RACSAM
Rev. R. Acad. Cien. Serie A. Mat.
VOL. **97 (**2), 2003, pp. 209–221
Análisis Matemático / Mathematical Analysis

On interpolation of tensor products of Banach spaces

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Dedicated to the memory of Klaus Floret

Abstract. For the complex interpolation method, Kouba proved an important interpolation formula for tensor products of Banach spaces. We give a partial extension of this formula in the injective case for the Gustavsson–Peetre method of interpolation within the setting of Banach function spaces.

Interpolación de productos tensoriales en espacios de Banach

Resumen. Para el método de interpolación complejo, Kouba probó una fórmula importante sobre productos tensoriales inyectivos de espacios de Banach. Nosotros damos una extensión parcial de esta fórmula para el método de interpolación de Gustavsson-Petree en el contexto de espacios de Banach de funciones.

1. Introduction

Recent progress in the local theory of Banach spaces allows to study interpolation between spaces of operators. The most striking result in this area is Kouba's [19] complex interpolation formula for tensor products of Banach spaces. A natural question that appears here is whether there are variants of Kouba's result for other methods of interpolation. Unfortunately, as one could expect, many difficulties appear, for instance a delicate problem related to the interpolation of bilinear operators.

The purpose of this note is to prove new abstract results on one-sided interpolation of injective tensor products of Banach spaces for the Gustavsson–Peetre method of interpolation. We believe that these results have interests in their own rights; one of our motivations for considering them was to use the powerful result of Kouba to prove extension and splitting theorems for Fréchet spaces of Rademacher type 2 (for details see [10]).

We now describe the main results of the paper. In Section 2 we introduce notations and basic facts. In Section 3 we prove a result (Lemma 1) which shows that under general assumptions on an exact interpolation functor the problem concerning interpolation of injective tensor products for Banach spaces can be reduced to finite-dimensional spaces.

In Section 4 we prove that for finite-dimensional Banach lattices the lower Ovchinnikov method of interpolation satisfies certain essential estimates, the key to apply Lemma 1. The proof involves bilinear interpolation theorems, Calderón-Lozanovsky spaces and the description of these spaces for couples of multipliers from the Hilbert sequence space into 2-concave Banach sequence spaces as well as well-known factorization theorems.

Presentado por José Bonet.
Recibido: 14 de Abril de 2003. *Aceptado*: 8 de Octubre de 2003.
Palabras clave / Keywords: interpolation, tensor products
Mathematics Subject Classifications: Primary 46M35; Secondary 46M05, 46E30

Finally in Section 5, we present applications of the obtained results for the Gustavsson–Peetre method of interpolation $\langle\cdot\rangle_\varphi$ generated by an appropriate function parameter φ. Our main result of this section states that under some mild assumptions on the function φ the following interpolation formula holds:

$$\langle X_0, X_1\rangle_\varphi \tilde{\otimes}_\varepsilon E = \langle X_0 \tilde{\otimes}_\varepsilon E, X_1 \tilde{\otimes}_\varepsilon E\rangle_\varphi,$$

where $\tilde{\otimes}_\varepsilon$ denotes the injective tensor product, (X_0, X_1) is a couple of 2-concave complex Banach function spaces and E a Banach space of cotype 2. For power functions $\varphi(s,t) = s^{1-\theta}t^\theta$, $0 < \theta < 1$, we have $\langle X_0, X_1\rangle_\varphi = [X_0, X_1]_\theta$ whenever (X_0, X_1) is a complex couple. Here as usual $[\cdot]_\theta$ denotes the complex method of interpolation (see [5]). Thus the obtained result is a partial extension of a well-known remarkable result due to Kouba: For a couple (X_0, X_1) of complex Banach function spaces and a complex Banach space E as above,

$$[X_0, X_1]_\theta \tilde{\otimes}_\varepsilon E = [X_0 \tilde{\otimes}_\varepsilon E, X_1 \tilde{\otimes}_\varepsilon E]_\theta.$$

In fact, Kouba even proved two-sided results of this type, and not only for ε but also for other norms as e.g. the projective norm. For further one-sided results on interpolation of spaces of operators and tensor products we refer to [7] and the references therein, where the authors deal with the real method of interpolation applied for a rather rare class of Banach couples called quasi-linearizable (for examples see, e.g., [3], pp. 464–465).

2. Preliminaries

We shall use standard notation and notions from Banach space theory, as presented e. g. in [16], [21], [22] and [31]; for tensor products of Banach spaces we refer to [9]. If E is a Banach space, then B_E is its (closed) unit ball and E' its dual, and $\mathcal{F}in(E)$ stands for the collection of all its finite-dimensional subspaces. As usual $\mathcal{L}(E, F)$ denotes the Banach space of all (bounded and linear) operators from E into F endowed with the operator norm. For a Banach space E of type 2 (resp., cotype 2) we write $\mathbf{T_2}(E)$ (resp., $\mathbf{C_2}(E)$) for its (Rademacher) type 2 constant (resp., cotype 2 constant), and for $1 \le r \le \infty$ we denote by $\mathbf{M^{(r)}}(X)$ (resp., $\mathbf{M_{(r)}}(X)$) the r-convexity (resp., r-concavity) constant of an r-convex (resp., r-concave) Banach lattice X. Recall that for Banach spaces E, F the injective norm on $E \otimes F$ is defined by

$$\|z\|_{E\otimes_\varepsilon F} := \sup\{|\langle x' \otimes y', z\rangle|;\ x' \in B_{E'}, y' \in B_{F'}\}, \quad z \in E \otimes F,$$

and with $E\tilde{\otimes}_\varepsilon F$ we denote the completion of $E \otimes F$ endowed with this norm. We will extensively use the fact that the equality $E \otimes_\varepsilon F = \mathcal{L}(E', F)$ holds isometrically whenever one of the two involved spaces is finite-dimensional.

Let (Ω, Σ, μ) (or shortly (Ω, μ)) be a σ-finite and complete measure space. As usual $L_0(\mu)$ denotes the vector lattice of all (equivalence classes of) μ-measurable real-valued functions defined on Ω, equipped with the topology of convergence in measure on μ-finite sets. A Banach space $X = X(\mu)$ is said to be a Banach function space on (Ω, μ) if X is a subspace of $L_0(\mu)$ with the following two properties:

(i) $|f| \le |g|$, with $f \in L_0(\mu)$ and $g \in X$ implies $f \in X$ and $\|f\|_X \le \|g\|_X$;
(ii) there exists $u \in X$ such that $u > 0$ on Ω.

