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Development of Sustainability Competencies in the Area of Didactics of Mathematics

El desarrollo de competencias para la sostenibilidad en el área de didáctica de las matemáticas

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Abstract ∞ This article presents the results of the analysis of the development of sustainability competencies of students enrolled in subjects in the area of Didactics of Mathematics of three different degrees in the Faculty of Educational Sciences at the Universidad de Cádiz. To carry out this analysis, the sustainability questionnaire of the EDINSOST project was used. This questionnaire consists of 18 items related to the development of the four sustainability competencies proposed by the Sectoral Commission of the Conference of Rectors of Spanish Universities. A total of 105 students responded to a questionnaire composed of items expressed like a four-point Likert scale. Three types of composite indicators were defined to analyse the students' answers in each competency, competency unit, and mastery level. The results reveal how the development of the sustainability in students depends on the degree analysed.

Keywords ∞ Critical Mathematics Education; Education for Sustainability; Teacher Training; Community Ethics; EDINSOST Project

Resumen ∞ Este artículo presenta los resultados del análisis sobre el desarrollo de competencias para la sostenibilidad de estudiantes matriculados en asignaturas del área de Didáctica de las Matemáticas de tres titulaciones diferentes de la Facultad de Ciencias de la Educación de la Universidad de Cádiz. Para realizar este análisis se ha utilizado el cuestionario de sostenibilidad del proyecto EDINSOST. El cuestionario consta de 18 ítems relativos al desarrollo de las cuatro competencias de sostenibilidad propuestas por la Comisión Sectorial de la Conferencia de Rectores de las Universidades Españolas. Un total de 105 estudiantes respondieron a un cuestionario compuesto de ítems expresados como escala Likert de cuatro puntos. Se definieron tres tipos de indicadores compuestos para analizar las respuestas dadas por los estudiantes en cada competencia, unidad de competencia y nivel de dominio. Los resultados muestran cómo el desarrollo de la sostenibilidad declarado por los estudiantes depende de la titulación analizada.

Palabras clave ∞ Educación Matemática Crítica; Educación para la Sostenibilidad; Formación de Profesores; Ética Comunitaria; Proyecto EDINSOST

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1. Introduction

Modern society was eager to prevail, and believed in the future, in science, and in technology. However, great modern ideas have been abandoned in a process of hedonistic personalisation (Lipovetsky, 2003). We are immersed in a phase of *desubstantialisation* or loss of solid foundations, the emergence of a new scenario that has been called liquid modernity (Bauman, 2015). Still, postmodernity should not be understood as a break with modernity, since its genesis lies at the core of modernity itself. However, the abandonment of a modern discourse has left postmodern man without major stimulating projects. Thus, as Lipovetsky (2003) claims, "postmodern society has no idol, no taboo, no more glorious image of itself, no inspiring historical project" (p. 9). According to authors such as Naredo (2022), postmodernity is nothing other than evidence of a lack of clear alternatives. Contrary to modernity, the pathological signs of current times are not prohibition or repression, but hyper-communication, hyper-consumption, excessive permissiveness, and the negativity of the other (Han, 2022) that dilutes the participatory civic spirit in favour of a collective narcissism (Lipovetsky, 2003). We agree with Han (2022) when he affirms that a system that rejects the negativity of the other develops self-destructive traits.

For its part, education in general and mathematics education in particular in the modern project was conceived as the acquisition of knowledge that should be treasured and preserved forever (Bauman, 2015). However, today's dominant neoliberal economy transfers to schools the reification of skills and competencies at the service of a consumer market (Radford, 2013). Knowledge today is conceived as merchandise (Bauman, 2015). In mathematics, this has led to an emphasis on what is "calculable", diluting the ethical and aesthetic, subjective, and social dimension of mathematics education (Radford, 2013). However, the socio-environmental, values, and knowledge crisis in which we find ourselves (Bonil et al., 2010) requires training critical, supportive, responsible, thinking, ethical human beings who transform their reality (Rodríguez, 2016). Educating should not be reduced to the transmission of knowledge, but should be a more encompassing project (Radford, 2013; Moreno-Pino et al., 2022). In this context, the National Council of Teachers of Mathematics affirms that mathematics education should be reviewed in depth (National Council of Teachers of Mathematics [NCTM], 1989). Mathematics and mathematics education should be configured around the development of solutions to problems created by human beings, from their lifestyles, which occur at a certain time, and in a certain context (Jaramillo, 2011). Wiek et al. (2011) point to sustainability in the educational field as an option for the construction of possible responses and oriented solutions from a planetary ethic. Authors such as Alsina and Mulà (2019) state that mainstreaming sustainability in teacher education has been identified as a key priority in authoritative international documentation, promoting a social transformation from the approach of sustainability through competency-based training of professionals (Tilbury and Wortman, 2004). Albareda et al. (2019) performed a compilation of studies in the Spanish University System that are already working in this line. The challenge of implementing suitable teaching strategies is thus presented for students –future professionals– to develop sustainability competencies (Albareda et al., 2019). However, there is a lack of literature unpacking the connections between mathematics education and sustainability (Alsina and Mulà, 2019).

In Spain, a basic reference in the field of integrating sustainability into the curriculum is the Sustainability group of the CRUE (Conference of Rectors of Spanish Universities). CRUE-Sustainability asks the entire university community to comprehensively review their curricula to ensure the integration of basic cross-curricular sustainability contents in all degrees of the Spanish university system consistent with four cross-curricular sustainability competencies (CRUE, 2012):

- SUS1: Competency in the critical contextualisation of knowledge through interrelating social, economic, and environmental issues at a local and/or global level.
- SUS2: Competency in the sustainable use of resources and in the prevention of negative impacts on the natural and social environment.
- SUS3: Competency to participate in community processes that promote sustainability.
- SUS4: Competency to apply ethical principles related to sustainability values in personal and professional behaviour.

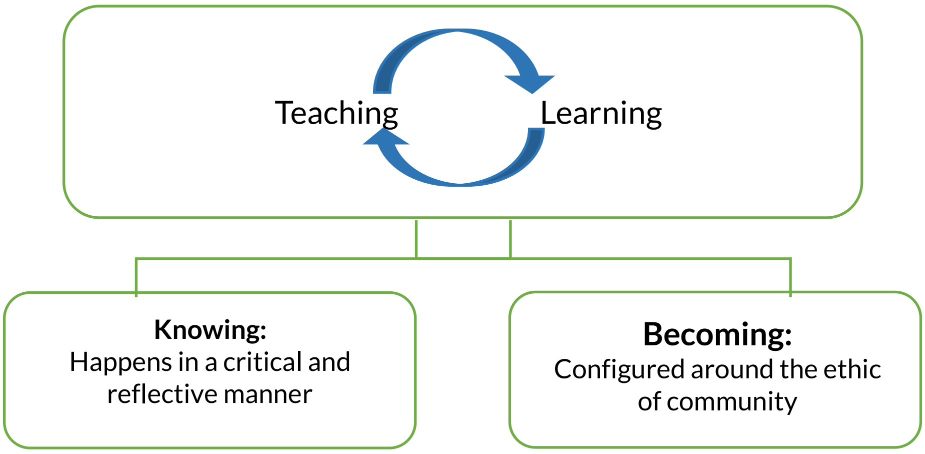
The research presented in this article aims to evaluate the development of these four sustainability competencies in accordance with the statements made by a group of students enrolled in subjects in the area of Didactics of Mathematics of three different degrees at the Universidad de Cádiz. This study is limited to the Professional Development of Teachers Research Group-HUM462 of the Universidad de Cádiz (Spain) within the framework of the EDINSOST research project in which we take part (Segalàs and Sánchez-Carracedo, 2019).

