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ANALYTICALLY IRREDUCIBLE POLYNOMIALS WITH COEFFICIENTS IN A REAL-VALUED FIELD

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Abstract. In this paper, we show a criterion of analytic irreducibility for polynomials with coefficients in a real-valued field. This generalizes previous criteria of Abhyankar as well as those of Granja.

1. Introduction

Throughout this paper, K will be a field and ν will be a non-trivial valuation of real rank one on K. We will denote by $\widehat{K}$ the completion of K with respect to the topology defined by ν, by $\widehat{\nu}$ the unique extension of ν to $\widehat{K}$ and by T an indeterminate over $\widehat{K}$.

Let L be an algebraic extension of K. A natural question is: *when does a valuation of finite real rank on K have a unique extension to L?*

Since an arbitrary algebraic field extension is a direct limit of finite extensions, we may assume that L is finite over K. Moreover, it is sufficient to solve the problem when ν has real rank one. The general case can be reduced to the real rank case by means of composition of valuations. (See [ZS, Corollary 1, p. 55].)

Furthermore, since for purely inseparable extensions, ν has a unique extension, we can also assume that L is a separable extension of K and write $L = K(\alpha)$ for some algebraic and separable element α over K. In this case, let $P \in K[T]$ be the minimal monic irreducible polynomial of α. Then ν has a unique extension to $K(\alpha)$ if and only if P is irreducible in $\widehat{K}[T]$ (i.e. P is analytically irreducible). (See [E, Theorem (2.12), p. 16].)

In [HOS] an algorithm is given to determine the extensions of a real valuation to a simple algebraic extension. In particular, Corollary 66 of [HOS] gives a characterization of when ν has a unique extension to $K(\alpha)$. Thus, the algorithm of [HOS] tests when a polynomial of $K[T]$ is irreducible in $\widehat{K}[T]$.

Here, we shall show a criterion of analytic irreducibility for polynomials of $K[T]$. (See Theorem 4.3 below.) In particular, if ν is discrete and K is complete, our criterion is nothing but Theorem 4.7 of [G]. Moreover, if $K = F((X))$ is the field of the Laurent power series with coefficients in an algebraically closed field F and ν is the usual order valuation on X, then our criterion also generalizes Abhyankar's criterion in [A].

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The paper is organized in four sections including this introduction. Section 2 is devoted to reviewing some concepts about valuations on rings. In section 3, we associate to each monic analytically irreducible polynomial $f \in K[T]$ a valuation ν_f on $K[T]$ such that ν_f is the unique extension of ν to $K[T]$ with $\nu_f(f) = \infty$. We show that $\nu_f(g) = \dfrac{\nu(\mathcal{R}(f,g))}{\deg(f)}$ for each $g \in K[T]$, where $\mathcal{R}(f,g)$ denotes the usual resultant of the polynomials f and g. (See Proposition 3.1.) The objective of the last section is to show the above-mentioned criterion of analytic irreducibility (Theorem 4.3).

2. Valuations on rings

In this section we review some concepts about valuations on rings.

Let R be a commutative ring with unit and Γ be a totally ordered abelian group. We extend Γ to an ordered monoid $\Gamma \cup \{\infty\}$ by the rules $\infty + x = x + \infty = \infty$ for all $x \in \Gamma \cup \{\infty\}$ and $x < \infty$ for all $x \in \Gamma$.

A *valuation* on R with values in Γ is a map $\nu : R \longrightarrow \Gamma \cup \{\infty\}$ such that

(V1) $\nu(xy) = \nu(x) + \nu(y)$ for all $x, y \in R$,
(V2) $\nu(x + y) \geq \min\{\nu(x), \nu(y)\}$ for all $x, y \in R$,
(V3) $\nu(1) = 0$ and $\nu(0) = \infty$.

If $\nu(R) = \{0, \infty\}$, ν is said to be *trivial*; otherwise ν is called *non-trivia*l or *proper*. Moreover, the prime ideal $\nu^{-1}(\infty)$ of R is called the *support* of ν and when $\Gamma = \mathbb{R}$ is the additive group of real numbers, ν is called a *real valuation* on R. In any case, the *group of values* Γ_ν of ν is the subgroup of Γ generated by $\nu(R) - \{\infty\}$.