A finite-dimensional real quasi-normed space $X = (\mathbb{R}^n, \|\cdot\|_X)$ is called an n-dimensional lattice if $\|\cdot\|_X$ is a lattice quasi-norm. Clearly, if $\|\cdot\|_X$ is a norm, then X is a Banach function space in the above sense.

A Banach function space X is said to be maximal if its unit ball B_X is closed in $L_0(\mu)$. It is well-known that X is maximal if and only if $X = X^{\times\times}$ holds isometrically, where $X^\times$ stands for the Köthe dual of X, i. e.,

$$X^\times := \Big\{ y \in L_0(\mu);\ \|y\|_{X^\times} := \sup_{x\in B_X} \int_X |xy|\, d\mu < \infty \Big\}.$$

Clearly, for an n-dimensional lattice X (not necessarily normed) the notion of the Köthe dual $X^\times$ as above (taking $\Omega = \{1, \dots, n\}$ and μ the counting measure) also makes sense and always gives an n-dimensional Banach lattice $X^\times$.

For basic results and notation from interpolation theory we refer e. g. to [2] and [3]. We recall that a mapping $\mathcal{F}$ from the category of all couples of Banach spaces into the category of all Banach spaces is said to be an interpolation functor (or a method of interpolation) if for any couple (X_0, X_1), the Banach space $\mathcal{F}(X_0, X_1)$ is intermediate with respect to (X_0, X_1) (i. e., $\Delta(\overline{X}) := X_0 \cap X_1 \hookrightarrow \mathcal{F}(X_0, X_1) \hookrightarrow X_0 + X_1$), and $T : \mathcal{F}(X_0, X_1) \to \mathcal{F}(Y_0, Y_1)$ for all $T : (X_0, X_1) \to (Y_0, Y_1)$; here as usual the notation $T :\ (X_0, X_1) \to\ (Y_0, Y_1)$ means that $T : X_0 + X_1 \to Y_0 + Y_1$ is a linear operator such that for $j = 0, 1$ the restriction of T to the space X_j is a bounded operator from X_j into Y_j. If additionally

$$\|T : \mathcal{F}(X_0, X_1) \to \mathcal{F}(Y_0, Y_1)\| \le \|T : \overline{X} \to \overline{Y}\| := \max\{\|T : X_0 \to Y_0\|,\ \|T : X_1 \to Y_1\|\}$$

holds, then $\mathcal{F}$ is called an exact functor (or an exact method of interpolation).

If X is a Banach space intermediate with respect to a couple $\overline{X} = (X_0, X_1)$, we let X° be the closed hull of $\Delta(\overline{X})$ in X. A Banach couple $\overline{X}$ is called regular if $X_j^\circ = X_j$ for $j = 0, 1$. If $\mathcal{F}$ is an interpolation functor, $\mathcal{F}^\circ$ is the interpolation functor defined by $\mathcal{F}^\circ(\overline{X}) = \mathcal{F}(\overline{X})^\circ$. If $\mathcal{F} = \mathcal{F}^\circ$, $\mathcal{F}$ is called a regular interpolation functor.

Throughout the paper the set of all functions $\varphi : [0, \infty) \times [0, \infty) \to [0, \infty)$ which are non-decreasing in each variable and homogeneous of degree one (i. e., $\varphi(\lambda s, \lambda t) = \lambda \varphi(s, t)$ for all $\lambda, s, t \ge 0$) is denoted by Φ, and if addition φ is concave, then we write $\varphi \in \mathcal{U}$. Recall that if $\overline{X} = (X_0, X_1)$ is a couple of Banach function spaces on (Ω, μ) and $\varphi \in \mathcal{U}$, the Calderón–Lozanovsky space $\varphi(\overline{X}) = \varphi(X_0, X_1)$ consists of all $x \in L_0(\mu)$ for which $|x| = \varphi(|x_0|, |x_1|)$ for some $x_j \in X_j$, $j = 0, 1$. Equipped with the norm

$$\|x\| := \inf \big\{ \max\{\|x_0\|_{X_0}, \|x_1\|_{X_1}\};\ |x| = \varphi(|x_0|, |x_1|),\ x_j \in X_j,\ j = 0, 1 \big\},$$

$\varphi(\overline{X})$ forms a Banach function space (see, e. g., [23], [30]). The class of these spaces includes the class of all Orlicz spaces; for instance, it is easy to see that for each Orlicz function $\psi : [0, \infty) \to [0, \infty)$ the equality $\ell_\psi = \varphi(\ell_1, \ell_\infty)$ holds isometrically, where $\varphi(s, t) := t\,\psi^{-1}(s/t)$ for $t > 0$ and $\varphi(s, 0) := 0$. In the case of the power function $\varphi_\theta(s, t) = s^{1-\theta} t^\theta$, $0 < \theta < 1$, we obtain the space $X_0^{1-\theta} X_1^\theta$ introduced by Calderón [5]. We note that by a result of Calderón (see [5], p.125), we have that the complex interpolation space $[X_0(\mathbb{C}), X_1(\mathbb{C})]_\theta = X_0^{1-\theta} X_1^\theta(\mathbb{C})$ whenever X_0 or X_1 is σ-order continuous. Here for a given Banach function space X on (Ω, μ), we denote by $X(\mathbb{C})$ the complexification of X, i. e., the space of all complex-valued measurable functions f on Ω such that $|f| \in X$.

For $\varphi \in \mathcal{U}$ we will also need the following duality formula:

$$\varphi(X_0, X_1)^\times = \varphi_*(X_0^\times, X_1^\times) \tag{1}$$

with

$$\|\cdot\|_{\varphi_*(X_0^\times, X_1^\times)} \le \|\cdot\|_{\varphi(X_0, X_1)^\times} \le 2\,\|\cdot\|_{\varphi_*(X_0^\times, X_1^\times)}.$$

Here, φ_* denotes the conjugate defined for any $\varphi \in \mathcal{U}$ by

$$\varphi_*(s, t) := \inf_{\alpha, \beta > 0} \frac{\alpha s + \beta t}{\varphi(\alpha, \beta)}.$$

We have $\varphi_* \in \mathcal{U}$ and $(\varphi_*)_* = \varphi$ (see [23]). For any $\varphi \in \Phi$, we define also the function φ^* by $\varphi^*(s, t) := 1/\varphi(1/s, 1/t)$ for any $s, t > 0$. It is easy to see that $(\varphi^*)^* = \varphi$, and $\varphi^* \asymp \varphi_*$ for any $\varphi \in \mathcal{U}$.