2. The socio-critical perspective of mathematics education

The formulation of new problems such as considering sustainability in the initial training of teachers in mathematics education calls for the search of conceptual frameworks that allow elucidating the ethical issue as an inherent competency in mathematical literacy. As Hernández (2022) maintains, in a process of mathematical literacy, students are not only expected to learn mathematics, but also to express themselves as human beings, as citizens.

This research is situated within the framework of critical mathematics education. The critical approach not only coincides with those points of view that understand the teaching and learning of mathematics as social phenomena, but also considers it essential to extend them to political-social aspects (Font, 2002; Godino, 2010). From critical mathematics education, mathematical literacy is conceptualised as a competency that integrates not only *mathematical knowledge*, referring to what we commonly understand as the development of mathematical skills, but also *reflective knowledge* (Skovsmose, 1994). One of the theories within the range of socio-cultural theories that seek to approach teaching and learning in terms of critical mathematics education is the theory of objectification (TO). The non-mentalistic conception of thought this educational theory assumes, far from those educational theories of an individualistic nature, implicitly frames the work presented here. The TO defines mathematics education as "a political, social, historical, and cultural effort whose purpose is to create ethical and reflective individuals who position themselves critically in historically and culturally constituted mathematical practices" (Radford, 2014, p. 135). To the TO, the teaching and learning of mathematics is conceived as a collective process through which students find systems of ideas of culture and cultural ways of being (Radford, 2014). Thus, in the TO, learning is defined in terms of two axes: the axis of *knowing* in mathematics learning and teaching, and the axis of *becoming* in mathematics learning and teaching, which run through what educational theory refers to as *joint labour* (Figure 1).

Figure 1. Joint labour (Radford, 2014)



3. Materials and Method

This study presents a methodology to evaluate the development of sustainability competencies achieved by a group of students enrolled in subjects related to the area of Didactics of Mathematics of three different degrees at the Universidad de Cádiz.

3.1. Sample

In this research, the sample consisted of 105 students of the Faculty of Educational Sciences of the Universidad de Cádiz who voluntarily responded to a questionnaire at the end of the semester. The total sample was distributed into three different degrees: Degree in Early Childhood Education (DECE), Degree in Primary Education (DPE) and Master's Degree in Teacher Training for Compulsory and Upper Secondary Education in the specialisation of Mathematics (MASE). At the time of the study, the students were enrolled in Development of Mathematical Knowledge in Early Childhood Education (DECE); Didactics of Mathematics 1 (DPE); Teaching Innovation and Initiation to Research in teaching of Science and Mathematics (MASE), the teachers of which were the first two authors of this article. Table 1 shows the distribution of student responses to the questionnaire organised by degrees.

Table 1. Distribution of student responses according to degrees

Degree	Nº of responses (%)
DECE	51 (48.57%)
DPE	32 (30.48%)
MASE	22 (20.95%)
TOTAL	**105 (100%)**

3.2. Research objectives

The general objective of this work was to enquire about the perception a group of students, enrolled in subjects related to the area of Didactics of Mathematics at the Universidad de Cádiz, has regarding their training in sustainability.

In order to respond to this general objective, we established the following three specific research objectives:

- O1: Measure the degree of agreement, using a Likert scale, with sustainability of the students of three education degrees at Universidad de Cádiz.
- O2: Analyse if the degree of agreement with sustainability of the education students is homogeneous in the three degrees or, if on the contrary, significant differences are perceived.
- O3: Measure the degree of development of sustainability competencies in the students of the three education degrees at Universidad de Cádiz, and identify if significant differences exist between the degrees.

3.3. Tools

In Spain, the Sectoral Commission of the CRUE-Sustainability (CRUE, 2012) has defined four sustainability competencies, described in section 1, which are the starting point of the sustainability competency map (SCM) for education degrees developed within the framework of the EDINSOST project (Segalàs and Sánchez-Carracedo, 2019).

An SCM is a double-entry matrix made up of a series of learning outcomes organised into a series of competency units, and a series of mastery levels for each competency unit. Table 2 shows the relationship between the competencies defined by CRUE-Sustainability and the six competency units defined within the framework of the EDINSOST project.

The SCM defined by the EDINSOST project is made up of a total of 18 learning outcomes that result from classifying the six competency units that make up the SCM into three mastery levels (L1: know, L2: understand, L3: show + do). The full version of this map for education degrees that contains the 18 learning outcomes (map cells) can be found in Muñoz-Rodríguez et al. (2020). In order to measure the degree of development of sustainability competencies of the students of the different education degrees object of this study, the tool used to gather data was the sustainability questionnaire designed within the framework of the same project.

Table 2. Competencies and competency units of the SCM. Albareda et al (2019)

SUS1	SUS1.1. Understands the functioning of natural, social and economic systems, as well as their interrelationships and problems, both at a local and global level.
	SUS1.2. Possesses critical thinking skills and creativity, taking advantage of the different opportunities that arise in planning a sustainable future.
SUS2	SUS2.1. Designs and develops actions, making decisions that take into account environmental, economic, social, cultural and educational impacts to improve sustainability (includes anticipatory thinking).
SUS3	SUS3.1. Promotes and participates in community activities that promote sustainability.
SUS4	SUS4.1. Is consistent in actions, respecting and valuing (biological, social and cultural) diversity and committed to improving sustainability.
	SUS4.2. Promotes an education in values oriented towards training responsible, active and democratic citizens.

The EDINSOST sustainability questionnaire includes one item for each of the learning outcomes that make up the SCM, that is, 18 items. Table 3 shows the relationship between the items of the questionnaire and the learning outcomes (map cells) that configure the SCM of the EDINSOST project.

Table 3. Relationship between the items of the questionnaire and the SCM

	SUS1		SUS2	SUS3	SUS4	
	SUS1.1	SUS1.2	SUS2.1	SUS3.1	SUS4.1	SUS4.2
Level 1 (L1)	Q1	Q4	Q7	Q10	Q13	Q16
Level 2 (L2)	Q2	Q5	Q8	Q11	Q14	Q17
Level 3 (L3)	Q3	Q6	Q9	Q12	Q15	Q18

The degree of agreement of a student with the items stated in the questionnaire is indicated by means of a 4-point Likert scale: "strongly disagree" (0), "disagree" (1), "agree" (2), and "strongly agree" (3). An even number of points on the Likert scale was chosen to encourage students to position themselves towards the "agree" or "disagree" option, thus avoiding the existence of neutral responses. The sustainability questionnaire defined by the EDINSOST project is shown in Appendix, and underwent a rigorous validation process (Muñoz-Rodríguez et al., 2020).

3.4. Statistical data analysis

Because of the quantitative nature of the study, the analysis system employed was based on the use of techniques characteristic of statistical analysis. The construction of composite indicators was therefore chosen. To do this, a confirmatory exploratory analysis was previously carried out in order to verify the relevance and feasibility of creating these indicators. The two stages that constitute this analysis are briefly explained below.