On the other hand, ν is called a *Krull valuation* on R when $\nu^{-1}(\infty) = \{0\}$. In this case, R is necessarily a domain and ν extends to a Krull valuation (also denoted by ν) on the quotient field $K(R)$ of R such that $\Gamma_\nu = \nu(K(R) - \{0\})$ is the group of values of ν. Furthermore, $V_\nu = \{x \in K(R);\ \nu(x) \geq 0\}$ is the *valuation ring* associated to ν, which is a local ring with maximal ideal $M(V_\nu) = \{x \in K(R);\ \nu(x) > 0\}$.

We also point out that any valuation ν on a ring R induces a Krull valuation $\overline{\nu}$ on $R/\nu^{-1}(\infty)$. We say that $V_\nu = V_{\overline{\nu}}$ is the *valuation ring* associated with ν and we have $\Gamma_\nu = \Gamma_{\overline{\nu}}$.

We recall that for a Krull valuation ν on a local domain R, the *real rank*, or simply the *rank* of ν (denoted $\mathrm{rank}(\nu)$), is the Krull dimension of the ring V_ν and that if this dimension is finite, then $\mathrm{rank}(\nu)$ is the least integer l so that $\Gamma_\nu = \nu(K(R) - \{0\})$ can be embedded as an ordered group into $(\mathbb{R}^l, +)$ endowed with the lexicographic order. Hence, $\mathrm{rank}(\nu)$ depends only on the group of values of ν. For details about the rank and other numerical invariants of Krull valuations, we refer to [ZS, Vol. II, Chapter VI, p. 50].

3. Valuation associated to analytically irreducible polynomials

Let K be a field and ν be a non-trivial real valuation on K. Let us denote by $\widehat{K}$ the completion of K with respect to the topology defined by ν and by $\widehat{\nu}$ the unique extension of ν to $\widehat{K}$. Let T be an indeterminate over $\widehat{K}$.

We say that a polynomial $f \in K[T]$ is **analytically irreducible** if f is irreducible in $\widehat{K}[T]$. Note that any analytically irreducible polynomial of $K[T]$ is an irreducible polynomial, but the converse is, in general, false.

Now, we shall introduce some notation for monic analytically irreducible polynomials.

Let $f \in K[T]$ be an analytically irreducible monic polynomial and let α be a root of f in some algebraic closure of $\widehat{K}$. Since $\widehat{K}$ is complete, there exists a unique extension, which we will denote by $\overline{\widehat{\nu}_f}$, of $\widehat{\nu}$ to $\widehat{K}(\alpha)$. (See [R, A, p. 127].) We shall also denote by $\widehat{\nu}_f$ the valuation on $\widehat{K}[T]$ given by $\widehat{\nu}_f(g) = \infty$ if $g \in f\widehat{K}[T]$ and by $\widehat{\nu}_f(g) = \overline{\widehat{\nu}_f}(\overline{g})$ if $g \notin f\widehat{K}[T]$, where $\overline{g}$ is the class of g in $\dfrac{\widehat{K}[T]}{f\widehat{K}[T]}$. We point out that $\widehat{\nu}_f(g) = \overline{\widehat{\nu}}_f(g(\alpha))$.

Finally, we will denote by $\overline{\nu_f}$ (resp. by ν_f) the restriction of $\overline{\widehat{\nu}_f}$ (resp. of $\widehat{\nu}_f$) to $K(\alpha)$ (resp. to $K[T]$). Then $\overline{\nu_f}$ (resp. ν_f) is the unique extension of ν to $K(\alpha)$ (resp. to $K[T]$ such that $\nu_f(f) = \infty$).

We can reduce the computation of $\widehat{\nu}_f$ (resp. ν_f) to $\widehat{\nu}$ (resp. ν) by means of the usual resultant of two polynomials $\mathcal{R}(P, Q)$ (see [L, p. 200]). Namely, we have the following:

Proposition 3.1. *With the above assumptions and notation, let $f \in K[T]$ be a monic analytically irreducible polynomial. Then $\widehat{\nu}_f(g) = \dfrac{\widehat{\nu}(\mathcal{R}(f,g))}{\deg(f)}$ for all $g \in \widehat{K}[T]$, where $\mathcal{R}$ denotes the usual resultant (*[L, p. 200]*). In particular, $\nu_f(g) = \dfrac{\nu(\mathcal{R}(f,g))}{\deg(f)}$ for all $g \in K[T]$.*

Proof. Let $\alpha_1, \dots, \alpha_r$ be the roots of f in an algebraic closure of $\widehat{K}$ ($r \le \deg(f)$). Since $\widehat{K}$ is complete, there exists a unique extension $\overline{\widehat{\nu}'_f}$ of $\widehat{\nu}$ to $\widehat{K}(\alpha_1, \dots, \alpha_r)$. (See [R, A, p. 127].)

