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Teacher mathematics in pre-service biology teachers: population growth' logistic model

Enseñar matemática en la formación de Profesores en Biología: modelo logístico de crecimiento poblacional

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Abstract

This paper describes and analyzes construction' process and reconstruction of mathematical and non-mathematical knowledge emerged in the study of the logistic model of population growth. This experience was carried out in first year pre-service university biology teacher (PUB) of the Facultad de Ciencias Exactas, Químicas y Naturales (FCEQyN) of the Universidad Nacional de Misiones (Argentine) in Mathematics course. This study is part of design and implementation of a didactic device called study and research paths (SRP). This device is framed in anthropological theory of didactics (ATD), which aims to overcome phenomenon called monumentalization of knowledge, and contribute to paradigm's change at way of mathematics' teaching. The aim of this experience is to transcend traditional approach where the teacher explains and students repeat mechanized techniques. So far, results obtained are encouraging, since this way mathematics' study has given back a sense and usefulness to mathematics studied by Biology teacher training program.

Keywords: population growth' logistic model; university; pre-service biology teachers; Mathematics; study and research paths.

Resumen

Este trabajo describe y analiza el proceso de construcción y reconstrucción de los saberes matemáticos y no matemáticos emergidos durante el estudio del modelo logístico de crecimiento poblacional. Esta experiencia se llevó a cabo por estudiantes de primer año del Profesorado Universitario en Biología (PUB) de la Facultad de Ciencias Exactas, Químicas y Naturales (FCEQyN) de la Universidad Nacional de Misiones (Argentina) en la asignatura Matemática. Este estudio forma parte del diseño e implementación de un dispositivo didáctico denominado Recorrido de estudio e investigación (REI). Este dispositivo, se encuadra en la teoría antropológica de lo didáctico (TAD), con el que se pretende soslayar el fenómeno denominado *monumentalización del saber,* y contribuir a la promoción de un cambio de paradigma en la forma de enseñar matemática. Se pretende, a partir de esta experiencia didáctica, trascender el enfoque tradicional donde el profesor explica y los estudiantes repiten técnicas mecanizadas. Hasta el momento los resultados obtenidos son prometedores pues, esta manera de estudiar Matemática ha permitido a los estudiantes para profesor en Biología encontrar un sentido y una utilidad a la Matemática estudiada.

Palabras clave: Modelo logístico de crecimiento poblacional; Universidad; Profesorado en Biología; Matemática; Recorrido de Estudio e Investigación.

Introduction

There are several pieces of research in the field of Didactics of Mathematics which refer to the different contributions that emerge from the analysis of practices involving the use and interpretation of mathematical models at the university (Bassanezi and Biembengut, 1997 [1]; Bosch et al., 2006 [2]; Barquero et al., 2006 [3], 2010 [4], 2014 [5]; Barquero, 2009 [6]; Biembengut and Hein, 2004 [7]; Ruiz Higueras and García García, 2011

[8]; Costa, 2013 [9]; Parra, 2013 [10]; Oliveira-Lucas, 2015 [11]; Sureda and Rossi, 2022 [12]). In Barquero et al. (2014) [5], it is stated that, presently, there is no discussion of the convergence of thought around the need to incorporate mathematical modelling in mathematical practices at all levels of schooling. However, these authors state that, under the influence of the dominant epistemology, this is the concrete way in which society, the university as a teaching institution and the study community (teachers and students) understand what mathematics is, how it is constructed and used; modelling has been restricted to the level of *applicationism*. From this perspective, mathematical modelling activity has been identified as "...a mere 'application' of previously constructed mathematical knowledge or, in its most extreme case, as a simple 'exemplification' of mathematical tools in certain extra-mathematical contexts artificially constructed for this purpose" (p. 89).

