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Abstract

In the analysis of time series, it is frequent to classify perturbations as *Additive Outliers* (AO), *Innovative Outliers* (IO), *Level Shift* (LS) outliers or *Transitory Change* (TC) outliers. When a time series with a clear seasonal behaviour is considered, this classification may be too restrictive since none of the four outlier types is adequate to model changes in the seasonal pattern of the series. In this paper, a new outlier type, the *Seasonal Level Shift* (SLS), is introduced in order to complete the usual classification. The iterative procedure for the detection of outliers in Chen and Liu (1993) is extended to detect SLS outliers. We use simulations and real examples to assess the properties of the new type of outlier.

1 Introduction

Outlier detection has become an important part of the analysis of time series and influences modelling, inference, and data processing, because outliers can lead to model misspecification, biased parameter estimation, and poor forecasts. Outlier detection has become a key feature in seasonal adjustment and in automatic time-series model identification; examples are the REG-ARIMA routine in the X-12 ARIMA program (see Findley et al., 1998); program TRAMO ("Time series Regression with Arima noise, Missing observations, and Outliers"; see Gómez and Maravall, 1996), and the time series module in the Scientific Computing Associates package (see Chen et al., 1990).

Four outlier types are traditionally considered (see, for instance, Fox, 1972, Tsay, 1986 or Chen and Liu, 1993): *Additive Outliers* (AO), *Innovative Outliers* (IO), *Level Shift* (LS) outliers and *Transitory Change* (TC) outliers. These four outlier types affect an observed time series in different ways. The effect of an AO, an LS or a TC on an observed series is independent of the ARIMA model; the effect of an IO on an observed series consists on an initial shock that propagates in the subsequent observations with the weights of the ARIMA model. When unobserved (trend-cycle, seasonal and irregular) components of the time series are considered, it is generally accepted (see Harvey, 1989) that AOs and TC outliers can be related with outliers affecting the irregular component, LS outliers can be associated with the trend-cycle component and, finally, IOs are the result of an outlier that simultaneously affects the trend-cycle and the seasonal components.

This classification is too restrictive when analysing time series with seasonal behavior since none of the considered outlier types can describe a perturbation mostly related to the seasonal component. In this paper, we present a new outlier type, the *Seasonal Level Shift* (SLS), which completes the previous classification. The iterative procedure for the detection of outliers, initially proposed by Chang (1982) and further modified by Tsay (1986), Chang et al. (1988), Chen and Liu (1993) and Gómez and Maravall(1998), is extended to consider the new outlier type. Finally, the consequences of ignoring (or not correcting) the presence of SLS outliers are investigated in simulated and real examples.

2 Outliers and unobserved components

Let y_t be a time series that can be described with the ARIMA (p,d,q)(P,D,Q) model

$$\phi(B)\nabla^d\Phi(B^s)\nabla_s^D y_t = c + \theta(B)\Theta(B^s)a_t, \quad t = 1, \ldots, N, \tag{2.1}$$

where N is the number of observations; s is the number of observations per year; c is a constant term; a_t is a white-noise process with zero mean and variance V_a; B is the lag operator, such that $By_t = y_{t-1}$; and ∇ and ∇_s are the regular and seasonal difference operators, such that $\nabla^d y_t = (y_t - y_{t-1})^d$ and $\nabla_s y_t = y_t - y_{t-s}$; $\phi(B)$ and $\theta(B)$ are regular polynomials in B of orders p and q, respectively; $\Phi(B^s)$ and $\Theta(B^s)$ are seasonal polynomials, of orders P and Q, respectively, with the roots of all 4 polynomials lying outside the unit circle. The observed series y_t can be additively decomposed (perhaps in the logs) into orthogonal unobserved components of trend-cycle (z_{pt}), seasonal (z_{st}) and irregular (z_{ut}) as,

$$y_t = \sum_i z_{it}, \quad i = p, s, u. \tag{2.2}$$

In this paper. we use the so-called ARIMA model-based approach to achieve the decomposition (2.2) (see, for example, Box, Hillmer and Tiao, 1978, Burman, 1980, Bell and Hillmer, 1984 or Maravall, 1995). Under this approach, the unobserved components can be formulated in an ARIMA-type format as:

$$\phi_i(B) z_{it} = \theta_i(B) a_{it}, \tag{2.3}$$

where $\phi_i(B)$ and $\theta_i(B)$ are finite polynomials in B with no root in common and with all roots on or outside the unit circle; and a_{it} are uncorrelated white-noise errors with zero mean and variance V_i. For ease of notation, let $\tilde{\phi}(B) = \phi(B)\nabla^d \Phi(B^s)\nabla_s^D$ and $\tilde{\theta}(B) = \theta(B)\Theta(B^s)$. Since aggregation of ARIMA models yields ARIMA models, the y_t series also follows an ARIMA model like the one in (2.1). It is straightforward to show that, under the assumption that the components share no AR root in common,

$$\tilde{\phi}(B) = \prod_i \phi_i(B), \tag{2.4}$$

$$\tilde{\theta}(B) a_t = \sum \phi_{ni}(B) \theta_i(B) a_{it}, \tag{2.5}$$

where $\phi_{ni}(B)$ is the product of all $\phi_j(B)$, $j \neq i$. Prior to the ARIMA modelling stage, some modifications to the series are often needed (see Gómez and Maravall, 1998). Outlier correction is essential since the presence of one or more outliers in the observed series, may have serious consequences on the identification and estimation of the ARIMA models (see Chang, 1982).

Following the seminal work of Fox (1972), four different types of outliers have been proposed, together with several procedures to detect them (see, for instance, Tsay, 1986, Chen and Liu, 1993, Gómez and Maravall, 1998, and Kaiser, 1998). The outliers are classified as *Additive Outliers* (AO), *Innovative Outliers* (IO), *Level Shift* (LS) outliers or *Transitory Change* (TC) outliers.

Assuming the observed series contains k outliers, their combined effect can be expressed in general as

$$y_t^* = \sum_{j=1}^{k} \xi_j(B)\omega_j I_t^{(\tau_j)} + y_t, \tag{2.6}$$

where y_t^* denotes the observed "contaminated" series; y_t follows the ARIMA process (2.1); ω_j is the initial impact of the outlier at time $t = \tau_j$; $I_t^{(\tau_j)}$ is an indicator variable such that it is 1 for $t = \tau_j$, and 0 otherwise; and $\xi_j(B)$ determines the dynamics of the outlier occurring at $t = \tau_j$ according to the following scheme:

$$\text{AO:} \quad \xi_j(B) = 1, \tag{2.7a}$$

$$\text{LS:} \quad \xi_j(B) = 1/(1 - B), \tag{2.7b}$$

$$\text{TC:} \quad \xi_j(B) = 1/(1 - \delta B), \quad 0 < \delta < 1, \tag{2.7c}$$

$$\text{IO:} \quad \xi_j(B) = \tilde{\theta}(B)/\tilde{\phi}(B). \tag{2.7d}$$

An AO represents an isolated spike, an LS a step function, a TC a spike that takes a few periods to disappear and an IO represents effects that depend on the ARIMA model for the observed series, as shown by expression (2.7d). Examples of the 4 types of outliers are displayed in Figure 1.

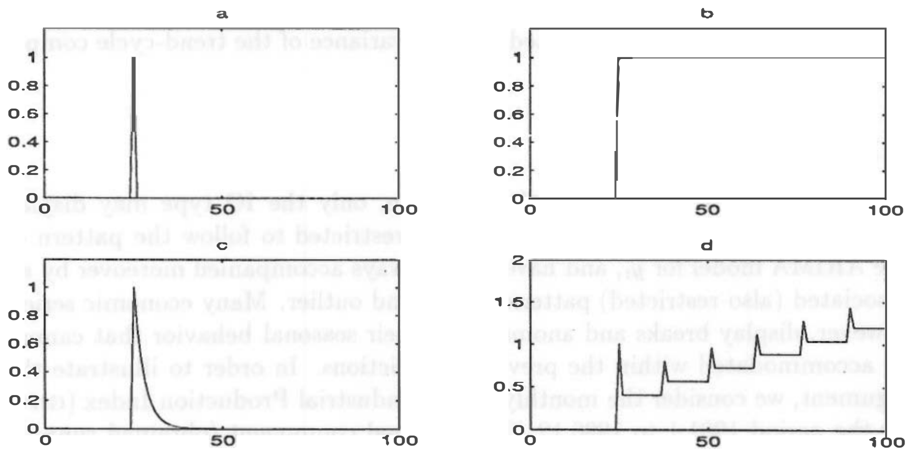

Figure 1. Effect of different types of outliers on the observed series a) AO, b) LS, c) TC and d) IO for an "Airline model" (see section 4).