For $1 \le i \le r$, let σ_i be a $\widehat{K}$-automorphism of $\widehat{K}(\alpha_1, \dots, \alpha_r)$ such that $\sigma_i(\alpha_1) = \alpha_i$. Then $\overline{\widehat{\nu}'_f} \circ \sigma_i$ is a valuation of $\widehat{K}(\alpha_1, \dots, \alpha_r)$ extending $\widehat{\nu}$ (see [R, I, (α), p. 118]). Therefore, $\overline{\widehat{\nu}'_f} = \overline{\widehat{\nu}'_f} \circ \sigma_i$ and $\overline{\widehat{\nu}'_f}(g(\alpha_1)) = \overline{\widehat{\nu}'_f}(\sigma_i(g(\alpha_1))) = \overline{\widehat{\nu}'_f}(g(\alpha_i))$, $1 \le i \le r$.

We point out that if $\overline{\widehat{\nu}^{(i)}_f}$ denotes the restriction of $\overline{\widehat{\nu}'_f}$ to $\widehat{K}(\alpha_i)$, then $\overline{\widehat{\nu}^{(i)}_f}$ is the unique extension of $\widehat{\nu}$ to $\widehat{K}(\alpha_i)$, $1 \le i \le r$. Hence, $\overline{\widehat{\nu}^{(1)}_f}(g(\alpha_1)) = \overline{\widehat{\nu}^{(i)}_f}(g(\alpha_i))$, $1 \le i \le r$.

On the other hand, we can write $f = \prod_{i=1}^r (T - \alpha_i)^{s_i}$, where s_i is the multiplicity of the root α_i, $1 \le i \le r$. (Note that, in fact, $s_i = 1$, if f is a separable polynomial and $s_i = p^s$, otherwise, $1 \le i \le r$, where p is the characteristic of $\widehat{K}$.) We have $\mathcal{R}(f, g) = \prod_{i=1}^r (g(\alpha_i))^{s_i}$ (see [L], Proposition 8.3, p. 202). Thus,

$$\begin{aligned}\widehat{\nu}_f(\mathcal{R}(f,g)) &= \sum_{i=1}^r s_i \overline{\widehat{\nu}'_f}(g(\alpha_i)) = \deg(f)\, \overline{\widehat{\nu}'_f}(g(\alpha_1)) \\ &= \deg(f)\, \overline{\widehat{\nu}^{(1)}_f}(g(\alpha_1)) = \deg(f)\, \widehat{\nu}_f(g). \qquad \square\end{aligned}$$

4. Analytic criterion of irreducibility

With the assumptions and notation as in the above section, we shall reduce the analytic irreducibility of polynomials of $K[T]$ to that of monic polynomials of $V_\nu[T]$, where V_ν is the valuation ring associated to ν.

Let $f \in K[T]$ be an analytically irreducible polynomial with $\deg(f) \geq 2$ and $\alpha \neq 0$ be a root of f. Without loss of generality we can assume that f is a monic polynomial and write $f = T^d + a_{d-1}T^{d-1} + \cdots + a_0 \in K[T]$.

If $f \in V_\nu[T]$, then $\nu_f(T) = \overline{\widehat{\nu}}_f(\alpha) \geq 0$. Otherwise, $\overline{\widehat{\nu}}_f(\alpha) < 0$ and $\overline{\widehat{\nu}}_f(\alpha^{-1}) > 0$. Since $\alpha + a_{d-1} + a_{d-2}\alpha^{-1} + \cdots + a_0(\alpha^{-1})^{d-1} = 0$, we have $\overline{\widehat{\nu}}_f(\alpha) \geq 0$, which is a contradiction.

If $\nu_f(T) = \overline{\widehat{\nu}}_f(\alpha) < 0$, then $\nu(a_0) < \nu(a_i)$, $1 \leq i \leq d-1$ and $\nu(a_0) < 0$. Otherwise, there exists $0 < s < d$ such that $\nu(a_s) \leq \nu(a_0)$ and $\nu(a_s) < 0$. (Note that $f \in K[T] - V_\nu[T]$.) Since $\widehat{K}$ is a Henselian field and f is an irreducible polynomial of $\widehat{K}[T]$, then $\nu(a_s) \geq ((d-s)/d)\nu(a_0)$. (See [R, Theorem 1, p. 128].) Therefore, $\nu(a_0) < 0$ and since $\nu(a_s) \leq \nu(a_0)$, we get $\nu(a_0) \geq ((d-s)/d)\nu(a_0)$, which is a contradiction.