First, an exploratory analysis of the responses to the questionnaire was carried out. On the one hand, it was found that there were no lost values. On the other hand, the normality of the distributions of the items of the questionnaire was analysed. The Shapiro-Wilk test indicates that statements 16 and 17 of the questionnaire are not normally distributed.

Secondly, in order to validate the construction of aggregate indicators, and as a consequence of the fact that not all the items in the questionnaire are normally distributed, the distributions of the 18 items were standardised. Next, reliability and validity tests were carried out on the entire questionnaire. With respect to the reliability analysis, Cronbach's alpha coefficient = 0.914 indicates that the items are internally consistent and, therefore, reliable. In addition, the withdrawal of any item from the questionnaire does not imply improvements in this coefficient greater than 0.01. As for the validity analysis, a factorial analysis was performed. This analysis is reliable since the KMO coefficient = 0.863, the determinant of the correlation matrix is 0.000, and Bartlett's test of sphericity indicates that the items are significantly related ($\chi 2=963.522$; $p=0.000$; $gL=153$). In addition, the result of the factorial analysis reveals that the items can be reduced to a factor that represents 69.32% of variability. Even so, the exploratory analysis of the questionnaire as a whole, shows that the construction of composite indicators is possible, reliable, and valid. Three types of composite indicators were developed based on the competencies, competency units, and mastery levels that make up the SCM. For the first indicator, the values of all the questionnaire items linked to the mastery levels of the respective four competencies in the SCM were added. For the second, the values of all the items of the questionnaire linked to the mastery levels of the respective six competency units of the SCM were added. For the third and last indicator, the values of all the questionnaire items related to each of the three mastery levels were added.

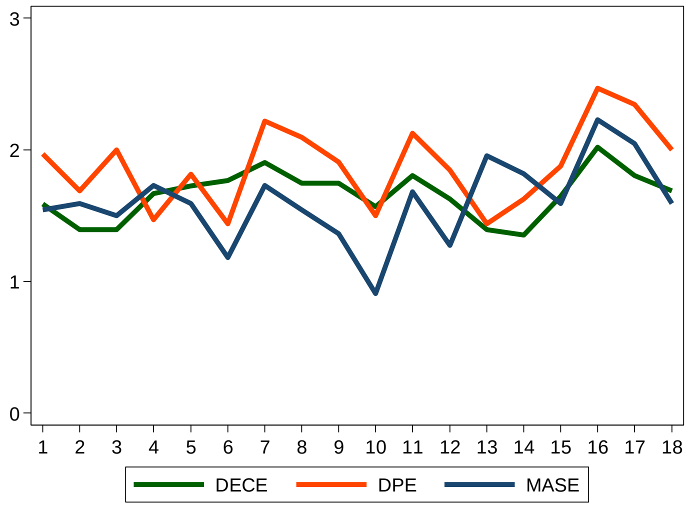
4. Results and discussion

The results and discussion of this study are presented below, following the order established for the research objectives stated in section 3.2.

4.1. Students' degree of agreement with sustainability

Figure 2 shows, in absolute terms, the degree of agreement of the students of the DECE, the DPE, and the MASE with each item of the sustainability questionnaire. The 18 items of the sustainability questionnaire are represented on axis X. The average value of the responses calculated for each item of the sustainability questionnaire is represented on axis Y (Likert scale 0-3).

Figure 2. Average degree of agreement with the 18 items of the sustainability questionnaire of the students of the three degrees analysed



In a first graphic interpretation of the results, as the three lines drawn seem to be contained in a "band", it can be said that the answers of the students of the three degrees analysed are similar. However, the MASE stands out because slightly lower scores were obtained in several items compared to the other two degrees (DECE and DPE). The minimum level of agreement of the MASE students is shown in item Q10: "I am aware of community educational programmes that encourage participation and commitment in socio-environmental improvement." It is worth noting that the two lines in the graphical representation corresponding to the DECE and the DPE present relative minima precisely in Q10. The DPE is the degree that obtains slightly higher scores than the other two degrees (DECE and MASE), but not for all the items. In the following section, we analyse whether the differences of means in the answers given by the students of the three degrees analysed are statistically significant or not.

4.2. Analysis of the differences of means between degrees

To respond to the second objective of our research (O2): "Analyse if the degree of agreement with sustainability of the education students is homogeneous in the three degrees or, if, on the contrary, significant differences are perceived", Table 4 was developed.

Table 4 shows the ANOVA test and the post-hoc analysis with multiple pairwise comparisons according to the degrees that were object of this study, and for each of the 18 items that make up the sustainability questionnaire. The p-values that identify significant differences ($p < 0.05$ for a confidence level of 95%) are shown in the table shaded in grey.

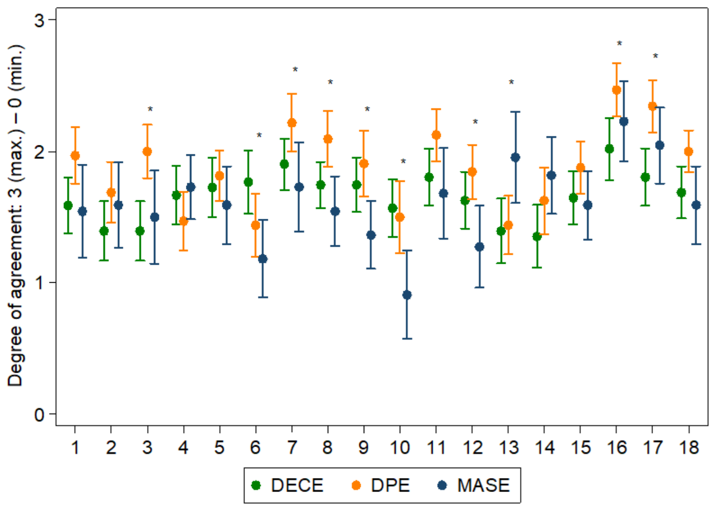
Table 4. Analysis of the differences of means (ANOVA) for each of the items of the sustainability questionnaire with multiple pairwise comparisons

Item	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
1	3.35	0.0388	DPE vs DECE	.380515	0.069
			MASE vs DECE	-.042781	0.973
			MASE vs DPE	-.423295	0.110
2	1.67	0.1942	DPE vs DECE	.295343	0.217
			MASE vs DECE	.198752	0.579
			MASE vs DPE	-.096591	0.896
3	6.91	0.0015	DPE vs DECE	.607843	0.002
			MASE vs DECE	.107843	0.849
			MASE vs DPE	-.5	0.055
4	1.13	0.3284	DPE vs DECE	-.197917	0.456
			MASE vs DECE	.060606	0.944
			MASE vs DPE	.258523	0.412
5	0.65	0.5249	DPE vs DECE	.08701	0.860
			MASE vs DECE	-.134581	0.755
			MASE vs DPE	-.221591	0.525
6	4.85	0.0098	DPE vs DECE	-.327206	0.174
			MASE vs DECE	-.582888	0.014
			MASE vs DPE	-.255682	0.489
7	3.70	0.0281	DPE vs DECE	.316789	0.130
			MASE vs DECE	-.174688	0.611
			MASE vs DPE	-.491477	0.040
8	5.83	0.0040	DPE vs DECE	.348652	0.044
			MASE vs DECE	-.199643	0.441
			MASE vs DPE	-.548295	0.007
9	4.05	0.0203	DPE vs DECE	.161152	0.592
			MASE vs DECE	-.381462	0.105
			MASE vs DPE	-.542614	0.022
10	5.97	0.0035	DPE vs DECE	-.068627	0.925
			MASE vs DECE	-.659537	0.005
			MASE vs DPE	-.590909	0.024
11	3.00	0.0541	DPE vs DECE	.321078	0.144
			MASE vs DECE	-.122103	0.800
			MASE vs DPE	-.443182	0.087
12	4.30	0.0161	DPE vs DECE	.216299	0.398
			MASE vs DECE	-.354724	0.147
			MASE vs DPE	-.571023	0.016
13	4.21	0.0174	DPE vs DECE	.045343	0.968
			MASE vs DECE	.562389	0.023
			MASE vs DPE	.517045	0.064