If y_t^* is to be decomposed into trend-cycle, seasonal, and irregular components, as in

$$y_t^* = z_{pt}^* + z_{st}^* + z_{ut}^*, \tag{2.8}$$

the effect of the outliers has to be assigned to the components in (2.2). Given that the irregular component is aimed at capturing transitory, mostly unsystematic, behavior, it seems natural that z^*_{ut} be equal to z_{ut} plus the AO and TC effects. On the other hand, given that an LS outlier represents a permanent effect on the mean level of the series, it also seems natural to associate its effects to the trend-cycle component. While the allocation of these three types of outliers to the components is straightforward, the allocation of the effects of an IO is not a clear-cut issue (see Bell, Hillmer and Tiao, 1983, and Kaiser, 1995); in series displaying trend and seasonality, both components will be affected by it.

From expression (2.6), an expression equivalent to (2.5) is given by,

$$\tilde{\theta}(B)a_t + \sum_{j=1}^{k} \xi_j(B)\tilde{\varphi}(B)\omega I_t^{(\tau_j)} = \sum_i \phi_{ni}(B)\left(\theta_i(B)a_{it} + \sum_{j=1}^{k_i} \xi_j(B)\phi_i(B)\omega I_t^{(\tau_j)}\right), \tag{2.9}$$

from which it is straightforward to see that an error in the specification or estimation stage of the model for the observed series (detecting spurious outliers or confusing the outlier type, for instance) results in errors in the specification and estimation of the correct models. If, for example, an AO is wrongly detected instead of one LS, the estimated components of the trend-cycle and the irregular component will be affected. In particular, even if the true ARIMA model and the coefficients involved are known, the variance of the irregular component will be underestimated and the variance of the trend-cycle component will be overestimated.

2.1 Seasonal outliers

Of the 4 types of outliers mentioned above, only the IO type may display seasonal features in its pattern, which are restricted to follow the pattern of the ARIMA model for y_t, and have to be always accompanied moreover by an associated (also restricted) pattern for a trend outlier. Many economic series, however, display breaks and anomalies in their seasonal behavior that cannot be accommodated within the previous restrictions. In order to illustrate the argument, we consider the monthly Italian Industrial Production Index (GIPI) for the period 1981.1 to 1996.12. The seasonal component (obtained running TRAMO and SEATS programs in an automatic manner) is presented in Figure 2. Direct inspection of the figure reveals that the level of the seasonal factors for August changes in 1988 and in 1994. The mean level for the August factor was 46.01 during the first seven years, it increased to 47.70 for the following six years and, it was 49.29 for the last three years (the discontinuous line in the figure represents the change in the mean factor for August). This seasonal level shift cannot be modelled as an IO outlier because the model for the series implies a $\nabla\nabla_{12}$ differencing, that would produce an IO of the type displayed in Figure 1. In the standard approach, these seasonal shifts could of course be handled with strings of AOs for the months of August, which would produce an unparsimonious model where, basically, one would be renouncing to model August, a rather unsatisfactory procedure. Addressing the same example of the August outliers in the Italian GIPI series, Proietti (1998) developed a nonlinear-nonstationary unobserved components model with heteroskedastic innovations in the seasonal component. An alternative approach, methodologically and computationally less complex, would be to extend outlier detection procedures to cover breaks in the seasonal component. Moreover, a proper outlier correction procedure should have ways to deal with some of the basic anomalies of a seasonal nature.

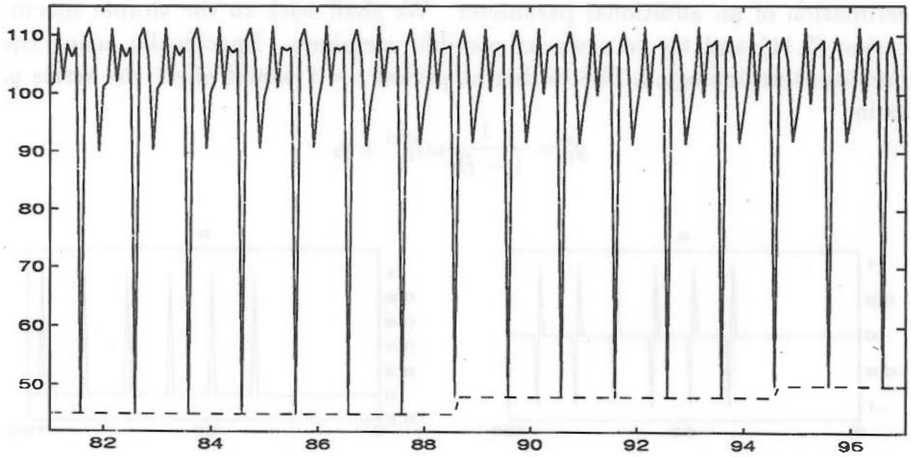

Figure 2. Italian industrial production index (GIPI). Seasonal factors.

Several basic structures seem possible for a seasonal outlier. If S denotes the annual aggregation operator, in terms of the representation (2.6), the simplest structure would be to set

$$\xi(B) = 1/S; \quad S = 1 + B + \ldots + B^{s-1}; \tag{2.10}$$

which would generate a "purely seasonal" outlier of the type displayed in Figure 3a. It is seen that this seasonal effect does not properly capture the seasonal level shift we aim to capture. An immediate extension of (2.10) is to set

$$\xi(B) = 1/\nabla_s, \tag{2.11}$$

which generates the outlier of Figure 3b. This type of outlier seems quite appropriate for our purpose, and is the one we should use. The outlier effect, however, is not anymore purely seasonal. From the factorization $\nabla_s = \nabla S$, the polynomial S will affect the seasonal component, but the ∇ factor will affect the trend. This is apparent from Figure 3b; because the sum of 12 (deterministic) seasonal outlier effects should be zero, the outlier effect for the last years of the figure have to be centered around zero. Therefore, the Seasonal Level Shift (SLS) outlier given by (2.11) has an effect on the trend given by the step function of Figure 3c, and an effect on the seasonal component shown in Figure 3d. Notice that both effects are permanent (due to the unit roots of ∇_s,) though not explosive.

More general expressions for $\xi(B)$ are obviously possible. An example could be

$$\xi(B) = (1 - \alpha B)/\nabla_s,$$

which, for $\alpha = 1$ yields (2.10), and for $\alpha = 0$ yields (2.11); it implies, however, estimation of an additional parameter. We shall stick to the simple specification (2.11) and try to asses its possible usefulness. Specifically. using the previous terminology, a SLS outlier at period $t = \tau$ would affect the series y_t as in

$$y_t^* = \frac{1}{1 - B^s}\omega I_t^{(\tau)} + y_t.$$

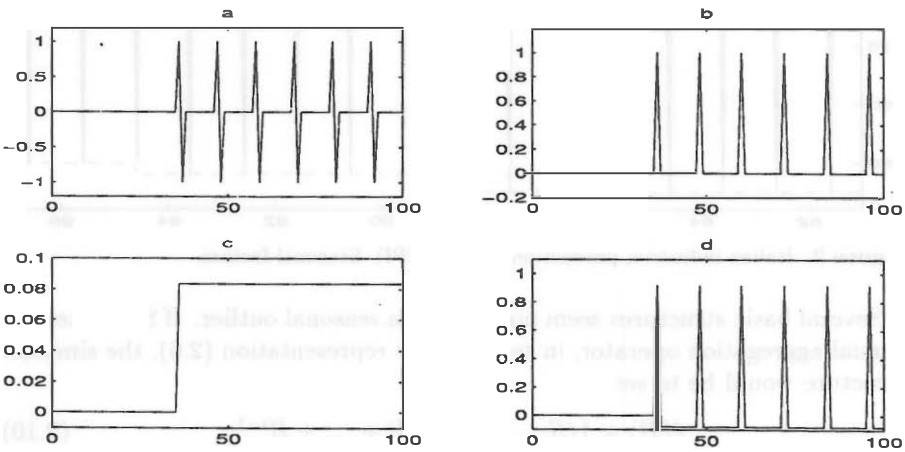

Figure 3. Seasonal Outliers effect. a) purely seasonal outlier b) Seasonal Level Shift c) effect of a SLS on the trend-cycle component d)effect of a SLS on the seasonal component

3 Detection and estimation

In this section we discuss how the iterative detection procedure described in Tsay (1986) and Gómez and Maravall(1998), to be denoted "standard procedure" can be extended to allow for both detection and estimation of SLS outliers ("extended procedure").