Didactic research linked to mathematical modelling within the framework of the Anthropological Theory of the Didactic (ATD) has been oriented to investigate, on the one hand, how mathematical modelling processes could improve the teaching and understanding of mathematical concepts. Moreover, on the other hand, how to get students to develop modelling skills associated with a specific professional field, if mathematical modelling is conceived as a means and an end to teaching mathematics, establishing a relationship with a particular discipline of the professional field in question (Bosch et al., 2006) [2]. In this way, mathematical modelling is thought of from a co-disciplinary perspective, in which intra- and extra-mathematical modelling converge.

Furthermore, Barquero et al. (2006) [3] emphasise that although the faculties linked to experimental sciences constitute an ideal space for teaching mathematical modelling, the knowledge taught generally represents finished, crystallised mathematical organisations. These make almost no reference to the context and origin in which they were created. In the process of didactic transposition, what is linked to the modelling activity is lost. Barquero (2009) [6] points out that research on mathematical modelling, in general, is divided into two categories: those that consider mathematical modelling as content to be taught and those that consider modelling as a means to teach mathematics. In this regard, Bassanezi and Biembengut (1997) [1] propose the concept of "simplified mathematical modelling" to distinguish the modelling process carried out in the scientific community from that carried out at the school level. To this end, the authors suggest adapting the mathematical modelling process according to the contents established in a regular course syllabus.

Following the points described above, the concern arises to investigate which mathematical, biological, chemical and physical organisations can be reconstructed through the study of a solid problematic question and, in turn, in what ways these would allow the development of mathematical modelling practices in the subject Mathematics of the Biology Teacher Training Course.

This paper aims to describe and analyse the process of construction and reconstruction of mathematical and non-mathematical knowledge that emerged during the study of the logistic model of population growth within the framework of the implementation of a didactic device called Study and Research Course (SRC) with first-year students of the Biology Teacher Training Course.

Theoretical framework: The Anthropological Theory of the Didactic (ATD)

The theoretical underpinnings that guided this research work are circumscribed in the ATD. This theory conceives mathematical activity as a human activity that can be modelled through praxeologies or mathematical organisations. This term includes two blocks of organisation of knowledge: that of praxis and that of logos. The first corresponds to 'know-how', specifically to the problems and techniques constructed and used for their treatment. Meanwhile, the second comprises the discourse that describes, explains, justifies and even produces the techniques used (Chevallard, 1999 [13], 2019 [14]).

ATD questions the approach to topics in school institutions since, in general, they do not provide a problematisation of the origin and functionality of their existence in mathematics curricula. Therefore, mathematics is conceived as a set of organisations or mathematical works already constructed, disconnected among themselves and from other disciplines, lacking meaning and functionality where it is only possible to visit them but not to question and construct them. Chevallard (2007 [15], 2017 [16], 2019 [14], 2022 [17]) explains this situation by using the analogy of *a visit to a museum*, where mathematics plays the role of a work of art, the students, the role of museum visitors, and the teacher, the tour guide. Under this conception, mathematics teaching is described as a visit to a work of art that the visitors cannot manipulate and is there to be admired and revered. This phenomenon is called *knowledge monumentalisation*. In this sense, ATD promotes a paradigmatic change oriented towards the paradigm of questioning the world, where mathematical modelling acquires a primordial place in the teaching of mathematics (Gascón, 2022) [18]. Here, mathematical modelling is thought of from a co-disciplinary perspective, in which intra- and extra-mathematical modelling converge.

The notion of *study and research courses* (SRC) is then presented as a didactic device whose primary function is to bring mathematical modelling to life in teaching systems and, in addition, to promote a paradigm shift in the way mathematics is taught. The SRC are characterised by the construction (and reconstruction) of mathematical (and non-mathematical) knowledge as an answer to a question in a strong sense called the *generative question*. Questions that come from the discipline in question (biology in this case) and whose answer is not the simple search for information on Internet sites, textbooks or by going to the teacher, but requires the generation of other sub-questions (called *derived questions*) and the construction of models that allow to bring into play, collectively, mathematical and non-mathematical knowledge.