It is well-known that the Calderón–Lozanovsky construction restricted to the class of couples of maximal Banach function spaces is an interpolation method in this class. There are many abstract extensions of this method to arbitrary Banach couples. Of particular interest for our paper are the following two constructions by Aronszajn and Gagliardo [1] (see also [2], [3]). Given a couple $\overline{A}$ and A an intermediate space with respect to this couple, two exact interpolation functors are defined by

$$G(\overline{X}) = G_A^{\overline{A}}(\overline{X}) := \Big\{ \sum_{n=1}^{\infty} T_n a_n;\ \sum_{n=1}^{\infty} \|T_n\|_{\overline{A}\to\overline{X}} \|a_n\|_A < \infty \Big\}$$

and

$$H(\overline{X}) = H_A^{\overline{A}}(\overline{X}) := \{x \in X_0 + X_1;\ Tx \in A \ \text{ for every } \ T : \overline{X} \to \overline{A}\}.$$

The norms are given by

$$\|x\|_{G(\overline{X})} = \inf \Big\{ \sum_{n=1}^{\infty} \|T_n\|_{\overline{A}\to\overline{X}} \|a_n\|_A;\ x = \sum_{n=1}^{\infty} T_n a_n \Big\}$$

and

$$\|x\|_{H(\overline{X})} = \sup\{\|Tx\|_A;\ \|T\|_{\overline{X}\to\overline{A}} \le 1\},$$

respectively. Note that G is the minimal interpolation functor satisfying $A \hookrightarrow G(\overline{A})$ and H is the maximal interpolation functor satisfying $H(\overline{A}) \hookrightarrow A$. In the particular case when $\varphi \in \Phi$ and $\overline{A} = (\ell_\infty, \ell_\infty(2^{-n}))$, $A = \ell_\infty(\varphi^*(1, 2^{-n}))$ (resp., $\overline{A} = (\ell_1, \ell_1(2^n))$ and $A = \ell_1(\varphi^*(1, 2^n))$), the interpolation functor $G_A^{\overline{A}}$ (resp., $H_A^{\overline{A}}$) is called the lower (resp., the upper) Ovchinnikov interpolation method and is denoted by φ_ℓ (resp., φ_u). These were intensively studied in [28] (see also the references therein).

Recall that if (X_0, X_1) is any couple of maximal Banach function spaces, then for any $\varphi \in \mathcal{U}$ the following identities hold:

$$\varphi_\ell(X_0, X_1) = \varphi_u(X_0, X_1) = \varphi(X_0, X_1)$$

with the the universal constants of equivalence of the norms not depending on φ and (X_0, X_1) (see [28]).

3. Approximation by finite-dimensional spaces

First we show—similar to [19, Section 4] and [14]—that interpolation formulas for tensor products as stated in the introduction are of a finite-dimensional nature. In order to make the following more readable, let us introduce another notation: If (X_0, X_1) is a Banach couple, $X \subset X_0 \cap X_1$ a subspace, and $\mathcal{A} \subset \mathcal{F}in(X)$ is cofinal (i. e., for every $G \in \mathcal{F}in(X)$ there exists $M \in \mathcal{A}$ with $G \subset M$), then the triple $((X_0, X_1), X, \mathcal{A})$ is called a cofinal interpolation triple (resp., a regular cofinal interpolation triple whenever X is dense in both X_0 and X_1).

For $M \in \mathcal{F}in(X)$ we denote by M_0 (resp., M_1) the subspace M of X_0 (resp., X_1) endowed with the induced norm. We call an exact interpolation functor $\mathcal{F}$ approximable on a cofinal interpolation triple $((X_0, X_1), X, \mathcal{A})$ if for any $\varepsilon > 0$ and any $H \in \mathcal{F}in(X)$, there exists $M \in \mathcal{A}$ such that $H \subset M$ and for all $x \in H$,

$$\|x\|_{\mathcal{F}(M_0, M_1)} \le (1 + \varepsilon)\|x\|_{\mathcal{F}(X_0, X_1)}.$$

Note that if in the above definition one considers one-dimensional subspaces H only as well as $X = X_0 \cap X_1$ and $\mathcal{A} = \mathcal{F}in(X)$, then approximability of $\mathcal{F}$ on the cofinal interpolation triple $((X_0, X_1), X_0 \cap X_1, \mathcal{A})$ would mean that $\mathcal{F}$ is computable on (X_0, X_1) in the sense of Brudnyi-Krugljak (see [3]). For general examples of computable orbit functors we refer to [20].

In the sequel we will need the following result.

Proposition 1 *Assume that $\mathcal{F}$ is an exact interpolation functor computable on a Banach couple $\overline{X} = (X_0, X_1)$. Then $\mathcal{F}$ is approximable on any cofinal interpolation triple $(\overline{X}, X, \mathcal{A})$.*

PROOF. Applying Lemma 2.5.27 in [3], we conclude that $\mathcal{F}$ is approximable on the cofinal interpolation triple $(\overline{X}, \Delta(\overline{X}), \mathcal{F}in(\Delta(\overline{X})))$. Thus, if $(\overline{X}, X, \mathcal{A})$ is a cofinal interpolation triple, then the interpolation property of $\mathcal{F}$ yields the required result. ■

Throughout the paper if (M_0, M_1) and (N_0, N_1) are finite-dimensional regular couples and $\mathcal{F}$ an interpolation functor, then we define the quantities

$$\ell_{\mathcal{F}}(M_0, M_1; N_0, N_1) := \|\mathcal{F}(M_0 \otimes_\varepsilon N_0, M_1 \otimes_\varepsilon N_1) \hookrightarrow \mathcal{F}(M_0, M_1) \otimes_\varepsilon \mathcal{F}(N_0, N_1)\|$$

and

$$r_{\mathcal{F}}(M_0, M_1; N_0, N_1) := \|\mathcal{F}(M_0, M_1) \otimes_\varepsilon \mathcal{F}(N_0, N_1) \hookrightarrow \mathcal{F}(M_0 \otimes_\varepsilon N_0, M_1 \otimes_\varepsilon N_1)\|.$$

Before we state and prove the following lemma which plays an essential role in the proof of the main result of the paper, we first note that if (E_0, E_1) and (F_0, F_1) are two Banach couples, then $E_j \tilde{\otimes}_\varepsilon F_j$, $j = 0, 1$, is continuously embedded in $(E_0 + E_1)\tilde{\otimes}_\varepsilon (F_0 + F_1)$ (see [9, 4.3]). In consequence, $(E_0 \tilde{\otimes}_\varepsilon F_0, E_1 \tilde{\otimes}_\varepsilon F_1)$ is a Banach couple.

