Analyzing transport phenomena in orbital conjunctions

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Abstract

Given the conjunction geometry between two objects in orbit is inherently defined by the intersection between their respective orbital planes, it is reasonable to study the dynamical evolution of their relative position when one of them is located along such intersection. Methods for determining the probability of collision are either computationally intensive or referred to a specific time of closest approach. Identifying the domain of attraction for a collision, i.e. the region within the initial probability distribution that leads to a minimum distance below a prescribed threshold, can aid in providing accurate estimates for collision probability computations.

1 Introduction

Computationally efficient methods for determining the probability of collision between two objects in orbit are crucial for the continuation of Earth orbital activities. The number of objects in Earth orbit capable of producing a catastrophic collision is currently on the order of 10^5. Continuously monitoring these objects and foreseeing close approaches among them thus requires a huge computational effort. Driven by this requirement, operational methods for determining the probability of collision between two objects typically depend on linear-gaussian and geometric assumptions that have been shown to fail for certain type of approaches, e.g. low velocity encounters. Various works have been proposed to overcome these limitations, but their associated computational cost is still beyond the current industry capabilities.

Within this work, the authors propose to study the transport phenomena in the dynamical system that models a collision in orbit. Through appropriate coordinate transformations, it is possible to efficiently explore the initial probability distribution function of the state of both objects with the aim of determining the domain of attraction of a potential collision. The latter could be extremely useful in the computation of the probability of collision as defined by the integral over all possible state realizations that lead to a minimum distance smaller than certain threshold.

2 Conjunction Geometry

For a collision to occur between two objects in orbit, both objects need to be located at one of the two orbital plane intersections. The direction of such intersections is commonly referred to as relative line of nodes and is defined by

$$\mathbf{n}_c = \pm \frac{\mathbf{h}_1 \times \mathbf{h}_2}{|\mathbf{h}_1 \times \mathbf{h}_2|}, \tag{1}$$

where $\mathbf{h}_i = \mathbf{r}_i \times \mathbf{v}_i$ is the angular momentum vector of object $_i$. Under the assumption of Keplerian dynamics, i.e. the only forces acting on the subjects are due to a central gravity field modeled as a restricted two-body problem, the necessary conditions for a collision reduce to:

1. The radii of both orbits along the collision direction should be coincident. This is measured by the quantity

$$\Gamma_c = 1 - \frac{r_{2,c}}{r_{1,c}}, \tag{2}$$

   being $r_{i,c} = (h_i^2/\mu)/(1 + e_i \cos \nu_{i,c})$ and $\cos \nu_{i,c} = \mathbf{n}_c \cdot \mathbf{e}_i / |\mathbf{e}_i|$. Note that in Keplerian motion this angle remains constant for each relative node.

2. Both objects need to be located at a common orbit intersection point, which can be measured by the angular distance

$$\Delta_c = \nu_2(t_{c,m}) - \nu_{2,c}. \tag{3}$$

   This distance corresponds to the phase between the collision anomaly of object two $\nu_{2,c}$, i.e. the angular position of the secondary that complies with the orbital intersection, and the actual angular position of the secondary $\nu_2(t_{c,m})$ when the primary is located at its respective collision anomaly, i.e. $\nu_1(t_{c,m}) = \nu_{1,c}$. Under Keplerian assumptions, the time invested by object 1 to reach the collision direction can be computed from Kepler's equation

$$\begin{aligned} &\sqrt{\frac{\mu}{a_1^3}}(t_{c,0} - t_0) = \\ &\quad (E_{1,c} - E_{1,0}) - e_1 (\sin E_{1,c} - \sin E_{1,0}), \end{aligned} \tag{4}$$

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where $t_{c,m} = t_{c,0} + mT_1$ and T_1 is the orbital period of the primary. In a similar fashion, the corresponding true anomaly of the secondary $\nu_2(t_{c,m})$ can be derived from

$$\sqrt{\frac{\mu}{a_2^3}}(t_{c,m} - t_0) = \\ (E_{2,c}^m - E_{2,0}) - e_2\left(\sin E_{2,c}^m - \sin E_{2,0}\right). \tag{5}$$

Moreover, the change in eccentric anomaly between two consecutive passages of the primary through a relative node can be computed as

$$2\pi\sqrt{\frac{a_1^3}{a_2^3}} = \\ (E_{2,c}^{m+1} - E_{2,c}^m) - e_2\left(\sin E_{2,c}^{m+1} - \sin E_{2,c}^m\right). \tag{6}$$

Appropriate thresholds may then be set for the distance in the conjunction map $\xi = [\Gamma,\ \Delta]^T$. Assuming a combined hard body radius $R_c = 10\ m$, such thresholds would be $\Gamma_{th} \sim 10^{-6}$ and $\Delta_{th} \sim 10^{-6}$ for a typical LEO encounter at an altitude of $1,000\ km$. Similarly, a conjunction in the Geostationary region with the same R_c would require distances in the conjunction map on the order of $\Gamma_{th} \sim 10^{-7}$ and $\Delta_{th} \sim 10^{-7}$.