Methodology

This is a qualitative, exploratory and descriptive research. This type of study allows us to gain in-depth knowledge of specific characteristics, properties or profiles of particular objects, groups of people or phenomena that we wish to investigate (Hernández Sampieri et al., 2014) [19]. In this case, the aim is to characterise the process of construction and reconstruction of mathematical and non-mathematical knowledge that emerged during the study of the logistic model of population growth in a regular course with first-year students of the Biology Teacher Training Course (PUB, for its name in Spanish) of the School of Exact, Chemical and Natural Sciences (FCEQyN, for its name in Spanish) of the National University of Misiones (Argentina) in the subject Mathematics. The subject is a year-long, and before the implementation of the SRC, the first units of the course syllabus had already been studied. The mathematics syllabus begins with an introduction to propositional logic and set theory, then continues with the study of real numbers, and ends the first four-month period with the development of real functions of real variables.

The information presented and analysed in this paper is part of a research project belonging to the FCEQyN of the National University of Misiones. It is also part of the development of a doctoral thesis project being carried out by one of the authors.

Additionally, the data analysed in the discussion section of the results come from the written or audio records of the observations made in 6 (six) classes out of 28 (twenty-eight) classes during the second four-month period of 2023. These six classes were selected rather than others because they were the ones where the construction of the logistic model of population growth was carried out. The study and investigation of this model are part of the design and implementation of the SRC, the genesis of which lies in the generative question:

***Q_0*: How can it be determined whether certain bodies of water are suitable for aquatic life?**

Based on the vast array of derived questions that emerged, one of the decisions taken by the study community was about which path to take to study the generative question. A collective decision was to study the dynamics of a population since the problem question refers to bodies of water that are suitable for aquatic life, so they considered it necessary to investigate the population behaviour of the different living organisms present in a body of water.

The implementation of the SRC began at the beginning of the second four-month period of the year 2023 in the subject of Mathematics in the first year of the PUB. To this end, we had the presence and collaboration of a graduate in Genetics, who contributed to the study of biological knowledge during the development of the lessons.

Twenty-three students participated in the implementation and were divided into seven study groups: four groups of four students, one group of three students and two groups of two students each. Data recording was carried out using different instruments. The classes were recorded in general videos of the course, field notes were taken, and each group had to prepare a field log of what was being investigated and studied regarding the problematic question and the derived questions. Throughout the four-month period, they made three digital deliveries per group, and, specifically in the case of the logistical model, they made multimedia presentations about what they had researched on this model, which were presented orally.

Finally, the extracts of the students' productions presented here correspond to the final presentation, in digital format, of a group field log made throughout the implementation of the SRC.

Analysis and discussion of the results

The classroom implementation of the generative question generated a vast array of derived questions that, for the sake of length, are not presented in this paper. This section only describes the students' responses to the derived questions associated with the logistic model of population growth. It should be clarified that prior to working with this model, the students asked several derived questions linked to the study and investigation of the exponential growth model, but due to the length of the manuscript, it is not possible to present their analysis in this paper.

After working with the exponential growth model, the teacher suggested the question in Figure 1. This derived question arose because the students stated that in reality, a population does not grow indefinitely over time since, if this were the case, the availability of resources in their environment would be unlimited.

What modifications will need to be made to the exponential growth model to take into account environmental constraints and resource availability?

Figure 1: Question derived from the Q_0 question, which emerged after studying the exponential growth model.

To provide an answer to this question, the students investigated how to represent, through a mathematical model, these environmental and resource constraints. Maximum carrying capacity was one of the key concepts explored for this response. Figure 2 contains part of one of the answers outlined by Group 4.

> The first thing we thought of was to use environmental and resource constraints, which would be to establish a maximum carrying capacity (K): carrying capacity is the maximum growth of any population that can be successfully maintained in a given environment over the very long term, taking into account the availability of resources needed for such species.

Figure 2: Excerpt from the answer to the derived question given by group 4.

Group 2, on the other hand, in addition to questioning about carrying capacity, asked other derived questions that allowed them to delve deeper into the problem under study. These questions refer, for example, to how carrying capacity is defined, what happens if the population exceeds the carrying capacity, what overpopulation means, and how this would affect the model, as shown in Figure 3.