Finally, we note that if $\nu_f(T) = \overline{\widehat{\nu}}_f(\alpha) < 0$, then $\widetilde{f} = a_0^{-1}T^d f(1/T) \in V_\nu[T]$ is a monic analytically irreducible polynomial. In fact, $\widetilde{f}$ is the minimal irreducible polynomial of α^{-1} over K (and $\widehat{K}$). In this form, we must only give a criterion of analytic irreducibility for monic polynomials of $V_\nu[T]$.

Now, we shall introduce some more notation. Let $f \in V_\nu[T]$ be a monic analytically irreducible polynomial. We denote by

$$\Omega_l(\nu_f) = \{\nu_f(g);\ g \in V_\nu[T] \text{ monic with } \deg(g) = l\}$$

and by $\omega_l(\nu_f) = \sup(\Omega_l(\nu_f))$, $0 \leq l < \deg(f)$. Note that $\omega_l(\nu_f) \in \mathbb{R} \cup \{\infty\}$ and $0 = \omega_0(\nu_f) \leq \omega_1(\nu_f) \leq \omega_2(\nu_f) \leq \cdots \leq \omega_{\deg(f)-1}(\nu_f) \leq \infty$.

Lemma 4.1. *With the above assumptions and notation, let $f \in V_\nu[T]$ be a monic analytically irreducible polynomial. Then $\nu_f(\phi) \leq \nu(b_h) + \omega_h(\nu_f) \leq \nu(b_h) + \omega_s(\nu_f)$, for each non-zero polynomial $\phi = \sum_{i=0}^{s} b_i T^i \in V_\nu[T]$ with $1 \leq \deg(\phi) = s < \deg(f)$, where h is the non-negative integer $0 \leq h \leq s$ such that $\nu(b_h) < \nu(b_i)$, $h < i \leq s$ and $\nu(b_h) \leq \nu(b_j)$, $0 \leq j \leq h$; i.e., h is the greatest index for which $\nu(b_h) = \min\{\nu(b_0), \ldots, \nu(b_s)\}$.*

Proof. Since $b_h \neq 0$, it is sufficient to show that $\nu_f((b_h)^{-1}\phi) \leq \omega_h(\nu_f)$. Therefore, we can assume $b_h = 1$ and $\nu(b_h) = 0$.

Now, we proceed by induction on s.

Assume $s = 1$. Then we have the following possibilities:

a) $h = s = 1$. In this case $\phi = T + b_0$ with $\nu(b_0) \geq 0$ and $\nu_f(\phi) \leq \omega_1(\nu_f)$ by definition of $\omega_1(\nu_f)$.

b) $h = 0 < 1$. In this case, $\phi = b_1 T + 1$ with $\nu(b_1) > 0$ and $\nu_f(\phi) = 0 \leq \omega_1(\nu_f)$. (Note that $\nu_f(\phi) = \widehat{\nu}_f(\phi(\alpha)) = 0$, where α is a root of f.)

Assume that the result is true for each $\phi \in V_\nu[T]$ with $\deg(\phi) \leq s-1$, $s \geq 2$.

If $h = s$, then $\nu_f(\phi) \leq \omega_h(\nu_f)$ by definition of $\omega_h(\nu_f)$; and if $h = 0$, then $\nu_f(\phi) = 0 = \omega_0(\nu_f)$. (Note that $\nu_f(\phi) = \widehat{\nu}_f(\phi(\alpha)) = 0$, where α is a root of f.) Therefore, we can assume $0 < h < s$, so $\nu(b_s) > 0$.

Since $\omega_s(\nu_f) = \sup(\Omega_s(\nu_f))$ and $\nu(b_s) > 0$, let us fix a monic polynomial $g \in V_\nu[T]$ such that $\deg(g) = s$ and $\nu(b_s) + \nu_f(g) > \omega_s(\nu_f)$ (for example, if $\omega_s(\nu_f)$ is reached, we may take g to be such that $\nu_f(g) = \omega_s(\nu_f)$).