Item	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
14	3.13	0.0477	DPE vs DECE	.272059	0.297
			MASE vs DECE	.465241	0.065
			MASE vs DPE	.193182	0.664
15	1.66	0.1952	DPE vs DECE	.227941	0.297
			MASE vs DECE	-.05615	0.943
			MASE vs DPE	-.284091	0.287
16	3.71	0.0278	DPE vs DECE	.449142	0.028
			MASE vs DECE	.207665	0.542
			MASE vs DPE	-.241477	0.495
17	6.08	0.0032	DPE vs DECE	.539828	0.003
			MASE vs DECE	.241533	0.391
			MASE vs DPE	-.298295	0.297
18	3.48	0.0345	DPE vs DECE	.313725	0.091
			MASE vs DECE	-.095365	0.838
			MASE vs DPE	-.409091	0.068

Figure 3 is a box chart that synthesises the values shown in Table 4. Again, each of the 18 items that make up the sustainability questionnaire are identified on axis X. Axis Y represents, in absolute terms (0-3 Likert scale), the average degree of agreement with each of the items in the sustainability questionnaire of the students of each of the three degrees analysed. The p-values that identified significant differences in Table 4 are marked with an asterisk (*) in Figure 3.

Figure 3. Box plot on the differences in mean values for each item of the sustainability questionnaire in the three degrees analysed



When analysing the differences of means of the responses of the students of the three degrees, statistically significant differences were found in 10 of the 18 items of the sustainability questionnaire. The analysis with multiple pairwise comparisons between degrees shows that 7 of these significant differences are identified in the MASE in items Q6, Q7, Q8, Q9, Q10, Q12, and Q13. Furthermore, the average scores in these items were clearly lower in the MASE compared to the mean scores identified in the DECE and the DPE, except for item Q13, in which the score in the MASE was higher. This fact could be explained, at least partially, by the effect described by Kruger and Dunning (1999) according to which the least prepared and least skilled individuals with limited knowledge have an illusory feeling of superiority. This could be the case for undergraduate students. However, the MASE is a degree characterised by professionalising students who already have of a bachelor's degree. This may be the reason why they are more critical, thus acknowledging they do not know about several issues regarding sustainability. Finally, it should be noted that items Q7, Q8, and Q9 are related to the development of competency SUS2, while Q10 and Q12 are linked to competency SUS3 (see Table 3).

4.3. Development of sustainability in the area of Didactics of Mathematics

To facilitate the reading of this section, the results are presented in three subsections in which, in accordance with the methodology described in section 3, the development of sustainability is analysed in:

1. Each of the four sustainability competencies.
2. Each of the six sustainability competency units.
3. Each of the three mastery levels (know, understand, show + do) of each sustainability competency.

4.3.1. Development of sustainability in each sustainability competency

Figure 4 shows the average development achieved, expressed in percentages, in each of the four sustainability competencies and its weighted average in the area of Didactics of Mathematics.

Figure 4. Average development achieved in each sustainability competency in the area of Didactics of Mathematics, and weighted average

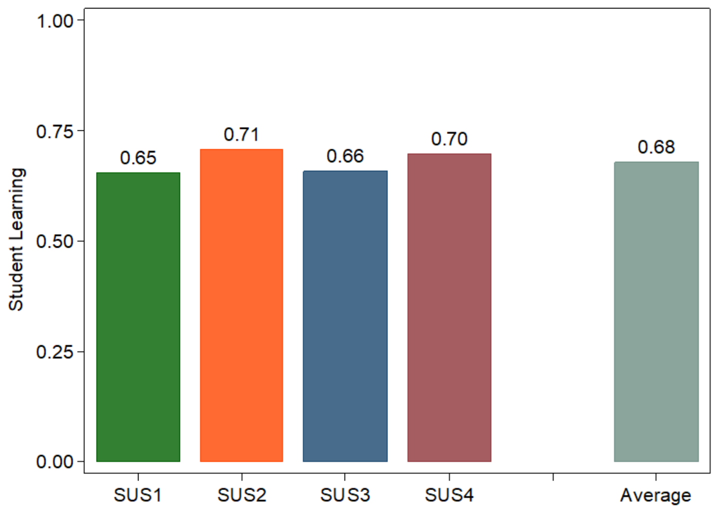
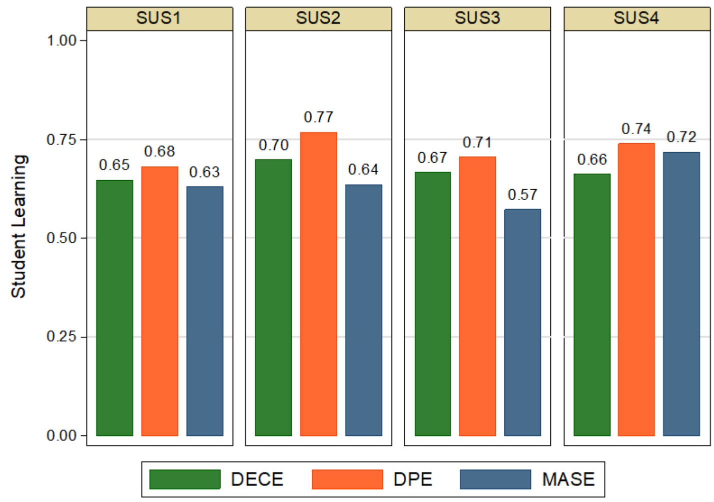


Figure 5 shows the development achieved, expressed in percentages, in each of the four sustainability competencies organised by degrees: DECE, DPE and MASE.

Figure 5. Development achieved in each sustainability competency in the area of Didactics of Mathematics organised by degrees



As shown in Figure 4, the average development of the four sustainability competencies is uniform and similar to the global average (68%). The analysis by degrees shows how the development of sustainability in the MASE is significantly lower in all the competencies except for competency SUS4. For its part, the DPE is the degree that achieves a greater development of sustainability in the four sustainability competencies (Figure 5). Table 5 shows the analysis of the differences of means in each of the four sustainability competencies with multiple pairwise comparisons according to the degrees. The p-values that identify a significant difference ($p < 0.05$) are shaded in grey.