During the sharing of each group's research, there was a discussion about what carrying capacity means for a population and how it is expressed in the mathematical model they presented. However, in the answers provided up to that point, there was no suggestion as to how this model was related to the one previously worked on for the exponential growth model. For this reason, the teacher in charge of implementing the SRC recommended working hypotheses linked to the birth rate and death rate of the size of a population, together with their respective variability. The teacher presented both rates and analysed how they behave in the exponential model (they remain constant) as opposed to the logistic model. Figure 4 presents this teaching intervention.

a)

The **birth rate** as a function of population size using the linear functional model can be expressed as:

$$b = b_0 - aN$$

Where b_0 is the birth rate under ideal conditions (unlimited resources and adequate space) used in the exponential growth model.

The **mortality rate** as a function of population size using the linear functional model is represented as:

$$d = d_0 + cN$$

Similarly, d_0 is the mortality rate under ideal conditions (unlimited resources and adequate space) used in the exponential growth model.

The variability of birth and death rates is based on the following assumptions:

- The per capita birth rate decreases with increasing population size.
- The per capita mortality rate increases with increasing population size.

b)

In the population model, both birth and death rates remain constant, with the model:

$$\frac{dN}{dt} = kN, \qquad N(0) = N_0$$

With $k = b - d$, where b and d are constant instantaneous birth and death rates (per capita), respectively.

Now, assuming that both rates are variable and that they respond to linear models, the exponential model can be reformulated as follows:

$$\frac{dN}{dt} = [(b_0 - aN) - (d_0 + cN)]N, \qquad N(0) = N_0$$

Figure 4: Hypotheses suggested by the teacher to study the logistic model of population growth.

By incorporating this hypothesis and these new variables, the logistic model of population growth could be worked on based on the modifications made to the exponential growth model.

One of the issues analysed by the class group arose from a new question derived question posed by the teacher: ***What does it mean that*** $\frac{dN}{dt} \to 0$ ***in the equation*** $\frac{dN}{dt} = [(b_0 - aN) - (d_0 + cN)]N$ ***?***

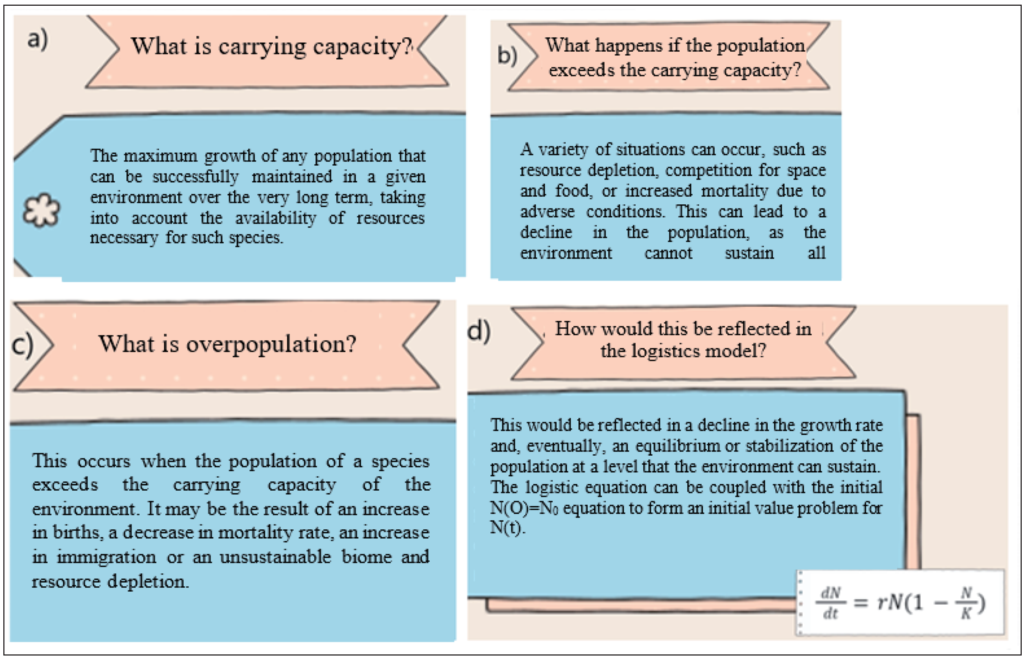

Figure 3: New derived questions asked by group 2 based on the question in Figure 1.

