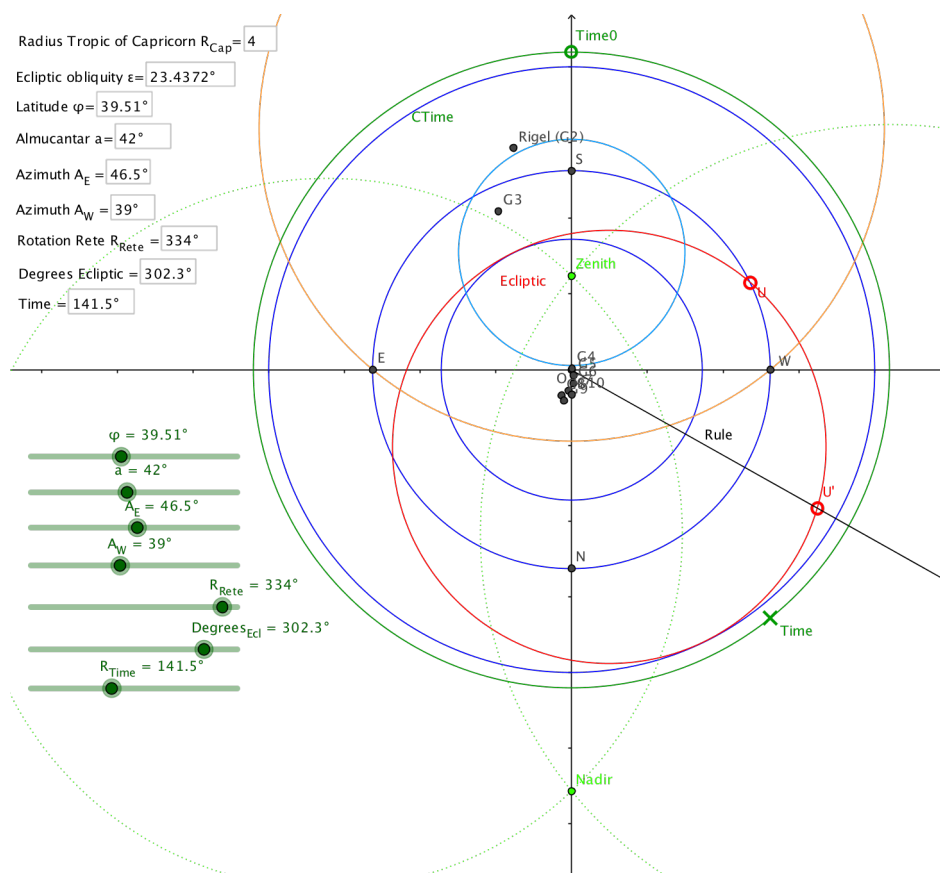


Figure 26: Rete stars.

Given the example of a particular star, let's put it into practice using a GeoGebra spreadsheet (*View menu, Spreadsheet option*). Download the file of section 4.7.1. in [1]. We will find out a series of values in the spreadsheet, we pass them to our file (Fig. 27):

1) Copy from the first to the fourth column, ie, cells A1:D10 and paste them into our spreadsheet.

	A	B	C	D	E	F	G	H
	Constelación	Estrella	Declinación (J2000.0)	Ascensión Recta (J2000.0)	Distancia	Ángulo	Punto (polares)	
2	Orión	Rigel	-8.2017	5.2423	3.0312	78.6345	(3.0312; 10.6345°)	
3	Orión	Betelgeuse	7.407	5.9195	2.3064	88.7925	(2.3064; 20.7925°)	
4	Osa Menor	Polaris	89.2803	2.5983	0.0165	38.9745	(0.0165; 330.9745°)	
5	Osa Menor	Yildun	86.5864	17.5369	0.0782	263.0535	(0.0782; 195.0535°)	
6	Osa Menor	ε UMi	82.0372	16.7661	0.1827	251.4915	(0.1827; 183.4915°)	
7	Osa Menor	ζ UMi	77.7945	15.7343	0.2807	236.0145	(0.2807; 168.0145°)	
8	Osa Menor	Kochab	74.1556	14.845	0.3654	222.675	(0.3654; 154.675°)	
9	Osa Menor	Pherkad	71.834	15.3455	0.4198	230.1825	(0.4198; 162.1825°)	
10	Osa Menor	η UMi	75.7553	16.2918	0.3281	244.377	(0.3281; 176.377°)	

Figure 27: Spreadsheet rete stars.

2) In the main command input (not the spreadsheet), we define the radius of the equator as $R_{Eq} = Distance[O, W]$, we set the title of the box E1 as *Distance* and in box E2 the formula:

$$= R_{Eq} \cdot tg\left(\left(\frac{90-C2}{2}\right)^{\circ}\right),$$

through the Input Bar of the spreadsheet. The boxes E3:E10 are obtained by dragging the E2 box down.