3.1 A single outlier

Let α be the vector of parameters in model (2.1) and let us suppose, for the moment, that it is known. Further, suppose that the observed series is subject to the influence of a perturbation at time $t = \tau$ such that,

$$y_t^* = \xi(B)\omega I_t^{(\tau)} + y_t, \tag{3.1}$$

where we first assume that the model $\tilde{\phi}(B)y_t = \tilde{\theta}(B)a_t$ is stationary. Model (3.1) can be rewritten as a linear regression model as follows,

$$y_t^* = Z_t^*(\tau)\omega + y_t \tag{3.2}$$

where $Z_t^*(\tau) = \xi(B)I_t^{(\tau)}$ is an $N \times 1$ vector. Let $\mathbf{y}^* = (y_1^*, \ldots, y_N^*)'$; $\mathbf{y} = (y_1, \ldots, y_N)'$ and $\mathbf{Z}^* = (Z_1^*(\tau), \ldots, Z_N^*(\tau))'$. Writing (3.2) in matrix terms yields,

$$\mathbf{y}^* = \mathbf{Z}^*\omega + \mathbf{y}. \tag{3.3}$$

The model in (3.3) is a regression model with autocorrelated residuals and, therefore, the problem of estimating ω can be solved by Generalized Least Squares (GLS). Let $var(\mathbf{y}) = V_a\mathbf{\Omega}$ with $\mathbf{\Omega}$ a $N \times N$ matrix which depends on α and which is assumed to be positive definite; and let $\mathbf{\Omega} = \mathbf{L}'\mathbf{L}$ be the Cholesky decomposition of $\mathbf{\Omega}$ with $\mathbf{L}$ lower triangular. Premultiplying (3.3) by $\mathbf{L}^{-1}$, and setting $\mathbf{e}^* = \mathbf{L}^{-1}\mathbf{y}^*$, $\mathbf{Z} = \mathbf{L}^{-1}\mathbf{Z}^*$ and $\mathbf{e} = \mathbf{L}^{-1}\mathbf{y}$, we obtain the Ordinary Least Squares (OLS) model,

$$\mathbf{e}^* = \mathbf{Z}\omega + \mathbf{e}, \tag{3.4}$$

where $var(\mathbf{e}) = V_a\mathbf{I}_N$. The OLS estimator of ω and its variance are obtained from (3.4) as,

$$\hat{\omega} = (\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{e}^* \qquad var(\hat{\omega}) = (\mathbf{Z}'\mathbf{Z})^{-1}V_a. \tag{3.5}$$

As argued in Gómez and Maravall (1994), to move from the GLS model in (3.3) to the OLS model in (3.4), there is no need to evaluate the matrix $\mathbf{\Omega}$, since the application of the Kalman filter on the observed series $\mathbf{y}^*$. yields the vector of standardized residuals $\mathbf{e}^* = \mathbf{L}^{-1}\mathbf{y}^*$. Similarly, the application of the same filter on vector $\mathbf{Z}^*$ provides the vector $\mathbf{Z} = \mathbf{L}^{-1}\mathbf{Z}^*$ from which (3.5) can be computed.

To test the null hypothesis that the observation at time $t = \tau$ is not an outlier, one can use the standardized statistic,

$$\lambda = \frac{\hat{\omega}}{\sqrt{var(\hat{\omega})}} \tag{3.6}$$

which, for known α, follows a standard normal distribution. By setting appropriate starting conditions (see, for example, Khon and Ansley, 1985, Bell and Hillmer, 1991, or Gómez and Maravall, 1994), the previous scheme extends in a straightforward manner to nonstationary series, for which y_t follows model (2.1).

If the objective of the analysis is to determine the type of the outlier at time $t = \tau$, one possibility, suggested by Chang et al. (1988), is to calculate the estimates $\hat{\omega}_i$ and their respective statistics λ_i, where the subscript i makes reference to the outlier type, i=AO,IO,LS,TC,SLS. The test statistic to use is

$$\eta(\tau, I) = \max_i \{| \lambda_i |\}. \tag{3.7}$$

If $\eta(\tau, I) > C$, where C is a predetermined critical value, then it is thought possible that the observed series is subject to the influence of an outlier of type i=I at time $t = \tau$.

The timing τ is seldom known a priori, but as suggested by Chang et al. (1988), the likelihood ratio criterion leads to,

$$\eta_{T,J} = \max_t \eta(\tau_t, I_t) \qquad t = 1, \ldots, N, \tag{3.8}$$

where T denotes the period at which the maximum of $\eta(\tau_t, I_t)$ occurs and J the associated type of outlier. Then, if $\eta_{T,J} > C$, there is a possibility that the observed series is subject to the influence of one outlier of type J at time $t = T$. In order to compute the $\eta_{T,J}$ statistic above, the Kalman filter should be run on the vector of observations to obtain the vector $\mathbf{e}^*$, and on $N \times 5$ different $\mathbf{Z}_i^*(\tau_t)$ vectors. for $i = AO, IO, LS, TC, SLS$ and $t = 1, \ldots, N$. The procedure is thus computationally cumbersome. This problem can be overcome by using the filter $\Pi(B) = \bar{\phi}(B)/\bar{\theta}(B)$, appropiately truncated. The GLS estimator of ω can be computed by using the Kalman filter to obtain the vector of exact residuals $\mathbf{e}^*$, and then the truncated filter $\Pi(B)$ can be applied on $\mathbf{Z}^*$ to obtain the vector $\mathbf{Z}$.

Once the location and the type of the outlier are determined, its effect can be adjusted from the residuals using (3.4); the adjusted series can also be obtained as

$$y_t = y_t^* - \xi(B)\omega I_t^{(\tau)}. \tag{3.9}$$

In practice, the true parameters in α are usually unknown in the modelling stage, although they can be estimated consistently; the λ -statistic given by (3.6) still has in this case an asymptotic normal distribution (see Chang et al., 1988).

3.2 Multiple outliers

In a more general framework, one can consider the observed series as being affected by k deterministic shocks at times $t = \tau_1, \ldots, \tau_k$. In this case, the representation of y_t^* consists of,

$$y_t^* = \sum_{j=1}^{k} Z_{j,t}^*(\tau_j)\omega_j + y_t. \tag{3.10}$$

where $Z_{j,t}(\tau_j) = \xi_j(B)I_t^{(\tau_j)}$ represents the effect of the outlier at time $t = \tau_j$. The extended iterative procedure described in the Appendix does not detect the k outliers at the same time but proceeds in several iterations detecting them one by one. In the detection stage, the procedure starts by applying the Kalman filter on the vector of observations to obtain the residuals and the truncated filter $\Pi(B)$ on the vectors $Z_{j,t}^*(j)$ to determine the location and type of the k outliers in (3.10). Following Chen and Liu (1993), once the detection stage is completed, in order to avoid possible masking effects, the final ω_js are obtained within the following multiple regression model,

$$\mathbf{y}^* = \mathbf{Z}^*\omega + \mathbf{y},$$

where $\mathbf{Z}^*$ is an $N \times k$ matrix with columns $\mathbf{Z}_j^*(\tau_j) = (Z_{j,1}^*(\tau_j), \ldots, Z_{j,N}^*(\tau_j))$ and ω is a $k \times 1$ vector with elements ω_j. The application of the Kalman filter recursions on the vector of observations $\mathbf{y}^*$ and on the k columns of the matrix $\mathbf{Z}^*$ allows the specification of an OLS model, from which the vector ω can be estimated as in (3.5). Kohn and Ansley (1985) proposed an efficient way to estimate the vector ω using the QR algorithm, in which an orthogonal $N \times k$ matrix $\mathbf{Q}$ is obtained, such that $\mathbf{Q}'\mathbf{L}^{-1}\mathbf{Z}^* = (\mathbf{R}', \mathbf{0})'$, where $\mathbf{R}$ is a non-singular $k \times k$ upper-triangular matrix. Then $\hat{\omega} = \mathbf{R}^{-1}\mathbf{v}_1$, where $\mathbf{v}_1$ consists of the first k elements of the vector $\mathbf{v} = \mathbf{Q}\mathbf{L}^{-1}\mathbf{y}^*$.