Students in group 1 stated that this meant that the birth and death rates were in equilibrium, as outlined in Figure 5 (students do not use subscripts in this case). Following the response given by the students, they were asked to expand on what it meant to them that the birth and death rates were in equilibrium. At that point, they could only state that the population growth rate $\frac{dN}{dt}$ was zero when the birth and death rates were equal, but for that part of the SRC, they were still unable to relate that information to the carrying capacity.

We said that when dN/dt → 0, the death and birth rates are in equilibrium, we also mentioned b0 and d0 being the death and birth rates.

Figure 5: Excerpt from the answer given by students in group 1 to the new derived question.

This question and the answer given by the students were taken up in later classes when studying the growth and decay of functions, critical points, and the concavity of a function.

In this part, it is essential to point out that the mathematical model presented by the students involves the study of mathematical organisations referring to initial value problems, non-linear first-order differential equations, average growth rate and instantaneous growth rate, derivatives of real functions of a real variable and indefinite integrals, among others. The students had already worked on some of these mathematical organisations during the development of the exponential growth model; however, from the investigation and study of the logistic model, this mathematical knowledge could acquire a new meaning for them. At the same time, this model enabled the construction of new knowledge linked to increasing and decreasing functions, critical points, extreme values of a function, concavity, inflection point and horizontal asymptote in real functions of a real variable.

The discussion around the information provided by the mathematical model of logistic growth $\frac{dN}{dt} = rN\left[1 - \frac{N}{K}\right], N(0) = N_0$ continued from what emerged from the presentation of group 2, who early on investigated what happens to population growth by analysing the sign of the first derivative. This is shown in Figure 6 and Figure 7.

The information presented by the group made it possible, on the one hand, to undertake a qualitative analysis of the logistic model. This means obtaining information from the mathematical model about the population's behaviour without the need to solve the associated differential equation. On the other hand, it also made possible the construction of mathematical knowledge linked to the growth of a real function of a real variable, which the class group had not studied until now. The latter is considered an extremely valuable contribution to the study community since it highlights the value of the qualitative treatment of a mathematical model in relation to analytical resolution techniques, which are generally prioritised in the teaching of differential equations, as pointed out by Moreno Moreno and Azcárate Giménez (2003) [21] and Artigue (1995) [22].

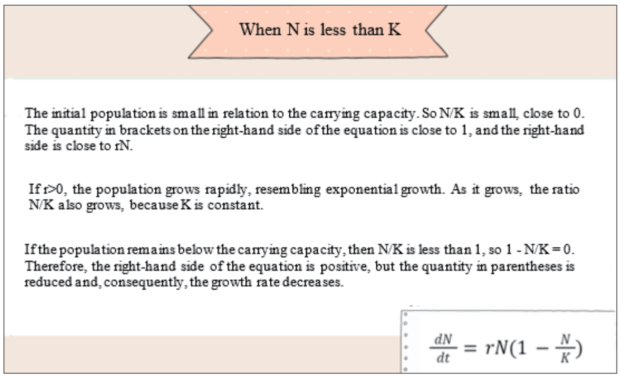

Figure 6: Excerpt from the presentation of group 2 on population growth in the logistic model.

A collective discussion about the meaning of the statements outlined both on the slide and on the blackboard was triggered following the presentation of group 2. Also, the question of where these statements are visualised in the graphical representation of the logistic model emerged. It should be noted that during the study and investigation of the exponential growth model, the graphical representation of the logistic model appeared in some groups.