Let us write $\phi = b_s g + c_{s-1}T^{s-1} + \cdots + c_0$. Since $\nu(b_s) > 0$ and $f, g \in V_\nu[T]$, $0 = \nu(c_h) < \nu(c_i)$, $h < i \leq s-1$ and $0 = \nu(c_h) \leq \nu(c_j)$, $0 \leq j < h$. Therefore, $\nu_f(c_{s-1}T^{s-1} + \cdots + c_0) \leq \omega_h(\nu_f) \leq \omega_s(\nu_f)$ by the induction hypothesis. Hence, since $\nu_f(b_s g) > \omega_s(\nu_f)$, we have $\nu_f(\phi) = \nu_f(c_{s-1}T^{s-1} + \cdots + c_0) \le \omega_h(\nu_f)$, and the proof is complete. $\square$

Lemma 4.2. *With the above assumptions and notation, let $f \in V_\nu[T]$ be a monic analytically irreducible polynomial. Then the set $\Omega_i(\nu_f)$ is bounded above for $1 \le i < \deg(f)$, i.e. $\omega_i(\nu_f) < \infty$ for $0 \le i \le \deg(f) - 1$. In particular, the set $\{\nu(\mathcal{R}(f,g));\ g \in V_\nu[T]$ monic with $\deg(g) = i\}$ is also bounded above for $1 \le i < \deg(f)$.*

Proof. Assume that $\Omega_i(\nu_f)$ is not bounded above for some $i < \deg(f)$ and that $\Omega_l(\nu_f)$ is bounded above for $0 \le l < i$. In particular, $\omega_{i-1}(\nu_f) < \infty$.

Let us consider a sequence $\{\phi_n\}_{n\ge 1} \subset V_\nu[T]$ such that $\nu_f(\phi_n) \ge n$ and $\phi_n = T^i + a_{i-1}^{(n)}T^{i-1} + \cdots + a_0^{(n)}$, $n \ge 1$. By Lemma 4.1, we have

$$n \le \nu_f(\phi_{n+1} - \phi_n) \le \min_{0\le j\le i-1}\{\nu(a_j^{(n+1)} - a_j^{(n)})\} + \omega_{i-1}(\nu_f).$$

Therefore, $n - \omega_{i-1}(\nu_f) \le \nu(a_l^{(n+1)} - a_l^{(n)})$ for $0 \le l \le i-1$ and $n \ge 1$. So, $\{a_l^{(n)}\}_{n\ge1}$ is a Cauchy sequence on K with respect to ν, $0 \le l \le i-1$. Let us write $a_l = \lim_{n\to\infty} a_l^{(n)} \in \widehat{K}$, $0 \le l \le i-1$ and $\phi = T^i + a_{i-1}T^{i-1} + \cdots + a_0 \in \widehat{K}[T]$.

We have $\widehat{\nu}(a_l - a_l^{(n)}) \ge n - \omega_{i-1}(\nu_f)$, $0 \le l \le s-1$, $n \ge 1$. Since $\nu_f(T) = \widehat{\nu}_f(T) \ge 0$, then $\widehat{\nu}_f(\phi - \phi_n) \ge n - \omega_{i-1}(\nu_f)$ and $\widehat{\nu}_f(\phi) \ge \min\{\widehat{\nu}_f(\phi-\phi_n), \nu_f(\phi_n)\} \ge n - \omega_{i-1}(\nu_f)$, $n \ge 1$. Hence, $\widehat{\nu}_f(\phi) = \dfrac{\widehat{\nu}(\mathcal{R}(f,\phi))}{\deg(f)} = \infty$ and $\widehat{\nu}\,(\mathcal{R}(f,\phi)) = \infty$. Thus, f and ϕ must have a common irreducible factor in $\widehat{K}[T]$, and since f is analytically irreducible, $\phi \in f\widehat{K}[T]$ and $\deg(f) \le \deg(\phi)$, which is a contradiction. $\square$

Next, we give our criterion of analytic irreducibility.

Theorem 4.3 (Criterion of Analytic Irreducibility)**.** *With the above assumptions and notation, let $P(T) \in V_\nu[T]$ be a monic polynomial. Then the following statements are equivalent:*

(a) *P is analytically irreducible.*
(b) *The set $\{\nu(\mathcal{R}(P,Q));\ Q \in K[T]$ monic with $\deg(Q) = i\}$ is bounded above for $1 \le i < \deg(P)$.*
(c) *The set $\{\nu(\mathcal{R}(P,Q));\ Q \in K[T]$ monic with $\deg(Q) = [\deg(P)/2]\}$ is bounded above.*

Here $[\deg(P)/2]$ denotes the greatest integer s such that $s \le \deg(P)/2$.