Once $\hat{\omega}$ is obtained, residuals are identified and corrected, the linear series obtained, a new estimator of α computed, and iterations proceed as described in the Appendix.

4 Performance of the extended procedure

In this section, we briefly investigate the performance of the standard procedure when it is extended to include the SLS outlier type. We focus our analysis on two aspects of the procedure: i) the relative frequency of detection of at least one outlier while no one is effectively present, which is a measure of a type I error; and ii) the relative frequency of correct detection, which is a measure of the power. The simulations in this section were performed using the monthly "Airline model", popularized by Box and Jenkins (1970), and found to be very often appropriate for series displaying trend and seasonality (see the large-scale study in Fischer and Planas, 1998). The model is given by

$$\nabla\nabla_{12}y_t = (1+\theta_1 B)(1+\theta_{12}B^{12})a_t. \tag{4.1}$$

with a_t being a white-noise innovation, $-1 < \theta_1 < 1$, and $-1 < \theta_{12} < 0$. The performance of the extended procedure in terms of type I errors was found to be associated with: i) the number of observations; and ii) the critical value. We performed a simulation in MATLAB, whereby, for each of the considered samples sizes (N=50,100, 200 or 400), 1000 noncontaminated "Airline" series were generated with $\theta_1 = \theta_{12} = -0.6$; the extended procedure, described in the Appendix. was then applied using critical values C=3.0,3.5,4.0 and 4.5. Table 1 gives the mean relative frequency of a type I error, decomposed into the different outlier types. for each combination of sample size and critical value.(Note that the total mean relative frequency of a type I error is the sum of columns 3 to 7 for C=3.0, 3.5 and the sum of columns 9 to 13 for C=4.0, 4.5)

		Outlier type						Outlier type				
		AO	IO	LS	TC	SLS		AO	IO	LS	TC	SLS
N=50	C=3.0	.05	.05	.07	.06	.08	C=4.0	.00	.00	.00	.00	.00
100		.13	.10	.11	.09	.16		.00	.00	.00	.00	.00
200		.16	.16	.17	.15	.22		.01	.01	.01	.00	.01
400		.18	.17	.21	.18	.24		.02	.03	.02	.01	.02
N=50	C=3.5	.01	.00	.01	.01	.01	C=4.5	.00	.00	.00	.00	.00
100		.03	.02	.02	.02	.01		.00	.00	.00	.00	.00
200		.06	.04	.06	.06	.06		.00	.00	.00	.00	.00
400		.11	.08	.10	.11	.12		.00	.00	.00	.00	.00

Table 1. Type I error in the extended iterative procedure

As expected, the relative frequency of a type I error is a decreasing function of the critical value, but an increasing function of the number of observations. When the critical value is too low for the sample size, an SLS outlier is spuriously detected with a mean relative frequency slightly higher than for the other outlier types. Nevertheless, when the critical value is adequate for the sample size ($C \geq 3.5$ for N=50, 100 and $C \geq 4.0$ for N=200,400), the mean relative frequency of a type I error is, for all cases, smaller than 5%

and there are not significant differences among the outlier types. In a second simulation exercise, the influence of parameters θ_1 and θ_{12} on the type I error was investigated. We considered values of $\theta_1 = 0.5, 0$ and -0.5 and $\theta_{12} = -0.1, -0.3, -0.5, -0.7$ and -0.9. As before, 1000 series of 100 observations each were generated. The results for this simulation exercise indicated that the relative frequency of a type I error is (almost) insensitive to changes in the parameters θ_1 and θ_{12}. (These results are not reported here but are available from the authors.)

Next, to investigate the power of the extended iterative procedure in terms of outlier detection, we study the relative frequency of correct detection (type and location are correctly identified) of one outlier. The "Airline" model with $\theta_1 = -0.6$ and $\theta_{12} = -0.1, -0.3, -0.5, -0.7, -0.9$ is used to generate simulated series of 100 observations. We consider one outlier affecting the observed series at the middle of the sample, $t = 50$. (Simulation of more than one outlier and different locations can be found in Kaiser, 1998.) The size of the initial impact ω is considered equal to 4 or 5. For each combination of outlier types and parameters θ_{12}, 1000 simulated series were generated. Table 2 reports the mean relative frequency of correct detection for critical values C=3.5 and C=4.0.

		Outlier type									
		AO		IO		LS		TC		SLS	
		C=3.5	4.0	3.5	4.0	3.5	4.0	3.5	4.0	3.5	4.0
θ_{12}=-.1	ω=4	.97	.93	.51	.32	.99	.99	.96	.94	.67	.50
	5	1	1	.79	.66	1	1	.99	99	.88	.88
-.3	4	.95	.85	.50	.33	.98	.95	.92	.85	.74	.60
	5	.99	.98	.79	.66	.99	.99	.98	.98	.92	.91
-.5	4	.87	.77	.50	.32	.95	.90	.87	.73	.90	.79
	5	.98	.97	.79	.66	.99	.99	.97	.95	.98	.98
-.7	4	.79	.62	.49	.31	.91	.84	.77	.65	.99	.97
	5	.95	.90	.78	.65	.99	.98	.93	.93	1	1
-.9	4	.69	.54	.44	.27	.87	.74	.71	.52	1	1
	5	.90	.86	.71	.58	.97	.95	.90	.84	1	1

Table 2. Mean relative frequency of correct detection using the extended procedure

The power of the extended procedure increases with the size of the initial impact and decreases with the critical value. Table 2 shows that the power is a decreasing function of the absolute value of the parameter θ_{12} for all outlier types except for the SLS. What Table 2 implies is that, when the seasonal component is highly erratic, (θ_{12} is close to zero), the outliers assigned to the trend or to the irregular component are correctly detected with a higher frequency than the SLS type. As the seasonal component approaches stability, the mean relative frequency of correct detection of an SLS increases. For the range $\theta_{12} = -0.6$ to -0.9, which is the value most often found in practice (see Fischer and Planas, 1998), the power of the iterative procedure in correctly detecting one SLS is superior to the power in detecting any other outlier type. The most insensitive outlier type is the IO, which is also the type for which the worst results for the power of the procedure are obtained (always less than 80% and, in 7 out of 20 cases, less than 50%).

5 Consequences of not including the SLS type

In this section, we compare the performance of the standard iterative procedure with that of the extended procedure when the observed series is subject to the presence of one SLS.

Using the monthly Airline model in (4.1) with $\theta_1 = -0.6$ and θ_{12} in the range [-0.1, -0.9], 1000 series of 100 observations each were generated for each combination of θ_1 and θ_{12}. Next, the series were contaminated at time $t = 50$ with one SLS of size $\omega = 4$ or $\omega = 5$ and the standard iterative procedure was applied three times with critical values C=3.5, C=4.0 and C=4.5. Table 3 presents the mean relative frequency of wrongly detecting one IO (columns labeled IO) or of not detecting any outlier (columns labeled NO). Notice that the relative frequency of detecting other outlier types, different from an IO, is extremely low.

		θ_{12}									
		-.1		-.3		-.5		-.7		-.9	
C	ω	NO	IO	NO	IO	NO	IO	NO	IO	NO	IO
3.5	4	.37	.56	.41	.58	.50	.50	.55	.45	.73	.26
	5	.11	.83	.14	.84	.17	.82	.28	.72	.53	.46
4.0	4	.65	.32	.68	.31	.69	.31	.80	.20	.90	.10
	5	.30	.67	.33	.66	.40	.60	.51	.49	.81	.19
4.5	4	.86	.13	.88	.12	.90	.10	.92	.08	.98	.02
	5	.56	.42	.58	.41	.67	.33	.79	.21	.97	.03

Table 3. Performance of the standard procedure for a SLS-contaminated series

The results in Table 3 indicate that, when the time series is subject to the presence of one SLS, the standard outlier detection procedure performs poorly. The mean relative frequency of wrongly detecting one IO is high, being larger than 50% in 9 out of 30 cases; this frequency increases with the size of the initial impact, ω. Table 3 shows a trade-off between not detecting any outlier or detecting one IO. This trade-off depends on the value of parameter θ_{12} and, therefore, on the stability or instability of the seasonal component. When the seasonal component is highly erratic, it is more likely to detect one IO than nothing and, in contrast, when the seasonal component is very stable, the procedure, most often will not detect any outlier.