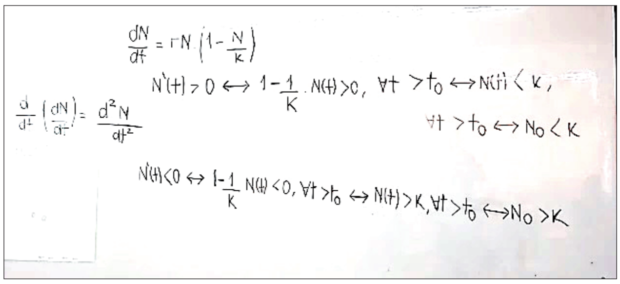

Figure 7: Excerpt from the study conducted by group 2 on the behaviour of a population under the logistic model.

Based on the discussion carried out by the study community, it was decided to introduce the derived question: ***What do*** $\frac{dN}{dt < 0}$ ***and*** $\frac{dN}{dt} \rightarrow 0$ ***mean?*** And to advance the study of the growth and decay of a function according to the sign of the first derivative.

For this question, the students stated that the information found on Internet sites was not complete, so it was suggested that they research a specific bibliography on differential calculus of real functions of a real variable, some of which are available in the university library and others in digital format.

Following the search for information on the sign of the first derivative, questions arose about whether the population growth rate behaves in the same way along the entire logistic growth curve or whether it undergoes changes. This led the students to investigate the derivative of the population growth rate and what information the sign of the second derivative in the logistic model provides. Mathematical knowledge also emerged concerning the inflection point and concavity of a real function of a real variable. Figures 9 and 10 show part of the study carried out by the students.

Figure 8 shows that the students state that 'the rate of change reaches its maximum' at the inflection point and indicate that a change of concavity occurs at that point; however, during the oral presentation of their research, they do not relate this statement to the study of the second derivative, nor what information the sign of this derivative provides concerning how the growth of a population occurs under the logistic model.

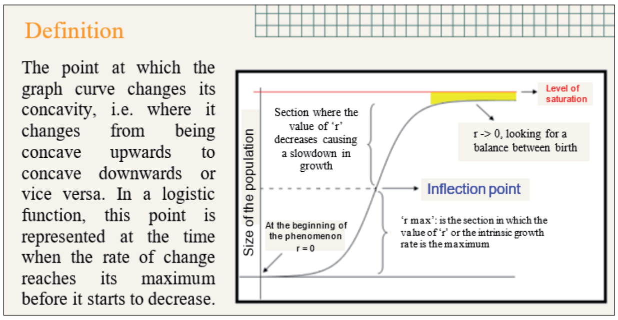

Figure 8: Excerpt from the presentation of group 4 on the study of concavity and the inflection point.

Group 2 states that the inflection point occurs in the middle of *K*, but they do not justify this statement. In addition, they indicate that 'the change in concavity is a change in the growth rate' and associate it with the second derivative. However, there is confusion with the information provided by the sign of this derivative. This is shown in Figure 9.

We look at the change in the growth rate, and what happens to growth.
The inflection point occurs when $\frac{dN}{dt}$ reaches its maximum value, at half of K, $N = \frac{K}{2}$.
The change in concavity is a change in the growth rate (second derivative).

$$\frac{d}{dt}\left(\frac{dN}{dt}\right) = \frac{d^2N}{dt^2}$$

$\frac{d^2N}{dt^2} > 0$ **the function is increasing**

$\frac{d^2N}{dt^2} < 0$ **the function is decreasing**

Figure 9: Excerpt from group 2 on the study of concavity and the inflection point.

Since the growth of a function had previously been studied by analysing the sign of the first derivative, group 2 relates the sign of the second derivative to the study of the growth of a function. Still, they do not indicate which function they are referring to: the population function N or the function associated with the derivative of this function.

Given these questions, the class group discussed together what it means that $\frac{d^2N}{dt^2} > 0$ and $\frac{d^2N}{dt^2} < 0$, and which function is increasing or decreasing, depending on the sign of this derivative. To do this, the students in group 1 made a graphical representation of the logistic curve and pointed out where the second derivative of the function N shows such behaviours. This is shown in Figure 10.