Table 5. Analysis of the differences of means (ANOVA) in each competency by degrees, and multiple pairwise comparisons

SUS	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
1	1.17	0.316	DPE vs DECE	.035	0.494
			MASE vs DECE	-.016	0.887
			MASE vs DPE	-.052	0.367
2	6.05	0.003	DPE vs DECE	.069	0.094
			MASE vs DECE	-.063	0.210
			MASE vs DPE	-.132	0.004
3	5.85	0.004	DPE vs DECE	.039	0.483
			MASE vs DECE	-.095	0.038
			MASE vs DPE	-.134	0.005

SUS	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
4	4.15	0.019	DPE vs DECE	.077	0.026
			MASE vs DECE	.055	0.224
			MASE vs DPE	-.021	0.819

It is interesting to note the significant differences identified in competency SUS2 and competency SUS3 in the analysis with multiple pairwise comparisons between the MASE and the bachelor's degrees. This result is consistent with the analysis made in section 4.2. Since competency SUS4 is configured around two different competency units: SUS4.1 and SUS4.2 (see Table 2), the analysis of the significant difference identified in SUS4 in the DECE and the DPE will be analysed in the next section.

SUS2 is a competency that encompasses anticipatory thinking, reason for which its strengthening inevitably involves, in initial teacher training, the approach of reflective mathematics ensuring consistency between the narrative language of reality and the paradigmatic language characteristic of mathematics (Bruner, 2010). As Moreno-Pino et al. (2022) point out, a powerful possibility to achieve this purpose is to involve pre-service teachers in mathematical modelling processes. However, we agree with Wittgenstein (2017) when he states that the act of formalising a narrative language is an important epistemological step, which may lead to a productive act or, on the contrary, may be extremely simplifying (Skovsmose, 1994). The mathematical modelling of problems is here assumed not from a conception of language as a univocal image of a certain reality (principle of isomorphy), but from the theory of the endeavour of language: of its uses and its possible games (Wittgenstein, 2017). From this perspective, the integration of principles of sustainability in mathematics education is possible if, first of all, students are engaged in problems that involve the construction of normative systems that bring about value judgments on a phenomenon or fact of interest. Secondly, as Skovsmose (1994) claims, we ensure addressing the problems, uncertainties and blind spots associated with the transitions between the different uses and language games involved in the cyclical, non-linear, and reversible process of mathematical modelling.

SUS3 is a competency in crisis in post-modern societies. According to Lipovetsky, we are currently experiencing a new phase in the history of Western individualism, a historical mutation still in progress that is associated with hedonistic, permissive, and psychological values that have generated a legitimate imprecision of beliefs, roles, and habits of the contemporary individual (Lipovetsky, 2003). This personalisation process, as the author calls it, is imposed on collective programmes and forms of actions, however attractive they may be. This does not mean that the contemporary individual is disconnected from the social, but it does mean that the personalisation process delves into sameness processes that encourage the individual to connect mostly with groups of peers, with whom they share the same interests or identical concerns. Collective narcissism that expels the other (Han, 2022) weakens the global perception. We consider that participation in processes of intellectual deconstruction, for instance from ethno-mathematics, would favour the development of cultural sustainability in mathematics education training; learning from what exists, and recognising the cultural and identity values of each society. It is by confronting the face of the other (the different) that true community ethic emerges (Lévinas, 1999; Radford and Lasprilla, 2022). "Its essence is pain. But sameness doesn't hurt" (Han, 2022, p. 12).

4.3.2. Development of sustainability in each sustainability competency unit

Figure 6 shows the average development achieved, expressed in percentages, in each of the six sustainability competency units in the area of Didactics of Mathematics.

Figure 6. Average development achieved in each sustainability competency unit in the area of Didactics of Mathematics

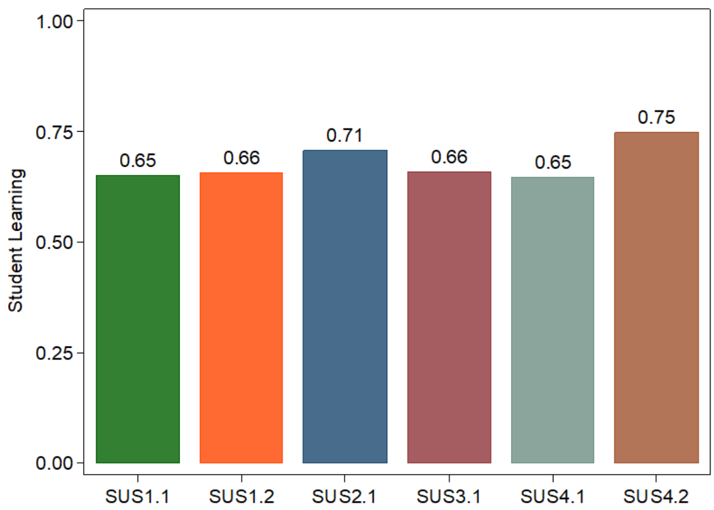


Figure 7 shows the development achieved, expressed in percentages, in each of the six sustainability competency units organised by degrees: DECE, DPE, and MASE.

Since SUS2 and SUS3 have a single competency unit (SUS2.1 and SUS3.1 respectively), the data shown for these competencies in Figures 6 and 7 are identical to those shown earlier in Figures 4 and 5. It is therefore not necessary to repeat their analysis. An analysis of competencies SUS1 and SUS4 is performed, as they are both configured around two different competency units: SUS1.1-SUS1.2 and SUS4.1-SUS4.2 (see Table 2).

As shown in Figure 6, the average development of the two competency units corresponding to SUS1 is practically identical (SUS1.1 = 65%, SUS1.2 = 66%), unlike what happens with the average development of the two competency units corresponding to SUS4 (SUS4.1 = 65%, SUS4.2 = 75%). However, the analysis by degrees reveals how the development achieved in competency units SUS1 and SUS4 is unequal in the degrees, with the exception of the two SUS1 competency units in the

MASE ($SUS1.1_{MASE}$ = 64%, $SUS1.2_{MASE}$ = 63%), where it is uniform and similar to the average.

Figure 7. Development achieved in each sustainability competency unit in the area of Didactics of Mathematics organised by degrees

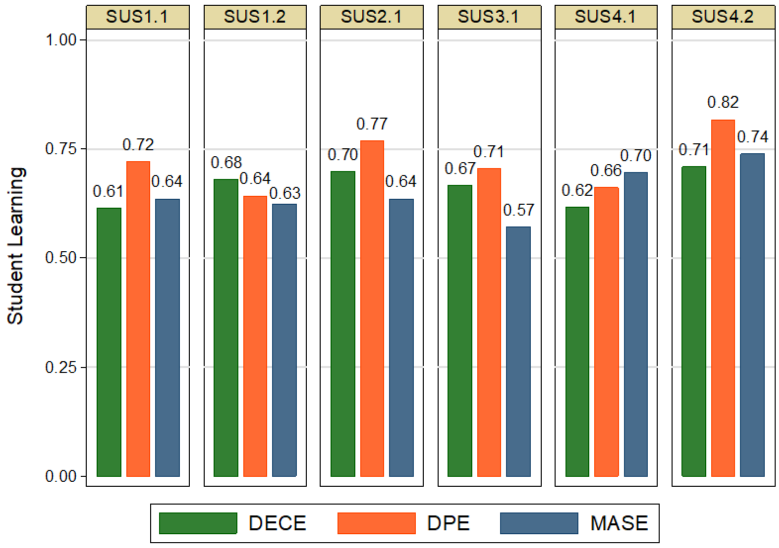


Table 6 shows the analysis of the differences of means in each of the six sustainability competency units with multiple pairwise comparisons according to degrees. The p-values that identify a significant difference ($p < 0.05$) are shaded in grey. Again, since SUS2 and SUS3 are configured around a single competency unit (SUS2.1 and SUS3.1 respectively), the data shown in Table 6 for SUS2.1 and SUS3.1 coincide with those data shown in Table 5 for competencies SUS2 and SUS3.