3) Column F is titled *Angle* and in cell F2 is entered the formula:

$$= 15D2$$

The boxes F3: F10 are obtained by dragging the box F2.

4) As for the column G, titled *Point (polar)*, gives us the position of the star by the formula:

$$= (E2, F2^{\circ} - R_{Rete}),$$

that dragging it to the rest of inferior boxes gives us the points that we wanted.

5) Finally, we have to graduate the hours in the astrolabe limb (we will support in the tropic of Capricorn), for this we will use the slider R_{Time} , of 0° to 360° , that will rotate clockwise a point on the circumference:

$$C_{Time} = Circumference [O, 1.05 \cdot R_{Cap}]$$

Now we define the point $Time0$ at the intersection of the positive axis OY with the circumference $CHora$, and we rotated this point with center in O and angle the defined by the slider R_{Time} clockwise:

$$Time = Rotate [Time0, -R_{Time}, O]$$

The $Hour$ point will give us the local time from the equivalence $1hour = 15^\circ$. In the end the obverse would look like Figure 4.

3.8. Calendar

The obverse of the astrolabe can be designed following the *eccentric* or *concentric* model, for the center of the calendar. The eccentric model was the most popular ([3]) and is the one we are going to explain. There are two circumferences, one inside for the calendar and another outside for the solar length (value of the Zodiac). The alidade relates a calendar date to a Zodiac point in its extension (Fig. 3).

In this case we have two fundamental centers, the *center of the calendar* and the *center of the instrument* (where the coordinate axes are cut). Both centers belong to the line that joins *aphelion* and *perihelion*, the first is the point farthest from the center of the instrument, while the second is the nearest.

The distance between both instruments is given by the formula:

$$\Delta = 2 \cdot e \cdot r_c,$$

where e is the eccentricity of the sun's elliptical orbit, while r_c is the radius of the calendar.

Thus, if ω is the angle that forms the axis with the line that joins aphelion and perihelion counterclockwise, then the coordinates of the center of the calendar, r_c would be given as:

$$C_{Cal} = (x_c, y_c) = (\Delta \cdot \cos(\omega), \Delta \cdot \sin(\omega))$$

The eccentric calendar is a recreation of the model of the Sun, from the Ptolemaic astronomy, which satisfied the medieval needs; at least until Copernicus, Galileo and, fundamentally Kepler, made their discoveries ([2]). The Ptolemaic model was sufficient in medieval times and more than meets the precision required for an astrolabe.

The Sun moves at a constant speed around the calendar circle in this model (as represented by the astrolabe) and is observed from the Earth in the center of the

instrument (although in fact it's just the opposite). The length for a date is displayed in the Zodiac when the alidade is set to a calendar scale date.

Let's look at a practical case of drawing the eccentric calendar of 2016 for the back of our astrolabe (Full astrolabe back data Fig. 28):

	A	B	C	D	E	F	G	H
1								
2	Date reference	Perihelion		Coord. X Center Calendar		Longitude Location		
3	31/12/2015 00:00		103.2124159529	-0.0763483058		1.8558333333		
4								
5	JD (Día Juliano) en http://aa.usno.navy.mil/data...	Aphelion		Coord. Y Center Calendar		Angle Aries-Enero0		
6	2457387.5		283.2124159529	0.3251956693		639.1162301218		
7								
8	T (measured in Julian centuries since J2000.0)	Eccentricity		Calendar center		Aries0-Enero0 Normalized		
9	0.1599589322		0.0167018926	(-0.0763483058, 0.3251956693)		279.1162301218		
10								
11	Radius of the calendar	Distance between centers		True anomaly (M)				
12	10		0.3340378529	6115.898743586				
13								
14	Days year			True anomaly normalized				
15	366			355.898743586				
16								
17								
18								
19								
20								

Figure 28: Calendar data.

1) Select 10 decimal places for rounding in *Options - Rounding*, then in the *Edit menu - Properties - Preferences - Spreadsheet* check the *Show Input Bar* box.

Our reference date will be the last day of the year before midnight, ie 12/31/2015 00:00 h. We need the Julian day for that date. We can obtain it in ([5]).

In our case, we obtain the value $DJ = 2457387.5$, which allows us to calculate the number of Julian centuries of 36525 days after noon of January 1, 2000, T , using the formula:

$$T = (DJ - 2451545) / 36525 = 0.159958932238193$$

In this column we complete more necessary data and we take 10 units for our calendar radius of 366 days (2016 is leap year).

2) In the second column we calculate the angles for the perihelion and aphelion, as well as the eccentricity and the distance to the center of the calendar.