Lemma 1 *Let $A := ((E_0, E_1), E, \mathcal{A})$ and $B := ((F_0, F_1), F, \mathcal{B})$ be regular cofinal interpolation triples and $\mathcal{F}$ be an exact interpolation functor.*

(i) *If $L_{\mathcal{F}}(A, B) := \sup_{M\in\mathcal{A}} \sup_{N\in\mathcal{B}} \ell_{\mathcal{F}}(M_0, M_1; N_0, N_1) < \infty$ and $\mathcal{F}$ is approximable on the cofinal interpolation triple $((E_0 \tilde{\otimes}_\varepsilon F_0, E_1 \tilde{\otimes}_\varepsilon F_1), E \otimes F, \mathcal{C})$ with $\mathcal{C} := \{M \otimes N;\ M \in \mathcal{A}, N \in \mathcal{B}\}$, then*

$$\mathcal{F}(E_0 \tilde{\otimes}_\varepsilon F_0, E_1 \tilde{\otimes}_\varepsilon F_1) \hookrightarrow \mathcal{F}(E_0, E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0, F_1).$$

(ii) *If $R_{\mathcal{F}}(A, B) := \sup_{M\in\mathcal{A}} \sup_{N\in\mathcal{B}} r_{\mathcal{F}}(M_0, M_1; N_0, N_1) < \infty$ and $\mathcal{F}$ is approximable on both cofinal interpolation triples A and B, then*

$$\mathcal{F}(E_0, E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0, F_1) \hookrightarrow \mathcal{F}(E_0 \tilde{\otimes}_\varepsilon F_0, E_1 \tilde{\otimes}_\varepsilon F_1).$$

PROOF. From the definition of an approximable interpolation functor and the density assumptions we conclude that $E \otimes F$ is dense in $\mathcal{F}(E_0, E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0, F_1)$ and in $\mathcal{F}(E_0 \tilde{\otimes}_\varepsilon F_0, E_1 \tilde{\otimes}_\varepsilon F_1)$, hence, in order to prove (i) and (ii), respectively, it is sufficient to show that for a given $z \in E \otimes F$

$$\|z\|_{\mathcal{F}(E_0,E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0,F_1)} \le L_{\mathcal{F}}(A, B)\, \|z\|_{\mathcal{F}(E_0\tilde{\otimes}_\varepsilon F_0, E_1\tilde{\otimes}_\varepsilon F_1)} \tag{2}$$

and

$$\|z\|_{\mathcal{F}(E_0\tilde{\otimes}_\varepsilon F_0, E_1\tilde{\otimes}_\varepsilon F_1)} \le R_{\mathcal{F}}(A, B)\, \|z\|_{\mathcal{F}(E_0,E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0,F_1)}, \tag{3}$$

respectively. We start with (2). By the assumption on $\mathcal{F}$ and the fact that the injective norm respects subspaces, there exist $M \in \mathcal{A}$ and $N \in \mathcal{B}$ such that $z \in M \otimes N$ and

$$\|z\|_{\mathcal{F}(M_0\otimes_\varepsilon N_0, M_1\otimes_\varepsilon N_1)} \le (1 + \varepsilon)\, \|z\|_{\mathcal{F}(E_0\tilde{\otimes}_\varepsilon F_0, E_1\tilde{\otimes}_\varepsilon F_1)}.$$

Here, by the mapping property of the injective norm,

$$\begin{aligned}
\|z\|_{\mathcal{F}(E_0,E_1)\tilde{\otimes}_\varepsilon \mathcal{F}(F_0,F_1)} &\le \|z\|_{\mathcal{F}(M_0,M_1)\otimes_\varepsilon \mathcal{F}(N_0,N_1)} \\
&\le L_{\mathcal{F}}(A, B)\|z\|_{\mathcal{F}(M_0\otimes_\varepsilon N_0, M_1\otimes_\varepsilon N_1)} \\
&\le (1 + \varepsilon)\, L_{\mathcal{F}}(A, B)\, \|z\|_{\mathcal{F}(E_0\tilde{\otimes}_\varepsilon F_0, E_1\tilde{\otimes}_\varepsilon F_1)}.
\end{aligned}$$

In order to show (3) let $z \in G \otimes H$ for some $G \in \mathcal{F}in(E)$, $H \in \mathcal{F}in(F)$, and choose by the assumption on $\mathcal{F}$ subspaces $M \in \mathcal{A}$ and $N \in \mathcal{B}$ such that $G \subset M$, $H \subset N$ and

$$\|(G, \|\cdot\|_{\mathcal{F}(E_0,E_1)}) \hookrightarrow \mathcal{F}(M_0, M_1)\| \le \sqrt{1 + \varepsilon},$$

$$\|(H, \|\cdot\|_{\mathcal{F}(F_0,F_1)}) \hookrightarrow \mathcal{F}(N_0, N_1)\| \le \sqrt{1+\varepsilon}.$$

Then, by the mapping property,

$$\|(G, \|\cdot\|_{\mathcal{F}(E_0,E_1)}) \otimes_\varepsilon (H, \|\cdot\|_{\mathcal{F}(F_0,F_1)}) \hookrightarrow \mathcal{F}(M_0, M_1) \otimes_\varepsilon \mathcal{F}(N_0, N_1)\| \le 1+\varepsilon,$$

hence, since the injective norm respects subspaces,

$$\|z\|_{\mathcal{F}(M_0,M_1)\otimes_\varepsilon \mathcal{F}(N_0,N_1)} \le (1+\varepsilon)\, \|z\|_{\mathcal{F}(E_0,E_1)\otimes_\varepsilon \mathcal{F}(F_0,F_1)}.$$