We now compare the estimates of the parameter θ_{12} when the "cleaning" or preadjustment of the observed series is carried out using the standard procedure or the extended procedures. For this comparison, we generated 1000 Airline series of 100 observations each, with $\theta_1 = -0.6$ and $\theta_{12} = -0.5$, and then included one SLS of size $\omega = 5$ at time t=50. Table 4 presents the first two moments of the distribution for $\hat{\theta}_{12}$ when the SLS is correctly adjusted (using the extended procedure) and, when it is treated as an IO or not adjusted (using the standard procedure). Figure 4 compares both densities.

	θ_{12}		σ_a	
	Mean	Std. Error	Mean	Std. Error
True parameter	-.500	-	1.000	-
Standard procedure	-.330	.097	1.081	.096
Extended procedure	-.536	.133	.977	.082

Table 4. θ_{12} and σ_a estimates for a SLS contaminated series

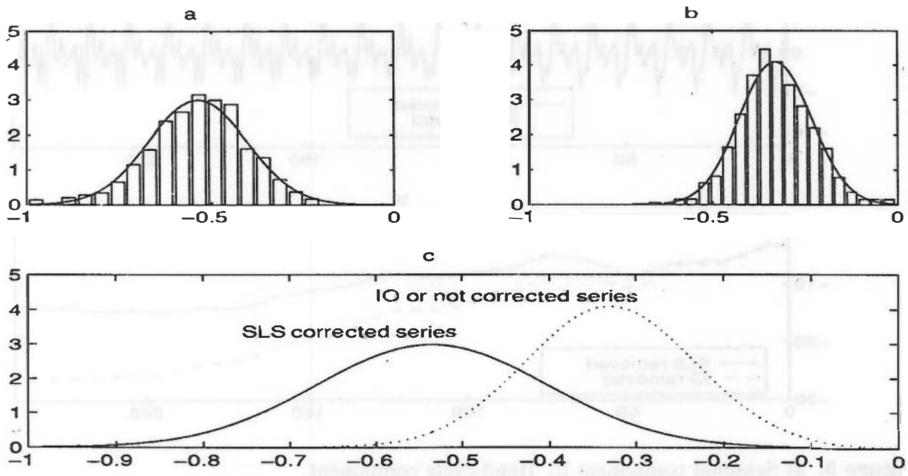

Figure 4. Densities for θ_{12} estimates. a) Histogram and normal approximation for the Extended procedure; b) Histogram and normal approximation for the Standard procedure; c) Comparison of the two normal approximations.

In the two top panels of Figure 4 the two histograms are displayed and approximated by the normal distribution. In the bottom panel, the two normal approximations are plot together, and it is seen that the density for the standard procedure is strongly biased. The estimator of θ_{12} under the extended procedure still contains some bias, although of a smaller size. The conclusion to this exercise is, therefore, that ignoring or missidentifying one SLS leads to an important bias in the estimation of the parameter θ_{12} towards the region of unstable seasonality, which would have the effect of yielding estimates of the seasonal component unreasonably erratic. The first two moments of the distribution of $\hat{\sigma}_a$ (the residual standard error) obtained with the standard and the extended procedures are also displayed in Table 4. It is seen that, on average, $\hat{\sigma}_a$ for the standard procedure is higher than for the extended one (in fact, the frequency of replications for which $\hat{\sigma}_a$ is higher for the former than for the latter is close to 80%).

One important consequence of fitting IO instead of SLS outliers is the effect on the underlying components of the series. Figure 5 displays the estimates of the stochastic seasonal and trend-cycle components for one of the series of the previous simulation, when an SLS outlier has been treated as an IO. Removal of the IO outlier produces a more unstable seasonal and strongly affects the trend-cycle component.

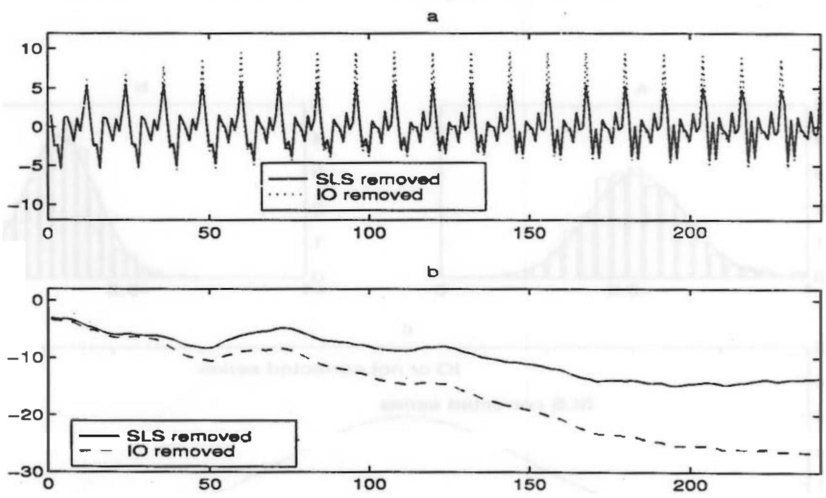

Figure 5. a) Seasonal component b) Trend-cycle component.

Finally, we investigate the consequences of ignoring or missidentifying one SLS on some diagnostics for the model. In particular, we consider the effects on the residual autocorrelation ρ_{12} and on the Box-Ljung statistic Q(36) for the residuals. Using the contaminated series that were generated for the simulation exercise of Table 3, the standard and extended iterative procedures were applied with critical value C=4.0. The values of $\hat{\rho}_{12}$ and Q(36) obtained for the residuals with the two procedures are compared in Figures 6 and 7. Each point in Figure 6 is a representation of the pair of values ($\hat{\rho}_{12}^{SLS}$, $\hat{\rho}_{12}^{IO}$) and each point in Figure 7 is a representation of the pair of values ($Q(36)^{SLS}$, $Q(36)^{IO}$). Points on the straight line (x=y) indicate cases for which the two procedures lead to the same value of $\hat{\rho}_{12}$ or Q(36); values over the line indicate cases for which the standard procedure leads to values of $\hat{\rho}_{12}$ or Q(36) larger than for the extended procedure. Figures 6 and 7 indicate that, as seasonality becomes more stable, the ρ_{12} and Q estimates obtained with the two procedures tend to diverge. The use of the standard iterative procedure, when an SLS is present on the observed series, may lead to significant $\hat{\rho}_{12}$ coefficients for the residuals, indicating that not all seasonality has been removed, and to high values of the Ljung-Box statistics, indicating the presence of autocorrelation in the residuals.

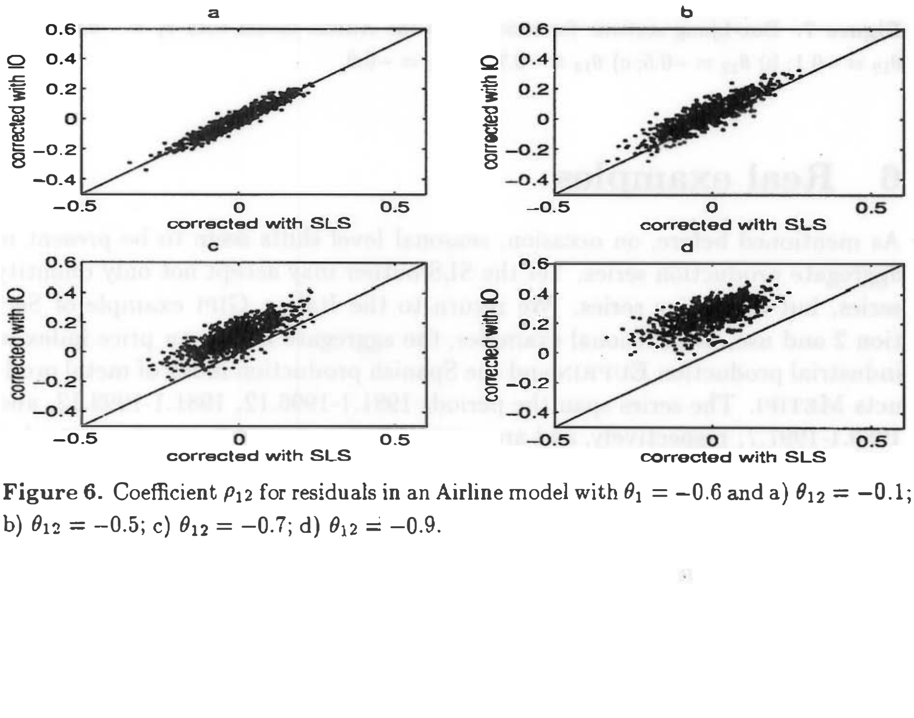

Figure 6. Coefficient ρ_{12} for residuals in an Airline model with $\theta_1 = -0.6$ and a) $\theta_{12} = -0.1$; b) $\theta_{12} = -0.5$; c) $\theta_{12} = -0.7$; d) $\theta_{12} = -0.9$.