The perihelion is given by the polynomial:

$$\omega(T) = 102.93735 + 1.71953 \cdot T + 0.00046 \cdot T^2 + 0.0000000499 \cdot T^3,$$

which in our case yields the value $\omega = 103.212415952911^\circ$.

As far as aphelion is concerned and, although it is not exact, we can assume without valuable loss that it will be $\omega + 180^\circ = 283.212415952912^\circ$

The eccentricity is calculated by:

$$E(T) = 0.01670862 - 0.000042037 \cdot T - 0.0000001236 \cdot T^2 + 0.00000000004 \cdot T^3,$$

which for our calendar has the value $e = 0.0167018926439933$

Once the eccentricity is calculated, we can find out the distance between the centers of the calendar and the instrument, using the relation $\Delta = 2 \cdot e \cdot r_c$, whose value in our example is $\Delta = 0.334037852879867$

3) We are now in a position to find the coordinates of the center of the calendar:

$$C_{Cal} = (x_c, y_c) = (\Delta \cdot \cos(\omega^\circ), \Delta \cdot \sin(\omega^\circ)) = (-0.0763483058073402, -0.0763483058073402),$$

It is compulsory to express the symbol $^\circ$ for the perihelion to find out cosine and sine.

To complete the data in this column we are missing the *true anomaly*; with it the point of the calendar can be located the beginning of counting days, that is, January 0, January 1, January 2...

The true anomaly, M , is obtained by the formula:

$$M(T) = 357.5291^\circ + 35999.0503^\circ \cdot T - 0.0001559^\circ \cdot T^2 - 0.00000048^\circ \cdot T^3,$$

which yields the value $M = 6115.89874358605$, normalized between 0° and 360° by means of the formula:

$$= M - \text{floor}\left(\frac{M}{360}\right) \cdot 360,$$

where the command $\text{floor}(arg)$ is the *largest integer less than or equal to arg*. Thus,

$$M = 355.898743586046$$

4) Now we need to enter the length of our reference location. For example, if we decide to do it for Albacete $1^\circ 51' 21''$ West we take

$$\lambda = 1 + \frac{51}{60} + \frac{21}{60^2} = 1.85583333333334$$

The clockwise angle α , between Aries 0° (intersection of the Zodiac with the positive OX axis) and January 0, is calculated by the following formula:

$$\alpha = Aphelion^\circ + M^\circ + \frac{\lambda}{\text{Days of the year}}$$

We take 366 because 2016 is a leap year, so $\alpha = 639.116230121835^\circ$, which is normalized as

$$= \alpha - \text{floor}\left(\frac{\alpha}{360}\right) \cdot 360 = 279.116230121835^\circ$$

5) All the values recorded in the spreadsheet must be transferred to the *Graphics View*. First, as in the algebraic view we have left a lot of text references, we select them all and in *Properties - Basic* we marked the option *Auxiliary Object* so that they disappear from the *Algebra View*.

We define the *EquinoxVernal* = (A12, 0) and plot the circumference corresponding to the calendar: *Circumference* = [E9, A12].

Points *Aries0* = (A12 + 2, 0) and *O* = (0, 0) are defined and we draw a circumference for the Zodiac: *Circumference* = [O, Aries0].

Now we define the *Perihelion* = *Rotate* [*EquinoxVernal*, $C3^\circ$, *O*] and we draw the ray: *RayPerihelionAphelion* = *Ray* [*Perihelion*, E9]

The aphelion is obtained by doing:

$$Aphelion = \text{Intersect} [\text{RayPerihelionAphelion}, \text{Calendar}]$$

We define *January0Aries0* = *Rotate* [*EquinoxVernal*, $G9^\circ$, *O*]; which will give us the start of the calendar if we do

$$Aries0 = \text{Intersect} [\text{Calendar}, \text{Ray} [O, \text{January0Aries0}]]$$

6) We establish the two main measurements of angles that we must make for the calendar and the Zodiac. We define the slider for the calendar, R_{Cal} , that goes from 0° to 366° in counter-clock wise; it will move the defined point according to the instruction:

$$DateCalendar = \text{Rotate} [Enero0, R_{Cal} \cdot 360, E9],$$

where $E9$ is the center of the calendar. The slider, R_{Zod} , between 0° and 360° counterclockwise, will rotate $PointZodiac = Rotate[Aries0, G_{Zod}, O]$

4. Conclusion

Astronomy is an inexhaustible source of resources for mathematics and viceversa. Knowing how the astrolabe works provides an implicit knowledge of basic celestial mechanics and, on the other hand, can be designed from fundamental mathematical properties. We believe that the construction of an astrolabe is a very enriching work, since several disciplines are involved.

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