By the usual interpolation theorem we obtain

$$\begin{aligned}
\|z\|_{\mathcal{F}(E_0\tilde{\otimes}_\varepsilon F_0, E_1\tilde{\otimes}_\varepsilon F_1)} &\le \|z\|_{\mathcal{F}(M_0\otimes_\varepsilon N_0, M_1\otimes_\varepsilon N_1)} \\
&\le R_{\mathcal{F}}(A,B)\, \|z\|_{\mathcal{F}(M_0,M_1)\otimes_\varepsilon \mathcal{F}(N_0,N_1)} \\
&\le (1+\varepsilon)\, R_{\mathcal{F}}(A,B)\, \|z\|_{\mathcal{F}(E_0,E_1)\otimes_\varepsilon \mathcal{F}(F_0,F_1)},
\end{aligned}$$

which proves (3). ■

4. The finite-dimensional case

The first result needed is based on bilinear Calderón–Lozanovsky interpolation, which was studied in [26].

Proposition 2 *Let $\varphi \in \mathcal{U}$ be such that for some $C > 0$ and all $s, t > 0$*

$$\varphi(1,s)\,\varphi(1,t) \le C\,\varphi(1,st).$$

Then there exists a constant $D > 0$ such that for each finite-dimensional Banach space N and each couple (M_0, M_1) of n-dimensional Banach lattices,

$$\ell_{\varphi_\ell}(M_0, M_1; N, N) \le D.$$

PROOF. Consider for $i = 0, 1$ the bilinear mappings $\Phi_i : \mathcal{L}(M_i', N) \times M_i' \to N$, $\Phi_i(T, x') := Tx'$. Note that $\varphi(1,s)\,\varphi_*(1,t) \le C\,\varphi_*(1,st)$; indeed,

$$\begin{aligned}
\varphi(1,s)\,\varphi_*(1,t) = \varphi(1,s) \inf_{\alpha,\beta>0} \frac{\alpha 1 + \beta t}{\varphi(\alpha,\beta)} &= \inf_{\alpha,\beta>0} \frac{\alpha 1 + \beta t}{\alpha\varphi(1,\beta/\alpha)} \varphi(1,s) \\
\le C \inf_{\alpha,\beta>0} \frac{\alpha 1 + \beta t}{\alpha\varphi(1,\beta/\alpha s)} &= \inf_{\alpha,\beta>0} \frac{\alpha 1 + \beta st}{\varphi(\alpha,\beta)} = C\,\varphi_*(1,st).
\end{aligned}$$

Then we may apply [26, 3.4] to obtain

$$\|\Phi : \varphi_\ell(\mathcal{L}(M_0', N), \mathcal{L}(M_1', N)) \times (\varphi_*)_\ell(M_0', M_1') \to N\| \le D,$$

where $\Phi(T, x') := Tx'$ and C_1 is some constant not depending on φ. Since $(\varphi_*)_\ell(M_0', M_1') = \varphi_\ell(M_0, M_1)'$ by (1), this shows that

$$\|\varphi_\ell(\mathcal{L}(M_0', N), \mathcal{L}(M_1', N)) \hookrightarrow \mathcal{L}(\varphi_\ell(M_0, M_1)', N)\| \le D,$$

the conclusion. ■

The counterpart of the preceding proposition for r_{φ_ℓ} is as in [14] based on factorization.

Proposition 3 *Let $\varphi \in \mathcal{U}$. Then there exists $C > 0$ such that for any finite-dimensional Banach space N and any couple (M_0, M_1) of n-dimensional Banach lattices,*

$$r_{\varphi_\ell}(M_0, M_1; N, N) \le C\,\mathbf{C_2}(N)^{3/2} \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}^{7/2}.$$

Before giving the proof we collect some facts about powers of finite-dimensional lattices. For $0 < r < \infty$ and an n-dimensional lattice X,

$$\|x\|_r := \||x|^{1/r}\|_X^r, \quad x \in \mathbb{R}^n$$

defines a lattice quasi-norm on $\mathbb{R}^n$; the n-dimensional lattice $(\mathbb{R}^n, \|\cdot\|_r)$ will be denoted by X^r. Note that in the normed case X^r is again normed whenever $\mathbf{M}^{(\max(\mathbf{1},\mathbf{r}))}(X) = 1$ (recall that $\mathbf{M}^{(\mathbf{1})}(X) = 1$). For an n-dimensional normed lattice X we denote by $M(\ell_2^n, X)$ the vector space $\mathbb{R}^n$ equipped with the corresponding multiplier norm, i. e., $\|\lambda\|_{M(\ell_2^n,X)} := \|D_\lambda : \ell_2^n \to X\|$ for $\lambda \in \mathbb{R}^n$, where $D_\lambda\mu := \lambda\mu$, $\mu \in \mathbb{R}^n$. It is easy to prove that

$$M(\ell_2^n, X) = (((X^\times)^2)^\times)^{1/2} \tag{4}$$

holds isometrically (see e. g. [11, 3.5]; note that there the assumption $\mathbf{M}_{(\mathbf{2})}(X) = 1$ is superfluous).

For any function $f : [0,\infty) \times [0,\infty) \to [0,\infty)$ and $r > 0$, we define the function $f^{(r)} := (f_r)^r$, where

$$f_r(s,t) := f(s^{1/r}, t^{1/r}),$$

and

$$f^r(s,t) := f(s,t)^r.$$

If additionally f is non-decreasing in each variable and homogeneous function of degree one, we define $\overline{f}$ by

$$\overline{f}(s,t) := \inf_{\alpha,\beta>0} \left(\frac{s}{\alpha} + \frac{t}{\beta}\right) f(\alpha,\beta)$$

for $s, t \geq 0$. Note that $\overline{f} \in \mathcal{U}$ and $f \leq \overline{f} \leq 2f$.