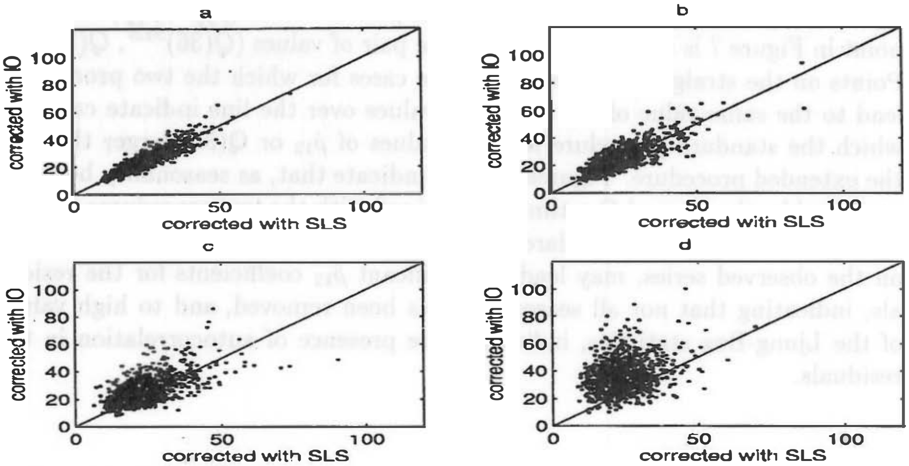

Figure 7. Box-Ljung statistic for residuals in the Airline model with $\theta_1 = -0.6$ and a) $\theta_{12} = -0.1$; b) $\theta_{12} = -0.5$; c) $\theta_{12} = -0.7$; d) $\theta_{12} = -0.9$.

6 Real examples

As mentioned before, on occasion, seasonal level shifts seem to be present in aggregate production series. Yet the SLS outlier may accept not only quantity series, but also price series. We return to the Italian GIPI example of Section 2 and use, as additional examples, the aggregate European price index of industrial production EUPRIN and the Spanish production index of metal products METIPI. The series span the periods 1981.1-1996.12, 1981.1-1993.12, and 1980.1-1997.7, respectively, and are represented in Figure 8, panels a, c and e. ARIMA identification of the three series (made with the program TRAMO run in the automatic mode) results in the identification of an ARIMA (0,1,1)(0,1,1) with significant trading and Easter effects for the GIPI and METIPI series, and an ARIMA (0,2,1)(0,1,1) for the EUPRIN series. The three original series were decomposed (using the program SEATS in an automatic mode) into unobserved components of trend-cycle, seasonal and irregular. Panels b, d and f in Figure 8 represent the three series of seasonal factors; it is seen that the three series are subject to some seasonal heteroscedasticity.

The standard procedure (included in the program TRAMO) was first applied with a critical value of C=3.5, the recommended (and default) value for medium sensitivity in series of moderate length. The detected outliers are listed in Table 5. The standard procedure finds two AOs in August for GIPI, only one IO in January of 1986 for EUPRIN, and three AOs and two IOs in August for METIPI .

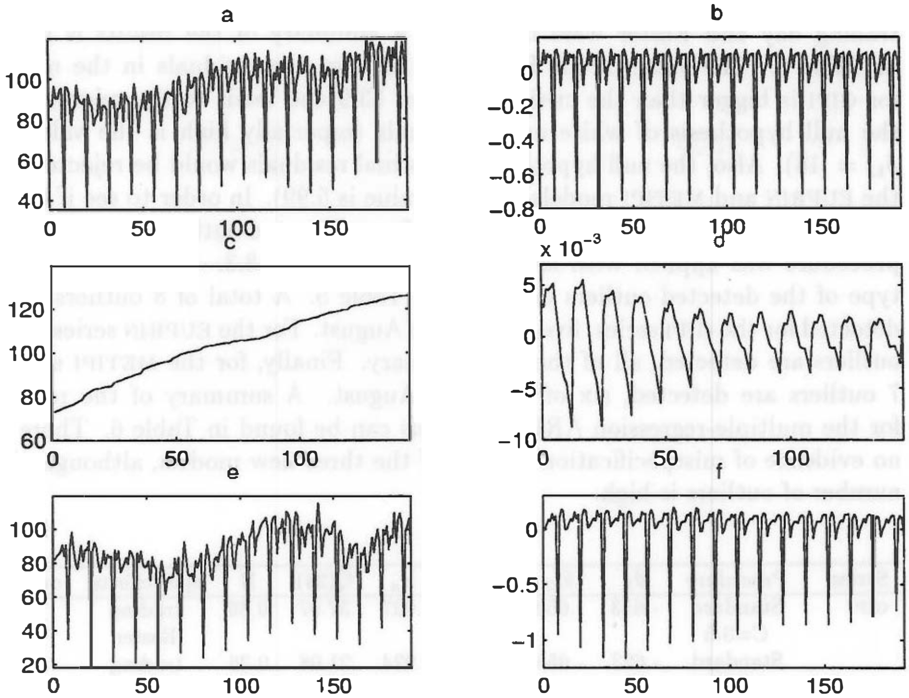

Figure 8. Italian industrial production index a) Original series b) Seasonal factors; European price index for industrial production . c) Original series d) Seasonal factors; Spanish industrial production index of metal products. e) Original series; f) Seasonal factors

Series	Procedure	C	Type	Date of Detected outliers
GIPI	Standard	3.5	AO	1984.8, 1995.8
	Standard	3.3	AO	1984.8, 1995.8, 1987.1,1990.8,1984.4
			TC	1989.8,1992.12
			IO	1994.8
	Extended	3.5	AO	1984.8, 1987.1
			SLS	1994.8, 1988.8
EUPRIN	Standard	3.5	IO	1986.1
	Standard	3.3	IO	1986.1, 1985.1, 1987.1
			LS	1988.1
	Extended	3.5	SLS	1985.12, 1986.12
METIPI	Standard	3.5	AO	1980.8, 1983.8, 1990.8
			IO	1985.8, 1994.8
	Standard	3.3	AO	1980.8, 1983.8, 1990.8,1982.8
			TC	1982.12
			IO	1985.8, 1994.8
	Extended	3.5	AO	1990.8
			SLS	1981.8, 1984.8, 1994.8

Table 5. Detected outliers using the standard and the extended procedures

Multiple regression-ARIMA models were fit, such that effects for outliers, trading day and Easter were included. A summary of the results is given in Table 6. The value of the Box-Ljung test for the residuals in the model for GIPI is bigger than the critical value of 33.9 and leads to the rejection of the null hypothesis of white noise residuals (especially high is the value of $\hat{\rho}_{12} = .18$). Also, the null hypothesis of normal residuals would be rejected for the EUPRIN and METIPI models (critical value is 5.99). In order to see if these misspecifications are due to the presence of non-detected outliers, the standard procedure was applied with lowered critical value C=3.3. The location and type of the detected outliers are listed in Table 5. A total of 8 outliers were detected for the GIPI series, five of them for August. For the EUPRIN series, four outliers are detected, all of them for January. Finally, for the METIPI series, 7 outliers are detected, six of them for August. A summary of the results for the multiple-regression ARIMA models can be found in Table 6. There is no evidence of misspecification for any of the three new models, although the number of outliers is high.

Series	Procedure	θ_1	θ_{12}	BIC	σ_a	Q(24)	N	spec.effects	outliers
GIPI	Standard C=3.5	-.613	-.650	-7.09	.027	37.97	0.86	trading Easter	2
	Standard C=3.3	-.623	-.655	-7.22	.024	21.08	0.36	trading Easter	8
	Extended C=3.5	-.602	-.781	-7.22	.024	29.58	0.33	trading Easter	4
EUPRIN	Standard C=3.5	-.461	-.535	-12.62	.002	16.00	7.03	no	1
	Standard C=3.3	-.462	-.594	-12.75	.001	16.72	1.64	no	4
	Extended C=3.5	-.339	-.729	-12.81	.001	13.76	1.34	no	2
METIPI	Standard C=3.5	-.552	-.324	-5.48	.062	15.36	6.87	trading Easter	5
	Standard C=3.3	-.541	-.406	- 6.06	.043	21.04	4.03	trading Easter	7
	Extended C=3.5	-.503	-.654	- 6.09	.043	25.09	4.42	trading Easter	4

Table 6. Summary of ARIMA estimation results.