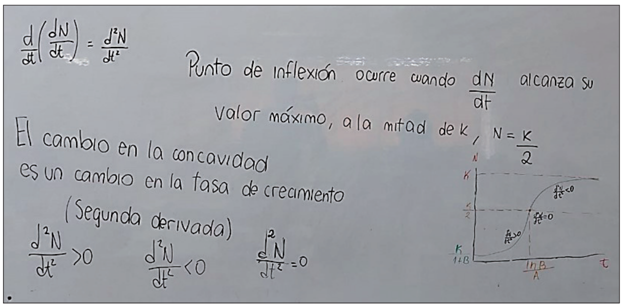

Figure 10: Graphical representation made by group 1 for the study of the concavity and the inflection point in the logistic model, which reads: *"The inflection point takes place when $\frac{dN}{dt}$ reaches its maximum value, at half of K, $N = \frac{K}{2}$. The change in concavity is a change in the growth rate (second derivative)."*

From the graphical representation, the students in group 1 pointed out that $\frac{d^2N}{dt^2} = 0$ at the inflection point, and that $\frac{d^2N}{dt^2} > 0$ is verified in the part of the curve that is concave upwards. Analogously, they said that in the part where the curve is concave downwards, it is realised that $\frac{d^2N}{dt^2} < 0$. This analysis allowed further exploration of the meaning of the statement 'the change in concavity is a change in the growth rate'. Collectively, it was concluded that the change in the growth rate is expressed by the derivative with respect to the time of the derivative of the function *N*. The teacher wrote the symbolic representation $\frac{d}{dt}\left(\frac{dN}{dt}\right) = \frac{d^2N}{dt^2}$ on the blackboard, following the conclusion developed by the study community.

Next, to analyse the statement that for $\frac{d^2N}{dt^2} > 0$ the function is increasing and that for $\frac{d^2N}{dt^2} < 0$ the function is decreasing, the teacher proposes that the students go back to the right-hand side of the equation $\frac{dN}{dt} = rN\left[1 - \frac{N}{K}\right]$ and analyse the function $f(N) = rN\left(1 - \frac{N}{K}\right)$.

It is worth mentioning that when studying the exponential population growth model, the students had studied initial value problems, so it was familiar to them to recognise in the expression on the right-hand side of the logistic equation a function that depends on . However, it was the teacher who introduced the discussion about the behaviour of this function. To do so, she wrote $f(N) = rN - \frac{r}{K}N^2$ on the board. This is illustrated in Figure 11a.

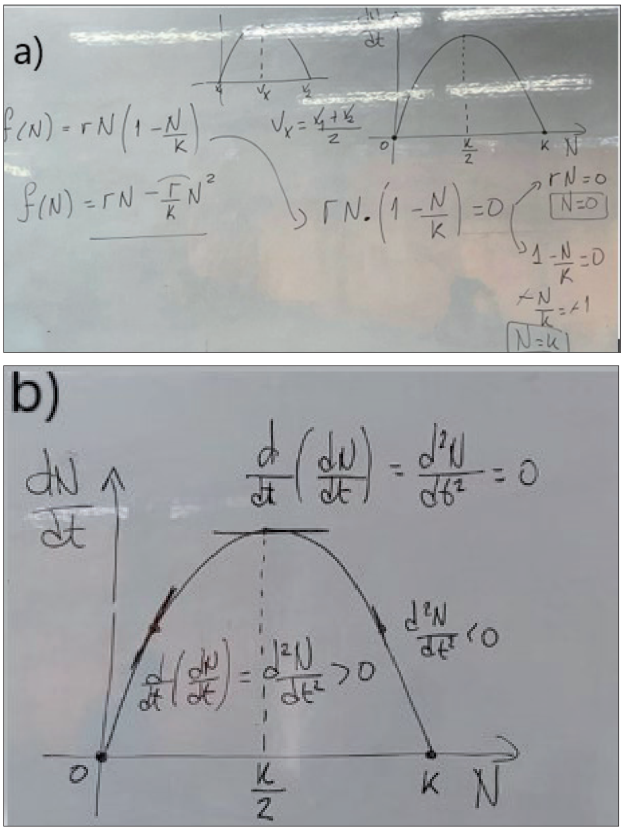

Figure 11: a) Graphical representation made by the teacher to analyse where the maximum population growth occurs. b) Analysis of the sign of the second derivative from the graphical representation of the function of the first derivative.