Lemma 2 *Let X, X_0, X_1 be n-dimensional normed lattices and $\varphi \in \mathcal{U}$. Then the following identities hold, with universal constants involved in the equivalence of norms only:*

(i) *For any $r > 0$, it is $\varphi(X_0, X_1)^r = \overline{\varphi^{(r)}}\,(X_0^r, X_1^r)$.*

(ii) $\varphi(M(\ell_2^n, X_0), M(\ell_2^n, X_1)) = M(\ell_2^n, \varphi(X_0, X_1))$ *whenever* $\mathbf{M}_{(\mathbf{2})}(X_0) = \mathbf{M}_{(\mathbf{2})}(X_1) = 1$.

PROOF. The proof of (i) is straightforward. To see (ii), first observe that part (i) and (4) together with (1) lead to

$$\begin{aligned} M(\ell_2^n, \varphi(X_0, X_1)) &= (((\varphi(X_0, X_1)^\times)^2)^\times)^{1/2} = (((\varphi_*(X_0^\times, X_1^\times))^2)^\times)^{1/2} \\ &= ((\overline{(\varphi_*)^{(2)}}\,((X_0^\times)^2, (X_1^\times)^2))^\times)^{1/2} \\ &= (\overline{(\varphi_*)^{(2)}})_*\,(((X_0^\times)^2)^\times, ((X_1^\times)^2)^\times))^{1/2} \\ &= \overline{(((\varphi_*)^{(2)})_*)^{(1/2)}}\,(M(\ell_2^n, X_0), M(\ell_2^n, X_1)). \end{aligned}$$

Here, the constants occurring in the equivalence of norms do not depend on the parameters n, X_0, X_1 and φ. Therefore, it is enough to show that the function

$$(((\varphi_*)^{(2)})_*)^{(1/2)} = (((((\varphi_*)_2)^2)_*)_{1/2})^{1/2}$$

is equivalent to φ. Indeed, taking into account that $\|\ell_2^2 \hookrightarrow \ell_1^2\| = \sqrt{2}$, we have

$$\begin{aligned} (\varphi_*)^{(2)}(s,t) &= (\varphi_*(s^{1/2}, t^{1/2}))^2 = \inf_{\alpha,\beta>0} \frac{(\alpha s^{1/2} + \beta t^{1/2})^2}{\varphi(\alpha,\beta)^2} \\ &\asymp \inf_{\alpha,\beta>0} \frac{\alpha^2 s + \beta^2 t}{\varphi(\alpha,\beta)^2} = \inf_{\alpha,\beta>0} \frac{\alpha s + \beta t}{\varphi(\alpha^{1/2}, \beta^{1/2})^2} \\ &= ((\varphi_2)^2)_*(s,t) = (\varphi^{(2)})_*(s,t), \end{aligned}$$

hence,

$$
\begin{aligned}
((\varphi_*)^{(2)})_*)^{(1/2)}(s,t) \asymp ((\varphi^{(2)})_*)_*)^{(1/2)}(s,t) &= (\varphi^{(2)})^{(1/2)}(s,t) \\
&= (((\varphi_2)^2)_{1/2}(s,t))^{1/2} = (((\varphi_2))^2(s^2,t^2))^{1/2} \\
&= \varphi_2(s^2,t^2) = \varphi(s,t),
\end{aligned}
$$

the conclusion. ■

Lemma 3 *If (X_0, X_1) is a couple of 2-concave Banach function spaces, and $\mathcal{F}$ an exact interpolation functor such that*

$$\mathcal{F}(\ell_2(X_0), \ell_2(X_1)) \hookrightarrow \ell_2(\mathcal{F}(X_0, X_1)), \tag{5}$$

then $\mathcal{F}(X_0, X_1)$ is also 2-concave, and in this case

$$\mathbf{M_{(2)}}(\mathcal{F}(X_0, X_1)) \le C \max\{\mathbf{M_{(2)}}(X_0), \mathbf{M_{(2)}}(X_1)\},$$

where $C > 0$ is a constant depending on the functor $\mathcal{F}$ and the norm of the embedding in (5) *only.*

PROOF. We denote by $X(\ell_2)$ as usual the Köthe–Bochner space of all strongly measurable functions x with values in ℓ_2 such that $\|x(\cdot)\|_{\ell_2} \in X$, endowed with the norm $\|x\|_{X(\ell_2)} := \|\|x(\cdot)\|_{\ell_2}\|_X$, and by $X[\ell_2]$ the space of all sequences $(x_n) \subset X$ such that $(x_n(t)) \in \ell_2$ for all $t \in \Omega$ and $\|(x_n(\cdot))\|_{\ell_2} \in X$, endowed with the norm $\|(x_n)\|_{X[\ell_2]} := \|\|(x_n(\cdot))\|_{\ell_2}\|_X$. Now for $j = 0, 1$, 2-concavity of X_j means that $X_j[\ell_2] \hookrightarrow \ell_2(X_j)$ and that the norm of this embedding then equals $\mathbf{M_{(2)}}(X_j)$. It is well-known that

$$\mathcal{F}(X_0(\ell_2), X_1(\ell_2)) = \mathcal{F}(X_0, X_1)(\ell_2)$$

holds isometrically and that for any Banach function space X, the spaces $X(\ell_2)$ and $X[\ell_2]$ are isomorphic to each other in a natural way, with universal constants (see [4] and also [6]), which gives

$$\mathcal{F}(X_0[\ell_2], X_1[\ell_2]) = \mathcal{F}(X_0, X_1)[\ell_2]$$

with only universal constants involved in the equivalence of norms. Hence, by the interpolation property of $\mathcal{F}$ and the assumption (5),

$$\mathcal{F}(X_0, X_1)[\ell_2] = \mathcal{F}(X_0[\ell_2], X_1[\ell_2]) \hookrightarrow \mathcal{F}(\ell_2(X_0), \ell_2(X_1)) \hookrightarrow \ell_2(\mathcal{F}(X_0, X_1)),$$

with norm less or equal than $C \max\{\mathbf{M_{(2)}}(X_0), \mathbf{M_{(2)}}(X_1)\}$, with C described as in the above. ■

Corollary 1 *Let (X_0, X_1) be a couple of 2-concave Banach function spaces and $\varphi \in \mathcal{U}$. Then $\varphi(X_0, X_1)$ is 2-concave, and*

$$\mathbf{M_{(2)}}(\varphi(X_0, X_1)) \le C \max\{\mathbf{M_{(2)}}(X_0), \mathbf{M_{(2)}}(X_1)\},$$

where $C > 0$ is some constant not depending on the couple (X_0, X_1).