Table 6. Analysis of the differences of means (ANOVA) in each competency unit according to degrees, and multiple pairwise comparisons

SUS	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
1.1	4.62	0.012	DPE vs DECE	.107	0.014
			MASE vs DECE	.0220	0.862
			MASE vs DPE	-.085	0.158
1.2	1.20	0.306	DPE vs DECE	-.037	0.566
			MASE vs DECE	-.055	0.370
			MASE vs DPE	-.018	0.910
2.1	6.05	0.003	DPE vs DECE	.069	0.094
			MASE vs DECE	-.063	0.210
			MASE vs DPE	-.132	0.004
3.1	5.85	0.004	DPE vs DECE	.039	0.483
			MASE vs DECE	-.095	0.038
			MASE vs DPE	-.134	0.005

SUS	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
4.1	2.56	0.082	DPE vs DECE	.045	0.394
			MASE vs DECE	.081	0.102
			MASE vs DPE	.036	0.684
4.2	6.47	0.002	DPE vs DECE	.109	0.002
			MASE vs DECE	.029	0.693
			MASE vs DPE	-.079	0.111

Focusing on the analysis of competencies SUS1 and SUS4, it is worth noting how the significant differences identified in the degrees of Early Childhood Education and Primary Education exist precisely in competency unit SUS1.1: "Understands the functioning of natural, social and economic systems, as well as their interrelationships and problems, both at a local and global level [...]" and SUS4.2: "Promotes an education in values oriented towards training responsible, active and democratic citizens". In both cases, the development of these competency units is higher in the DPE than in the DECE (see Figure 7). It should be pointed out (see section 4.3.1) that the development of competency units SUS2.1 and SUS3.1 is significantly higher in the DPE than in the MASE. In conclusion, it is clear that the development of sustainability in the DPE is superior to the DECE and the MASE, at least, in some of the competency units in which each of the four sustainability competencies is configured. This allows us to understand why the development of sustainability in the DPE is superior to the development in the DECE and the MASE in the four sustainability competencies, as seen in the previous section (see Figure 5). The two competency units in which the development of sustainability in the DPE is lower are SUS1.2: "Possesses critical thinking skills and creativity [...]" and SUS4.1: "Is consistent in actions, respecting and valuing (biological, social and cultural) diversity and committed to improving sustainability".

However, we believe that education in general and mathematics education in particular cannot adopt the form of an inconsequential, thoughtless, and uncritical delivery of information. True thinking should have the nature of an event that, by interrupting the same, turns out to be transformative (Han, 2022). The belief in possibilities, and the need to build alternative and better futures lies precisely at the heart of critical and creative thinking (Cebotarev, 2003).

We agree with Vila (2017) when he states that ethical principles are a main reference in education degrees. A greater effort is required in the ethical and deontological training of future teachers in order to enable them to develop respectful and harmonious educational work from the perspective of otherness (Muñoz-Rodríguez et al., 2020).

4.3.3. Development of sustainability in each of the mastery levels of each sustainability competency: L1, L2, L3

Figure 8 shows the average development achieved, expressed in percentages, in each of the three mastery levels of the four sustainability competencies in the area of Didactics of Mathematics and the weighted average.

Figure 8. Average development of sustainability competencies in the area of Didactics of Mathematics according to mastery levels, and weighted average

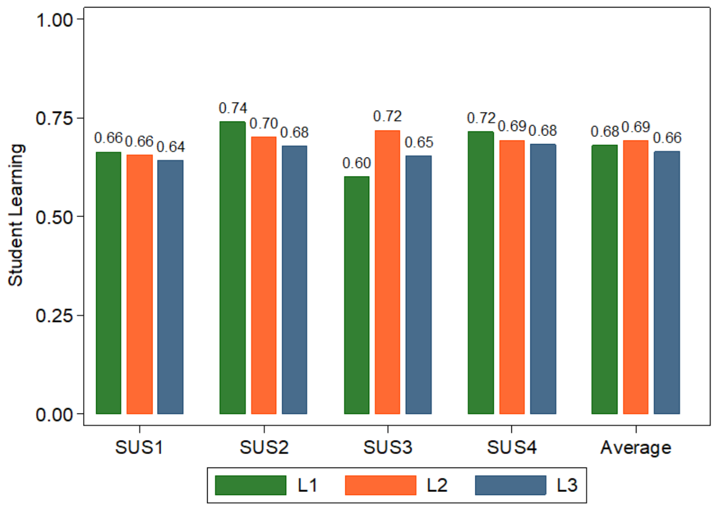


Figure 9 shows the development achieved, expressed in percentages, in each of the three mastery levels of each of the four sustainability competencies organised by degrees: DECE, DPE, and MASE.

Figure 9. Development of sustainability competencies in the area of Didactics of Mathematics according to mastery levels organised by degrees

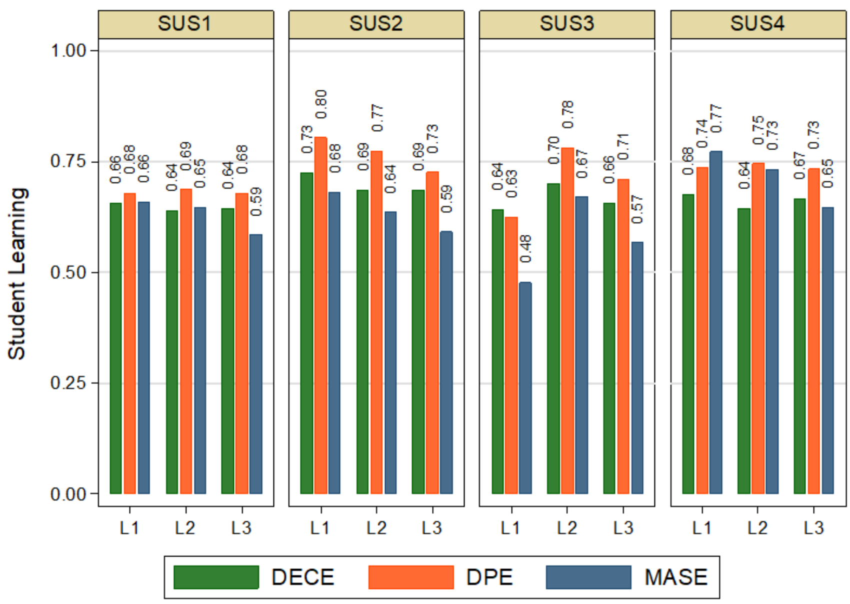


Figure 8 shows how the average development in the three mastery levels of all competencies is more or less homogeneous and similar to the global average, except for level L1 of competency SUS3: "Competency to participate in community processes that promote sustainability" in which a lower development is identified (60%). The analysis by degrees (Figure 9) reveals that this is due to the lower level of development identified in the MASE for this mastery level in said competency (48%). It should also be mentioned that, in the analysis with multiple pairwise comparisons according to degrees, the development of competency SUS3 in the MASE is significantly lower than in the other two degrees (see section 4.3.1). It is also worth noting that level L1 of SUS3 is related to item Q10 of the sustainability questionnaire in which the two bachelor's degrees obtained relative minimum values and the master's degree obtained an absolute minimum value (see section 4.1).