This equivalent symbolic expression of the function $f(N)$ was recognised by the students as a quadratic function whose graphical representation is given by a downward concave parabola since the quadratic coefficient is negative. However, for the analysis of the roots and the determination of the vertex of the parabola, it was necessary to remember how to find such points of the curve for a quadratic function in general $y = f(x)$. Thus, the students and the teacher determined that the roots were at $N_1 = 0$ and $N_2 = K$, and that the coordinate corresponding to the horizontal axis of the vertex was at the midpoint of the roots, given by the expression $V_N = \frac{K}{2}$.

Based on this, the statement written on the blackboard was brought up again: '*The inflection point occurs when* $\frac{dN}{dt}$ *reaches its maximum value, at half of K,* $N = \frac{K}{2}$'. This was analysed in the graphical representation of the function $f(N)$ as the value that the vertex of the parabola reaches for the coordinate corresponding to N. Then, a student from group 1 suggested drawing the tangent line at the vertex of the parabola and affirmed that the derivative at that point is zero. In response to this statement, the teacher asked which derivative is zero. A member of group 2 said that it is the derivative of the derivative that cancels out. As shown in Figure 12b), the teacher wrote the following on the blackboard: $\frac{d}{dt}\left(\frac{dN}{dt}\right) = \frac{d^2N}{dt^2} = 0$. Then, similarly, what happens with the slope of the tangent line for those points of the parabola that are at $0 < N < \frac{K}{2}$ and for $\frac{K}{2} < N < K$, as shown in Figure 12b) was analysed. Based on this analysis, it was concluded that for values of N within $0 < N < \frac{K}{2}$ the derivative function $\frac{dN}{dt}$ is increasing and that for $\frac{K}{2} < N < K$ the derivative function $\frac{dN}{dt}$ is decreasing.

To conclude the class, this analysis was retrieved in the logistic curve graph shown in Figure 11. This allowed us to justify the statement that, although the function $N(t)$ is increasing throughout its domain, its maximum growth occurs for $\frac{d^2N}{dt^2} = 0$, and that in the part where $\frac{d^2N}{dt^2} < 0$, although the population presents an increasing behaviour, it does so at a lower rate than for those values located to the left of the inflection point. It was also concluded that, for these values, where the function is concave upwards, the curve follows an exponential growth, as had been studied in the classes where population dynamics under the exponential growth model was discussed.

Conclusions

According to the results obtained with the implementation of the generative question in general and what emerged from the study and research of the logistic model in particular, we conclude that this type of didactic experiences are aimed at transcending the traditional approach to the teaching of mathematics. From this point of view, it is understood that this way of studying has allowed the students of the Biology Teacher Training Course not only to find meaning and utility in the mathematical knowledge studied, particularly in this case, in the logistic model but also to construct the model themselves throughout the study process. A certain autonomy has even been achieved on the part of the students and an interest in learning mathematics. On the other hand, this change of paradigm promotes substantial modifications in the teaching practices of mathematics in the university environment, questioning not only the way in which it is conceived as a science but also the epistemology that underlies the usual practices of teaching mathematics.

The study and research of the logistic model of population growth, in this implementation, has contributed to overcoming the inherent problem of the loss of meaning that frames the study of Mathematics in degree courses where the study of Mathematics is not an end in itself, but a modelling tool that allows answers to problems specific to the area of training, as in the case of PUB. It is understood that future implementations of the generative question will enrich the construction and reconstruction of new mathematical models in addition to those mentioned in this work, which will make it possible to deepen the research being carried out.

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