PROOF. This an immediate consequence of the above lemma and the following facts: any 2-concave Banach function space contains no isomorphic copy of c_0, so it is a maximal space (see, e. g., [21]), and next (see, e. g., [24]), for any $\varphi \in \mathcal{U}$ and any Banach couple (X_0, X_1)

$$\varphi_u(\ell_2(X_0), \ell_2(X_1)) \hookrightarrow \ell_2(\varphi_u(X_0, X_1)),$$

and (see [28])

$$\varphi_u(X_0, X_1) = \varphi(X_0, X_1)$$

whenever (X_0, X_1) is a couple of maximal Banach function spaces. Here the constants of the norms of the involved inclusion maps do not depend on φ and (X_0, X_1). ■

Now we are prepared to give the proof of Proposition 3. We have to show that for given $\varphi \in \mathcal{U}$, there exists $C > 0$ such that for any finite-dimensional Banach space N and any couple (M_0, M_1) of n-dimensional Banach lattices,

$$\begin{aligned}&\|\mathcal{L}(N', \varphi_\ell(M_0, M_1)) \hookrightarrow \varphi_\ell(\mathcal{L}(N', M_0), \mathcal{L}(N', M_1))\| \\ &\qquad\qquad\qquad\qquad \leq C\, \mathbf{C_2}(N)^{3/2} \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}^{7/2}.\end{aligned}$$

Let $T \in \mathcal{L}(N', \varphi_\ell(M_0, M_1))$ and $n := \dim(M_0) = \dim(M_1)$. Then by Pisier's factorization theorem (see [29]), T factors as follows:

$$\begin{tikzcd} N' \arrow[rr, "T"] \arrow[dr, "T_1"'] & & \varphi_\ell(M_0, M_1) \\ & \ell_2^n \arrow[ur, "T_2"'] & \end{tikzcd}$$

where $\|T_1\|\,\|T_2\| \leq (2\,\mathbf{C_2}(N)\,\mathbf{C_2}(\varphi_\ell(M_0, M_1)))^{3/2}$. Furthermore, by a variant of the Maurey–Rosenthal factorization theorem (see [8]), T_2 factors as follows:

$$\begin{tikzcd} \ell_2^n \arrow[rr, "T_2"] \arrow[dr, "R_0"'] & & \varphi_\ell(M_0, M_1) \\ & \ell_2^n \arrow[ur, "D_\lambda"'] & \end{tikzcd}$$

where $\|R_0\|\|D_\lambda\| \leq \sqrt{2}\,\mathbf{M_{(2)}}(\varphi_\ell(M_0, M_1))$. Taking $R := R_0 T_1$ and using Corollary 1, this gives us a factorization

$$\begin{tikzcd} N' \arrow[rr, "T"] \arrow[dr, "R"'] & & \varphi_\ell(M_0, M_1) \\ & \ell_2^n \arrow[ur, "D_\lambda"'] & \end{tikzcd}$$

where $\|R\|\,\|D_\lambda\| \leq C\,\mathbf{C_2}(N)^{3/2} \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}^{5/2}$, with $C > 0$ some constant not depending on φ. With this, consider for $i = 0, 1$ the mappings $\Phi_i : M(\ell_2^n, M_i) \to \mathcal{L}(N', M_i)$, $\Phi_i(D_\mu) := D_\mu R$, with norm less or equal $\|R\|$ each. Then by interpolation and Lemma 2 (ii),

$$\|\Phi : M(\ell_2^n, \varphi(M_0, M_1)) \to \varphi_\ell(\mathcal{L}(N', M_0), \mathcal{L}(N', M_1))\| \leq D\, \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}\,\|R\|,$$

where $\Phi(D_\mu) := D_\mu R$, and $D > 0$ is some universal constant. Hence,

$$\begin{aligned}\|T\|_{\varphi_\ell(\mathcal{L}(N', M_0), \mathcal{L}(N', M_1))} &= \|D_\lambda R\|_{\varphi_\ell(\mathcal{L}(N', M_0), \mathcal{L}(N', M_1))} \\ &\leq D\, \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}\,\|R\|\,\|D_\lambda\| \\ &\leq C\,D\,\mathbf{C_2}(N)^{3/2} \max\{\mathbf{M_{(2)}}(M_0), \mathbf{M_{(2)}}(M_1)\}^{7/2},\end{aligned}$$

which gives the claim. ■

We conclude this section with a technical result needed in the proof of our main result. Two Banach couples $\overline{X}$ and $\overline{Y}$ are called isomorphic if there exist operators $T : \overline{X} \to \overline{Y}$ and $T^{-1} : \overline{Y} \to \overline{X}$ such that $TT^{-1}|_{Y_j} = \mathrm{id}_{Y_j}$ and $T^{-1}T|_{X_j} = \mathrm{id}_{X_j}$ ($j = 0, 1$). Given isomorphic Banach couples $\overline{X}$ and $\overline{Y}$, $d(\overline{X}, \overline{Y})$ is defined by

$$d(\overline{X}, \overline{Y}) := \inf\{\|T\|_{\overline{X} \to \overline{Y}}\,\|T^{-1}\|_{\overline{Y} \to \overline{X}}\},$$

where the infimum is taken over all isomorphisms between $\overline{X}$ and $\overline{Y}$. We omit the easy proof of the following lemma.

Lemma 4 *Let (M_0, M_1), (U_0, U_1) and (N_0, N_1), (V_0, V_1) be pairs of finite-dimensional regular Banach couples of the same dimensions, respectively. Then the following inequalities hold true for any exact interpolation functor $\mathcal{F}$:*

(i) $\ell_{\mathcal{F}}(M_0, M_1; N_0, N_1) \le d(\overline{M}, \overline{U})\, d(\overline{N}, \overline{V})\, \ell_{\mathcal{F}}(U_0, U_1; V_0, V_1)$.

(ii) $r_{\mathcal{F}}(M_0, M_1; N_0, N_1) \le d(\overline{M}, \overline{U})\, d(\overline{N}, \overline{V})\, r_{\mathcal{F}}(U_0, U_1; V_0, V_1)$.

5. The Gustavsson–Peetre method

In this section we prove our main result on interpolation formulas for injective tensor products with respect to the Gustavsson–Peetre functor G_φ.