It is curious how this result is consistent with other studies carried out in broader contexts and in other universities, in which the low learning value reported by students for level L1 of this competency implied that SUS3 was the sustainability competency with a lower average learning percentage in education degrees, regardless of the area of study (Sánchez-Carracedo et al., 2021).

Finally, Figure 10 shows the average development, expressed in percentages, achieved in each of the three mastery levels L1, L2, L3.

Figure 10. Average development of the mastery levels of the sustainability competencies in the area of Didactics of Mathematics organised by degrees

Figure 10 shows how the average development of the three mastery levels is similar in the three degrees, it being higher in the DPE and lower in the MASE.

Table 7 shows the analysis differences of means in each of the three mastery levels in which the sustainability competencies are developed with multiple pairwise comparisons according to degrees. The p-values that identify a significant difference ($p < 0.05$) are shaded in grey.

Table 7. Analysis of the differences of means (ANOVA) in each mastery level according to degrees, and multiple pairwise comparisons

Nivel	Anova test		Post-hoc Scheffe test		
	F	p	Comparisons	Diff. of means	p
L1	2.02	0.138	DPE vs DECE	.037	0.393
			MASE vs DECE	-.028	0.661
			MASE vs DPE	-.064	0.152
L2	4.43	0.014	DPE vs DECE	.079	0.021
			MASE vs DECE	0.004	0.992
			MASE vs DPE	-.075	0.095
L3	5.21	0.007	DPE vs DECE	.049	0.240
			MASE vs DECE	-.066	0.140
			MASE vs DPE	-.115	0.007

As shown in the table, no significant differences of means are identified in the analysis by mastery levels, except for level L2 of the competencies for which the data reported by the students of the DPE are higher than those of the DECE, and for level L3 of the competencies for which the data of the Master's degree students are clearly lower than those of the students of the DPE (see Figure 9). No significant differences of means were identified at level L1 of the competencies in the analysis with multiple pairwise comparisons according to degrees.

5. Conclusions

This article analysed the development of the CRUE-Sustainability competencies of the students who took the following subjects: Development of Mathematical Knowledge in Early Childhood Education; Didactics of Mathematics 1; Teaching Innovation and Initiation to Research in teaching of Science and Mathematics in the area of Didactics of Mathematics in the Faculty of Educational Sciences at the Universidad de Cádiz. Overall, the average development of the four sustainability competencies is uniform and similar to the global average (68%). The analysis by degrees reveals how the development of sustainability in the MASE is significantly lower than in the other degrees in all the competencies except for competency SUS4: "Competency to apply ethical principles related to sustainability values." On the contrary, the development of sustainability in the DPE is higher than in the DECE and in the MASE, at least in some of the competency units in which each of the four sustainability competencies is configured. Finally, the average development in the three mastery levels of all the competencies is more or less homogeneous, except for level L1 of competency SUS3: "Competency to participate in community processes that promote sustainability" in which a lower development is identified.

We would like to conclude this study with the following thought: as Han (2022) points out, "calculating" is an endless repetition of the same. Information (big data) will always be available but, unlike knowledge, it is blind to events. True thinking shows a very different temporality: it needs a period of maturation, but it is going through a crisis today. To Bauman (2015), the main challenge of education in liquid modernity is not only learning to live or survive in a world oversaturated with information, but what is harder, learning the "art of preparing the next generations to live in such a world" (p. 46). We are convinced that integrating sustainability in mathematics education will create an awareness of the complexity of conflicts and the development of social criticism in future teachers.

Finally, this research has several limitations that should be taken into account when evaluating the results:

- The sample of students was reduced. However, the aim of this research was not to generalise results, but to know and understand the phenomenon under study in order to be able to characterise it in its context: the Universidad de Cádiz. In any case, the study provides a methodology applicable to other contexts and to other universities.
- The development of sustainability competencies was measured from the responses given by the students to a single tool for collecting data: the sustainability questionnaire, which does not necessarily coincide with the competency level they really have.
- This study has helped us collect preliminary data for future research. However, it is necessary to carry out qualitative longitudinal studies that allow analysing the cause-and-effect relationships in the variable of "development of sustainability competencies" of university students. This is an objective that will be part of our future work at UNIEDINSOST.

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Appendix. Student questionnaire for education degrees

Q1	I know the interrelationship between natural, social and economic systems
Q2	I understand and can analyse the relationships between natural systems and social and economic systems
Q3	I am able to foresee the repercussions of changes in natural, social and economic systems
Q4	I know procedures and resources to integrate sustainability in the subjects
Q5	I know how to analyse the opportunities provided in the subjects to plan educational projects that integrate sustainability
Q6	I know how to design educational projects from the perspective of sustainability
Q7	I know how to identify the possible socio-environmental impacts derived from my educational actions
Q8	know how to develop educational actions that minimise negative socio-environmental impacts
Q9	I know how to design and develop educational actions taking into account the negative socio-environmental impacts and incorporating corrective actions

Q10	I am aware of community educational programmes that encourage participation and commitment in socio-environmental improvement
Q11	I know how to perform satisfactorily in community educational projects encouraging participation
Q12	I know how to design and carry out socio-educational activities in participatory community processes that promote sustainability, feeling an integral part of my environment
Q13	I know the ethical principles of sustainability
Q14	I understand the ethical principles of sustainability and I am able to integrate them into my professional and personal actions
Q15	I know how to design and/ or manage educational projects, taking into account ecological ethics to improve the quality of life and promote the common good
Q16	I consider the promotion of sustainable human development as a key purpose of citizen training
Q17	I know how to critically assess and analyse the consequences that my personal and professional performance may have on the comprehensive development of students and on the promotion of sustainable human development
Q18	I know how to design and develop educational intervention proposals that integrate sustainability values and result in justice and the common good

References

Albareda Tiana, S., Azcárate Goded, P., Muñoz Rodríguez, J. M., Valderrama Hernández, R., & Ruiz Morales, J. (2019). Evaluar competencias en sostenibilidad en los grados y posgrados de educación: propuesta de un instrumento. *Enseñanza de las Ciencias, 37*(3), 11-29. https://doi.org/10.5565/rev/ensciencias.2670

Alsina, Á., & Mulà, I. (2019). Advancing towards a transformational professional competence model through reflective learning and sustainability: The case of mathematics teacher education. *Sustainability, 11*(15), 4039. https://doi.org/10.3390/su11154039

Bauman, Z. (2015). *Los retos de la educación en la modernidad líquida*. Gedisa.

Bonil, J., Junyent, M., & Pujol, R. M. (2010). Educación para la sostenibilidad desde la perspectiva de la complejidad. *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias,* 7(Nº Extraordinario), 198-215. https://doi.org/10.25267/Rev_Eureka_ensen_divulg_cienc.2010.v7.iextra.05

Bruner, J. (2010). *Realidad mental y mundos posibles. Los actos de la imaginación que dan sentido a la experiencia*. Gedisa.

Cebotarev, E. A. (2003). El enfoque crítico: una revisión de su historia, naturaleza y algunas aplicaciones. *Revista Latinoamericana de Ciencias Sociales, Niñez y Juventud, 1*(1), 17-56.