The extended procedure, including the SLS outlier type, was then applied with the default value for the critical level (C=3.5). The location and type of the detected outliers can also be found in Table 5. In the three cases, SLS outliers are detected. The aggregate effect of the outliers detected using the two (standard with lowered critical value and extended) procedures is represented in Figure 9. The results for the estimation of the ARIMA models are summarized in Table 6. There is no evidence of misspecification in any of the new models; the θ_{12} estimates are closer to -1 indicating more stable seasonal components, which possibly result from a better modelling of the seasonal outlier. This effect can be seen in Figure 10 which compares the stochastic seasonal component obtained with the standard and the extended procedures. For the EUPRIN and METIPI series, the BIC criterion indicates preference for the model with SLS outliers; for the GIPI series, the BIC criterion do not distinguish between the two models. Comparing Figures 8 and 10 it is seen how the extended procedure has removed heterocedasticity from the seasonal component, which behaves now in a more regular manner.

To further compare the models, two out-of-sample forecasting exercises were performed. First, for each of the three series, multiple regression-ARIMA models were re-estimated with 20 fewer observations (C=3.3 for the standard procedure; C=3.5 for the extended one). One-period-ahead forecasts were then computed for the last 20 months. Table 7 gives, for the two models of each series, the out-of-sample variance and the value of the F-test for the equality of the variances (approximate critical value 1.6). The comparison of these magnitudes reveals that the models that include SLS outliers have: (1) smaller out-sample variance; and (2) smaller values for the F-test.

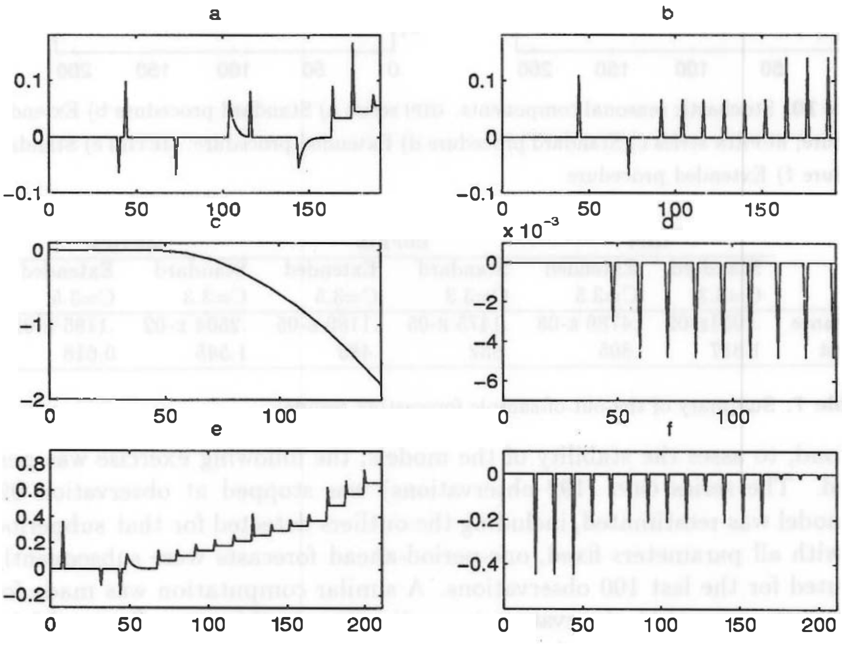

Figure 9. Effect of detected outliers on the observed series. GIPI series a) Standard procedure b) Extended procedure; EUPRIN series c) Standard procedure d) Extended procedure; METIPI e) Standard procedure f) Extended procedure

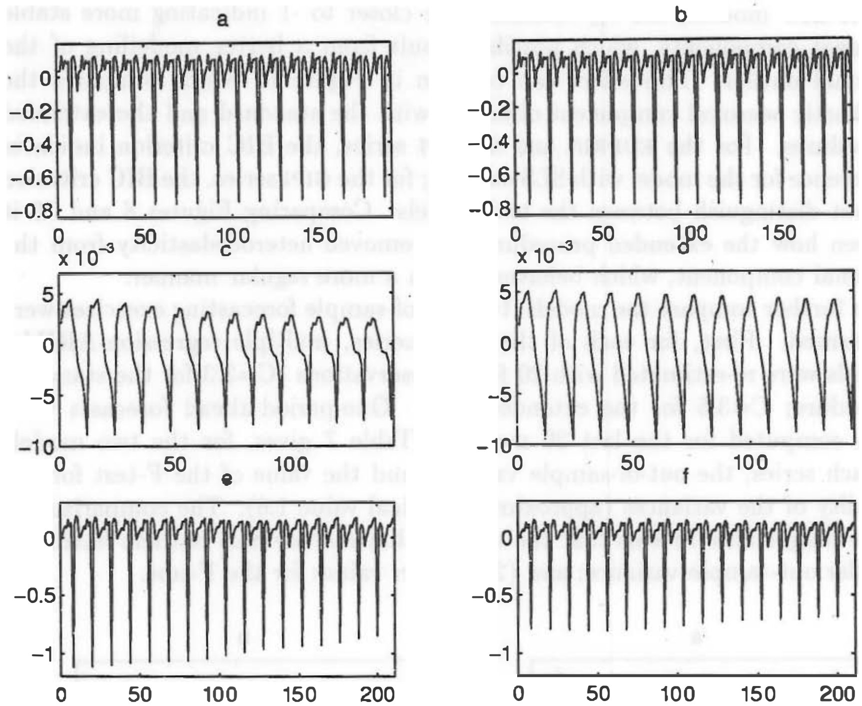

Figure 10. Stochastic seasonal components. GIPI series a) Standard procedure b) Extended procedure; EUPRIN series c) Standard procedure d) Extended procedure: METIPI e) Standard procedure f) Extended procedure

	GIPI		EUPRIN		METIPI	
	Standard C=3.3	Extended C=3.5	Standard C=3.3	Extended C=3.5	Standard C=3.3	Extended C=3.5
Variance	.1051E-02	.4780 E-03	.1475 E-05	.1189 E-05	.2504 E-02	.1185 E-02
F-test	1.817	.805	.632	.483	1.545	0.618

Table 7. Summary of the out-of-sample forecasting results.

Second, to asses the stability of the models, the following exercise was performed. The series GIPI (192 observations) was stopped at observation 92. The model was reestimated, including the outliers detected for that subperiod and, with all parameters fixed, one-period-ahead forecasts were subsequently computed for the last 100 observations. A similar computation was made for the EUPRIN series (144 observations), reestimating in this case, the model for the first 64 observations, and obtaining the one-period-ahead forecast errors for the next 80 observations. Finally, for the METIPI series (211 observations), the model for the first 121 observations was reestimated, and one-period-ahead forecast errors were obtained for the next 90 observations. Table 8 compares the forecast standard errors, and Figure 11 the forecast errors. From the table, it is seen that the extended method decreases, in the three cases, the forecasts standard error, although the differences are small. Comparing the forecast standard errors for the subperiods with the residual standard error for the full sample given in Table 6, the proximity of the two is remarkable. This model stability is further confirmed by Figure 11, which displays (considering the length of the out-of-sample forecasting period) very well-behaved errors. Although moderate, the superiority of the extended approach is noticeable.