A Banach couple (E_0, E_1) is said to have an unconditional basis if there is a sequence $\{e_n\}$ in $E_0 \cap E_1$ which forms an unconditional basis in E_0 and E_1. Further, following [25], a Banach couple $\overline{X} = (X_0, X_1)$ is said to have local unconditional structure (l.u.st.) if there exists a positive constant $\lambda = \lambda(\overline{X})$ such that for any regular finite-dimensional sub-couple (A_0, A_1) of $\overline{X}$ there is a regular finite-dimensional sub-couple $\overline{B} = (B_0, B_1) \supset (A_0, A_1)$ (i. e., $B_j \supset A_j$ for $j = 0, 1$) of $\overline{X}$, isomorphic to a Banach couple $\overline{E}$ with a monotone unconditional basis and such that $d(\overline{B}, \overline{E}) \le \lambda$. The smallest λ with this property is called the l.u.st. constant of $\overline{X}$ and is denoted by $\mathrm{lu}(\overline{X})$.

In what follows for any $\varphi \in \Phi$ we denote by G_φ the exact interpolation functor $G_A^{\overline{A}}$ with $\overline{A} := (c_0, c_0(2^{-n}))$ and $A := c_0(\varphi^*(1, 2^{-n}))$ defined on $\mathbb{Z}$. Note that if $\varphi \in \Phi$ is a non-degenerate function (i. e., $\varphi(1, t) \to 0$ and $\varphi(t, 1) \to 0$ as $t \to 0$), then G_φ coincides with the Gustavsson–Peetre method of interpolation $\langle\cdot\rangle_\varphi$ studied in [17] (see also [18], [28]).

Theorem 1 *Let $\varphi \in \Phi$ be a non-degenerate function, (X_0, X_1) a couple of Banach function spaces and E a Banach space.*

(i) *If (X_0, X_1) is regular and $\varphi(1, s)\, \varphi(1, t) \le C\, \varphi(1, st)$ for some $C > 0$ and all $s, t > 0$, then*

$$G_\varphi(X_0 \tilde{\otimes}_\varepsilon E, X_1 \tilde{\otimes}_\varepsilon E) \hookrightarrow G_\varphi(X_0, X_1) \tilde{\otimes}_\varepsilon E.$$

(ii) *If X_0, X_1 are 2-concave and E has cotype 2, then*

$$G_\varphi(X_0, X_1) \tilde{\otimes}_\varepsilon E \hookrightarrow G_\varphi(X_0 \tilde{\otimes}_\varepsilon E, X_1 \tilde{\otimes}_\varepsilon E).$$

In particular, if φ, X_0, X_1 and E satisfy the assumptions of (i) and (ii), then we have the equality

$$G_\varphi(X_0, X_1) \tilde{\otimes}_\varepsilon E = G_\varphi(X_0 \tilde{\otimes}_\varepsilon E, X_1 \tilde{\otimes}_\varepsilon E).$$

PROOF. Note first that since $\varphi \asymp \overline{\varphi}$ we have $G_\varphi = G_{\overline{\varphi}}$, hence, we may assume without loss of generality that $\varphi \in \mathcal{U}$. Moreover, since it is well-known that a Banach function space of non-trivial concavity has an order-continuous norm, the simple functions in such a space are dense. In consequence, the couple $\overline{X}$ in both considered cases is regular.

We start with the proof of (ii): It follows from Proposition 3 in [25] that any couple of Banach function spaces has l.u.st. with l.u.st. constant 1. This shows that there exists a regular cofinal interpolation triple

$$A = (\overline{X}, \Delta(\overline{X}), \mathcal{A})$$

with $\mathcal{A} \subset \mathcal{F}in(\Delta(\overline{X}))$ containing such finite-dimensional subspaces M for which the couple (M_0, M_1) with $M_0 = M_1 = M$ is isomorphic to a Banach couple (N_0, N_1) with a monotone unconditional basis, and $d(\overline{M}, \overline{N}) \le 2$. It is clear that

$$B := ((E, E), E, \mathcal{F}in(E))$$

forms a regular cofinal interpolation triple. Since φ is non-degenerate, we conclude from [18] or [20] that G_φ is a computable functor. By Proposition 1 this yields that G_φ is approximable on any cofinal interpolation triple, in particular on A and B. By Lemma 1, (ii) it remains to show that

$$R_{G_\varphi}(A,B) = \sup_{M\in\mathcal{A}} \sup_{N\in\mathcal{F}in(E)} r_{G_\varphi}(M_0,M_1;N,N) < \infty.$$

Again, since φ is non-degenerate, for any couple $\overline{Y}$ of Banach function spaces

$$G_\varphi(\overline{Y}) = (\varphi_\ell)^\circ(\overline{Y}),$$

with universal constants for the equivalence of norms neither depending on φ nor on $\overline{Y}$ (see [18] or [28], p. 466). This implies that

$$G_\varphi = \varphi_\ell$$

on the class of all regular finite-dimensional couples of Banach spaces, with universal constants for the equivalence of norms. Now the conclusion follows from Lemma 4, (ii) and Proposition 3 (note that $\mathbf{M}_{(\mathbf{2})}$ respects sublattices and $\mathbf{C_2}$ subspaces).

The proof of (i) is similar: We now apply the first instead of the second part of Lemma 1. In the proof of (ii) we saw that G_φ is approximable on every cofinal interpolation triple, in particular on

$$((X_0\tilde{\otimes}_\varepsilon E, X_1\tilde{\otimes}_\varepsilon E), \Delta(\overline{X})\otimes E, \mathcal{C}),$$

where

$$\mathcal{C} := \{M\otimes N;\ M\in\mathcal{A},\ N\in\mathcal{F}in(E)\}$$

and $\mathcal{A}$ as in the proof of (ii). Hence, by Lemma 1, (i) it suffices to check that

$$L_{G_\varphi}(A,B) := \sup_{M\in\mathcal{A}} \sup_{N\in\mathcal{F}in(E)} \ell_{G_\varphi}(M_0,M_1;N,N) < \infty.$$

But this, similar to what was done in the proof of (ii), follows from Lemma 4, (i) and Proposition 2. ■

Acknowledgement. The research of Mieczysław Mastyło was supported by KBN Grant 2 PO3A 042 18.

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