Proof. $(a) \Rightarrow (b)$ follows from Lemma 4.2 and obviously $(b) \Rightarrow (c)$.

To see $(c) \Rightarrow (a)$, let us assume that P is not analytically irreducible. We can write $P = P_1P_2$, where $P_1, P_2 \in \widehat{K}[T]$ are monic polynomials, $h_1 = \deg(P_1) \le [\deg(P)/2]$ and $P_1(T) = T^{h_1} + a_{h_1-1}T^{h_1-1} + \cdots + a_0$ with $a_j \in \widehat{K}$, $0 \le j \le h_1 - 1$.

For each integer $n \ge 0$ and for each $0 \le j \le h_1 - 1$, let us consider $a_j^{(n)} \in K$ such that $\widehat{\nu}(a_j - a_j^{(n)}) \ge n$ and write $P_1^{(n)}(T) = T^{h_1} + a_{h_1-1}^{(n)}T^{h_1-1} + \cdots + a_0^{(n)}$.

We recall that if $U_0, \ldots, U_{h_1}$ are indeterminates over $\widehat{K}[T]$, then

$$\mathcal{R}(P,\underline{U}) = \mathcal{R}(P, U_0T^{h_1} + U_1T^{h_1-1} + \cdots + U_{h_1})$$

is a homogenous polynomial on $U_0, \ldots, U_{h_1}$ of degree $\deg(P)$.

Therefore, $\mathcal{R}(P, P_1^{(n)}) = \mathcal{R}(P, P_1) + \Lambda_n$ such that $\Lambda_n \in (a_{h_1-1}^{(n)} - a_{h_1-1}, a_{h_1-2}^{(n)} - a_{h_1-2}, \ldots, a_0^{(n)} - a_0)V_{\widehat{\nu}}$, where $V_{\widehat{\nu}}$ is the valuation ring associated with $\widehat{\nu}$. Hence, $\widehat{\nu}(\Lambda_n) \geq n$, $n \geq 1$.

Since $\mathcal{R}(P, P_1) = 0$, then $\nu(\mathcal{R}(P, P_1^{(n)})) = \widehat{\nu}(\mathcal{R}(P, P_1^{(n)})) = \widehat{\nu}(\Lambda_n) \geq n$, $n \geq 1$.

In particular, $\nu(\mathcal{R}(P, T^{[\deg(P)/2]-h_1} P_1^{(n)})) \geq n$ for $n \geq 1$ and the set

$$\{\nu(\mathcal{R}(P, Q));\ Q \in K[T] \text{ monic with } \deg(Q) = [\deg(P)/2]\}$$

is not bounded above, which is a contradiction. □

Remark 4.4. We point out that the irreducibility criterion given in Theorem 4.7 of [G] is a particular case of our Theorem 4.3. In fact, in [G], the valuation ν is assumed to be discrete and the field K to be complete with respect to the topology defined by ν. Moreover, if $K = F((X))$ is the field of the Laurent power series with coefficients in an algebraically closed field F and ν is the usual order valuation on X, then our criterion also generalizes Abhyankar's criterion in [A].

References

[A] S.S. Abhyankar, *Irreducibility criterion for germs of analytic functions of two complex variables*. Adv. Math., **74**, no. 2 (1989), 190–257. MR997097 (90h:32018)

[E] O. Endler, *Valuation Theory*. Springer-Verlag, Berlin, 1972. MR0357379 (50:9847)

[G] A. Granja, *Irreducible polynomials with coefficients in a complete discrete valuation field*. Adv. Math., **109**, no. 1 (1994), 75–87. MR1302757 (95j:13006)

[HOS] F. J. Herrera Govantes, M. A. Olalla Acosta and M. Spivakovsky, *Valuations in algebraic field extensions*. J. Algebra, **312** (2007), 1033–1074. MR2333199 (2008e:12007)

[L] S. Lang, *Algebra (Revised Third Edition)*. Graduate Text in Mathematics, 211, Springer-Verlag, New York, 2002. MR1878556 (2003e:00003)

[R] P. Ribenboim, *The Theory of Classical Valuations*. Springer-Verlag, New York, 1999. MR1677964 (2000d:12007)

[ZS] O. Zariski and P. Samuel, *Commutative Algebra*. Vols. I and II. Reprint of the 1958-60 edition, Graduate Texts in Math., 28-29, Springer-Verlag, New York, 1979. MR0384768 (52:5641), MR0389876 (52:10706)

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