CRUE. (2012). *Directrices para la introducción de la Sostenibilidad en el Currículum*. Comité Ejecutivo del Grupo de Trabajo de Calidad Ambiental y Desarrollo Sostenible de la CRUE.

Font, V. (2002). Una organización de los programas de investigación en Didáctica de las Matemáticas. *Revista Ema,* 7(2), 127-170.

Godino, J. (2010). *Perspectiva de la Didáctica de las Matemáticas como Disciplina Tecnocientífica*. Universidad de Granada.

Han, B. C. (2022). *La expulsión de lo distinto*. Herder.

Hernández, J. C. (2022). El estado de la alfabetización matemática: análisis desde las producciones y las subjetividades. *Revista Venezolana de Investigación en Educación Matemática,* 2(3), 1-31. https://doi.org/10.54541/reviem.v2i3.31

Jaramillo, D. (2011). La educación matemática en una perspectiva sociocultural: tensiones, utopías, futuros posibles. *Revista Educación y Pedagogía, 23*, 13-36.

Kruger, J., & Dunning, D. (1999). Unskilled and unaware of it: how difficulties in recognizing one's own incompetence lead to inflated self-assessments. *Journal of personality and social psychology*, 77(6), 1121. https://doi.org/10.1037/0022-3514.77.6.1121

Lévinas, E. (1999). *Totalidad e infinito. Ensayo sobre la exterioridad*. Ediciones Sígueme.

Lipovetsky, G. (2003). *La era del vacío. Ensayos sobre el individualismo contemporáneo*. Anagrama.

Moreno-Pino, F.M., Jiménez-Fontana, R., Cardeñoso Domingo, J. M. & Azcárate Goded, P. (2022). Training in Mathematics Education from a Sustainability Perspective: A Case Study of University Teachers' Views. *Education Sciences, 12*(3), 199. https://doi.org/10.3390/educsci12030199

Muñoz-Rodríguez, J. M., Sánchez-Carracedo, F., Barrón-Ruiz, Á., & Serrate-González, S. (2020). Are we training in sustainability in higher education? Case study: Education degrees at the university of Salamanca. *Sustainability, 12*(11), 442. https://doi.org/10.3390/su12114421

Naredo, J.M. (2022). *La crítica agotada*. Siglo XXI.

National Council of Teachers of Mathematics [NCTM] (1989). *Curriculum and Evaluation Standards for School Mathematics*. Author.

Radford, L. (2013). Sumisión, alienación y (un poco de) esperanza: hacia una visión cultural, histórica, ética y política de la enseñanza de las matemáticas. In A. Ramírez & Y. Morales (Eds.), *Memorias del I Congreso de Educación Matemática de América Central y el Caribe*. Santo Domingo, República Dominicana, November 2013, 6-8.

Radford, L. (2014). De la teoría de la objetivación. *Revista Latinoamericana de Etnomatemática,* 7(2), 132-150.

Radford, L. & Lasprilla, A. L. (2022). Ética en la Enseñanza-Aprendizaje de las Matemáticas. *Revista de Investigación y Desarrollo en Educación Matemática, 1*(1).

Rodríguez, M. E. (2016). La Función Social de la Enseñanza de la Matemática desde la Matemática-Cotidianidad-y Pedagogía Integral. *Revista Eleuthera, 15*, 34-45. https://doi.org/10.17151/eleu.2016.15.3

Sánchez-Carracedo, F., Romero-Portillo, D., Sureda, B. & Moreno-Pino, F. M. (2021). Education for Sustainable Development in Spanish Higher Education: an Assessment of Sustainability Competencies in Engineering and Education Degrees. *International Journal of Sustainability in Higher Education, 23*(4), 940-959. https://doi.org/10.1108/IJSHE-02-2021-0060

Segalàs, J. & Sánchez-Carracedo, F. (2019). The EDINSOST Project: Improving Sustainability Education in Spanish Higher Education. *In Proceedings of the 19th European Roundtable for Sustainable Consumption and Production (ERSCP 2019),* Institute for Sustainability Science and Technology: Univesitat Politècnica de Catalunya (Barcelona), October, 217-240.

Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education*. Kluwer. https://doi.org/10.1007/978-94-017-3556-8

Tilbury, D., & Wortman, D. (2004). *Engaging people in sustainability*. IUCN.

Vila, E. S. (2017). *Competencias éticas y deontología profesional en la universidad*. Aljibe.

Wiek, A., Withycombre, L. & Redman, C. (2011). Key competencies in sustainability: a reference framework for academic program development. *Sustainability Science, 6*(2), 203-218. https://doi.org/10.1007/s11625-011-0132-6

Wittgenstein, L. (2017). *Tractatus lógico-philosophicus. Investigaciones filosóficas*. Gredos.

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El desarrollo de competencias para la sostenibilidad en el área de didáctica de las matemáticas

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Situamos esta investigación en el marco de la educación matemática crítica. El enfoque crítico coincide con aquellos puntos de vista que entienden la enseñanza y el aprendizaje de las matemáticas no sólo como fenómenos sociales, sino que considera esencial ampliarlos a los aspectos políticos. La educación matemática crítica conceptualiza la alfabetización matemática como competencia que integra no sólo un *conocer matemático* sino también un *conocer reflexivo*. Desde estos presupuestos, este estudio presenta los resultados del análisis sobre el desarrollo de competencias para la sostenibilidad de estudiantes matriculados en asignaturas del área de Didáctica de las Matemáticas de tres titulaciones diferentes de la Universidad de Cádiz: Desarrollo del Conocimiento Matemático en Educación Infantil (Grado en Educación Infantil), Didáctica de las Matemáticas 1 (Grado en Educación Primaria) e Innovación Docente e Iniciación a la Investigación en la enseñanza de las Ciencias y de las Matemáticas (Máster en Profesorado de Secundaria y Bachillerato en la especialidad de matemáticas). Para recoger la información se ha utilizado el cuestionario de sostenibilidad para estudiantes del proyecto EDINSOST. El cuestionario consta de 18 ítems relativos al desarrollo de las cuatro competencias de sostenibilidad propuestas por la Comisión Sectorial de la Conferencia de Rectores de las Universidades Españolas: (1) Competencia en la contextualización crítica del conocimiento estableciendo interrelaciones con la problemática social, económica y ambiental, a nivel local y/o global; (2) Competencia en la utilización sostenible de recursos y en la prevención de impactos negativos sobre el medio natural y social; (3) Competencia en la participación en procesos comunitarios que promuevan la sostenibilidad; y (4) Competencia en la aplicación de principios éticos relacionados con los valores de la sostenibilidad en los comportamientos personales y profesionales. En el estudio se definieron tres tipos de indicadores compuestos para analizar las respuestas dadas por los estudiantes en cada competencia, unidad de competencia y nivel de dominio de cada competencia. Los resultados globales muestran un desarrollo medio de las cuatro competencias para la sostenibilidad uniforme y similar al promedio (68%). El análisis por titulaciones revela cómo el desarrollo de la sostenibilidad en la titulación del Máster en Profesorado de Secundaria y Bachillerato en la especialidad de matemáticas es inferior, respecto de las otras titulaciones, en todas las competencias salvo para la competencia relacionada con los aspectos éticos.