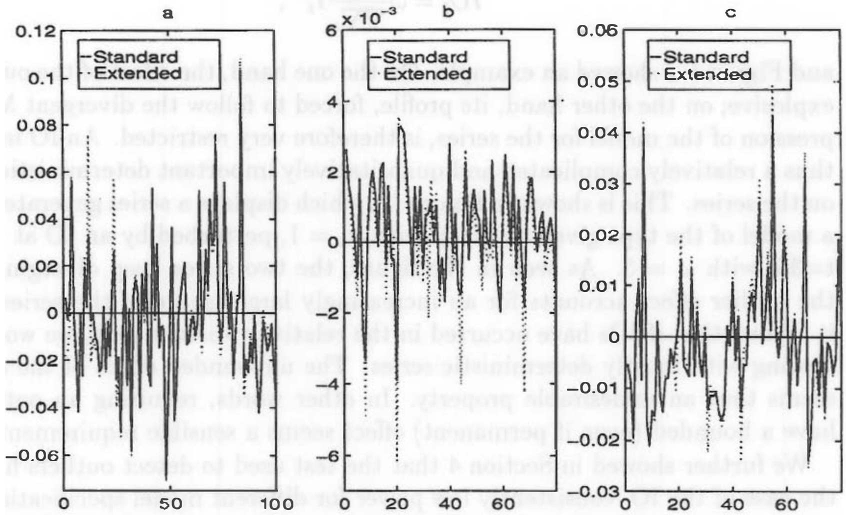

Figure 11. One-period-ahead forecast errors a)series GIPI b) series EUPRIN c) series METIPI

Series	Procedure	Forecast SE
GIPI	Standard	.0288
	Extended	.0276
EUPRIN	Standard	.0018
	Extended	.0015
METIPI	Standard	.0139
	Extended	.0125

Table 8. Standard error of the one-period-ahead forecast.

The three real examples in this section suggest that the consideration of the SLS outlier in the extended outlier detection procedure yields more parsimonious models, more stable seasonal components, and better forecasting performance. The improvements are not dramatic, but they appear to be quite general.

7 A final remark: innovational versus seasonal outlier

With series containing trend and seasonality, that follow, for example, a model of the type

$$\nabla\nabla_s y_t = \theta(B)a_t, \tag{7.1}$$

where $\theta(B)a_t$ is a stationary process, the IO has the form

$$IO_t = \omega\frac{\theta(B)}{\nabla\nabla_s}I_t^{(\tau)},$$

and Figure 1.d showed an example. On the one hand, the effect of the outlier is explosive; on the other hand, its profile, forced to follow the divergent MA expression of the model for the series, is therefore very restricted. An IO imposes thus a relatively complicated and quantitatively important deterministic effect on the series. This is shown in Figure 12, which displays a series generated with a model of the type given by (7.1), with $\sigma_a = 1$, perturbed by an IO at period t=36, with $\omega = 5$. As seen in the figure, the two series keep diverging, and the outlier effect accounts for an increasingly larger share of the series level; it follows that if IOs have occurred in the relatively distant past, we would be dealing with mostly deterministic series. The unbounded effect of the outlier seems thus an undesirable property. In other words, requiring an outlier to have a bounded (even if permanent) effect seems a sensible requirement.

We further showed in Section 4 that the test used to detect outliers has, for the case of the IO, consistently low power for different model specifications, in particular, considerably lower than for the other types of outliers: AO, LS, TC or SLS (further limitations of innovational outliers in the time series context are discussed in Peña, 1990). It is a fact, besides, that IO may produce, on occasion, unattractive decomposition of series and, for example, by default, IOs are removed from the automatic outlier detection and correction procedure of the TRAMO-SEATS methodology. Removing IOs from the procedure, however, presents a drawback: none of the remaining types of outliers is capable of explaining any seasonal structure.

Considering the examples of Section 6, in all cases, the standard procedure yields innovational outliers. The extended procedure (which simply adds the SLS outlier to the four considered by the standard method) besides a moderate (though general) improvement in the results and a more parsimonious outlier representation, yields no IO. In its place, SLS outliers appear. (We have checked with more examples that this feature happens often.) IO outliers present another relative disadvantage. If an IO were to perturbate the series under consideration, a combination of LS and SLS outliers should approximate

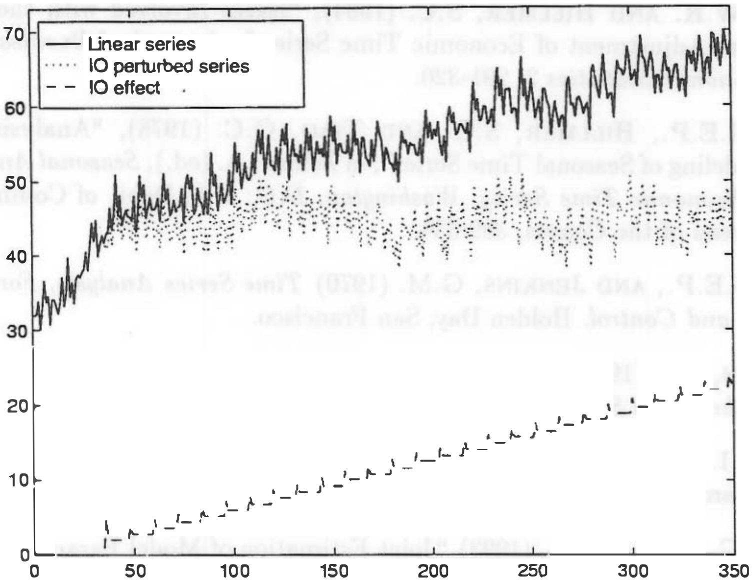

Figure 12. Linear and IO perturbated series.

it well. On the contrary, if an SLS outlier is present, its replacement by an IO outlier, as seen in Section 5, can be seriously damaging. All considered, a conclusion emerges: instead of complicating the standard method by adding an additional outlier, the procedure could be simply modified by replacing the IO outlier with the SLS one. In this way, all outlier effects are bounded, outliers are modelled in a more parsimonious manner, and diagnostics and forecasts are likely to improve. One last point should be mentioned: we have looked at the effect of a very particular outlier structure (associated with the polynomial ∇_s^{-1}). Other specifications may well be appropriate, and this is a point of further research.

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Appendix

A extended procedure for outlier detection

The procedure starts with the specification of the model for the observed series as if there were no outliers, and the following stages are followed:

I.1 Obtain the maximum likelihood estimators for the unknown parameters in vector α based on the vector of standardized residuals $\mathbf{e}^*$. In the first iteration, the residuals obtained from the application of the Kalman filter on the observed series are used; after the first iteration, the adjusted residuals are used to evaluate the likelihood function.

Detection inner loop

I.2 For $t = 1, \ldots, N$ and $i = AO, IO, LS, TC, SLS$, compute $\lambda_i(t)$ using (3.6), and the statistic $\eta(t) = max_i \{|\lambda_i(t)|\}$. If $\eta_\tau = max_t \eta(t) > C$, where C is a predetermined critical value, then there is a possibility of one outlier at time $t = \tau$.

In the presence of outlying observations, the standardized residuals, obtained with the Kalman filter, are contaminated and, hence, $\hat{\sigma}_a^2 = \{1/N\}\mathbf{e}'^*\mathbf{e}^*$ may be overestimated. One method to overcome this problem, which is not time consuming, is the omit-one method, in which $\hat{\sigma}_a^2$ is calculated, such that the residual at time $t = \tau$ is omitted. Other alternatives include the MAD method or the α % trimmed method, see Chen and Liu (1993).

I.3 If a possible outlier is found, remove its effect from the residuals and obtain the adjusted residuals $\mathbf{e}$ using,

$$\mathbf{e} = \mathbf{e}^* - \mathbf{Z}\hat{\omega},$$

and go back to I.2 to iterate. Otherwise, proceed to I.4.

I.4 If no outlier is found in the first iteration, then stop. If one or more outliers have been detected in the previous iterations from steps I.2-I.3, then go back to I.1 to revise the parameter estimates. Continue to repeat I.1-I.3 until no new outliers are found, then go to II.

Joint estimation stage

II The effects of the identified outliers are jointly estimated in the multiple regression model in (3.10) by applying the Kalman filter on the vector of observations and on each column in matrix $\mathbf{Z}$; and applying the QR algorithm on $\mathbf{Z}$. This provides an efficient estimator for vector ω. Compute the t-statistic for the estimated effects and check if there is any outlier for which the t-statistic is smaller than C, where C is the critical value used in I.2. If there are not, then obtain the adjusted series, check whether the initial specification of its ARIMA model is still valid, apply the Kalman filter on the adjusted series, obtain the new residuals and go back to stage I to repeat the complete process until no new outliers can be detected. Otherwise, delete one by one the insignificant effects and re-estimate the multiple regression model until all the ω_is are significant, then obtain the vector of adjusted observations, apply the Kalman filter on it, obtain the new residuals and go back to stage I to iterate until no new outliers can be detected.

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