3 Assessment of the approach

The proposed mapping based on relative orbital geometry has been successfully applied to collision risk analysis, allowing to efficiently determine the Earth's orbital congestion (see [3]). Within this work, we want to explore the capabilities of the method and, in particular, determine the ability to approximate the dynamical evolution of the conjunction geometry in an efficient manner.

In fact, we seek to perform low order approximations to the difference between the predicted mapping at the closest approach $\xi(t_{c,m})_{kep}$ under Keplerian motion, and the reference mapping $\xi(t_{c,m})$ computed using high-fidelity propagation. To validate the approach, a Monte Carlo simulation has been carried out for a representative LEO test case based on the Iridium-Kosmos 2009 collision [1]. The initial state of the objects is assumed to follow a Gaussian distribution with mean states at the initial epoch t_0 = 2009 FEB 03 20:01:28.126 UTC

$$\mathbf{r}_1^{ECI}(t_0) = \begin{bmatrix} 1286.102 \\ -1129.618 \\ -6957.400 \end{bmatrix},\ \mathbf{v}_1^{ECI}(t_0) = \begin{bmatrix} -3.970654 \\ 6.062485 \\ -1.718518 \end{bmatrix}$$

$$\mathbf{r}_2^{ECI}(t_0) = \begin{bmatrix} 6308.427 \\ 3294.617 \\ -916.8711 \end{bmatrix},\ \mathbf{v}_2^{ECI}(t_0) = \begin{bmatrix} -0.158786 \\ 2.243546 \\ 7.103635 \end{bmatrix}$$

expressed in km and km/s. The initial co-variance is assumed equal for both objects and given by

$$\mathbf{P}^{RTN}(t_0) = \text{diag}\begin{bmatrix} 41.42 \\ 2533 \\ 70.98 \\ 5.744 \cdot 10^{-3} \\ 1.049 \cdot 10^{-5} \\ 1.091 \cdot 10^{-6} \end{bmatrix}$$

expressed in m^2 and m^2/s^2. $N = 500$ samples are drawn from a Gaussian distribution combining both objects, and then propagated with a deterministic dynamical model considering drag [4], solar radiation pressure, Earth's non spherical gravity up to degree and order 10 and the Sun and Moon as third bodies. A total propagation time of 7 days is set in order to cover the reference collision epoch t_c^* = 2009 FEB 10 16:55:59.806 UTC.

Figure 1 depicts the distribution of the distance of closest approach with respect to the occurrence epoch. Note there is a color code indicating the Mahalanobis distance of each sample with respect to the initial distribution, defined as

$$d_{M,i} = \sqrt{\begin{bmatrix} \mathbf{x_{1,i}} - \mathbf{x_1} \\ \mathbf{x_{1,i}} - \mathbf{x_1} \end{bmatrix}^T \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{P} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{x_{1,i}} - \mathbf{x_1} \\ \mathbf{x_{1,i}} - \mathbf{x_1} \end{bmatrix}}$$

where both $\mathbf{x}$ and $\mathbf{P}$ are referred to t_0. While most of the particles that lead to minimum distances lower that 1 km correspond to high probability (low d_M) regions, there is an even higher portion of particles that lead to higher distances within the gross of the initial distribution. This suggests that the domain of attraction of a potential collision may not present a smooth behavior in the probability space. In addition, the scatter in the time of closest approach (TCA) suggests that analyses based on a single reference epoch may not be sufficient to determine the set of states that lead to collision, note that relative velocities are of the order of 10 km/s for this particular case.

Fig. 2 shows the conjunction geometry at the first node passage with a colormap indicating the distance of closest approach. Note that at this specific pass (referred to $t_{c,0}$), the two objects are separated by an angular distance of 75^o and there is not a clear region that leads to (future) lower minimum distances. Nonetheless, one can apply the same mapping to a future conjunction (referred to $t_{c,m}$) assuming Keplerian motion, obtaining the distribution shown in Fig. 3. Therein, one can see that the samples are somehow ordered so there is a clear relation between $||\xi_{kep}(t_{c,m})||$ and $||\mathbf{r}_{1,c} - \mathbf{r}_{2,c}||$. Thereafter, the transport of particles that lead to collision cannot be characterized as a two-dimensional transformation $\mathcal{T}' : \mathbb{R}^2 \to \mathbb{R}^2$ but

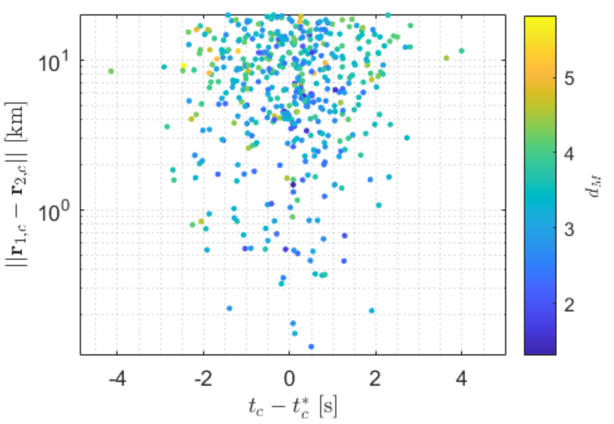

Figure 1: Minimum distance as a function of time. Here t_c^* is the reference collision epoch and d_M is the Mahalanobis distance with respect to the initial sampling distribution.

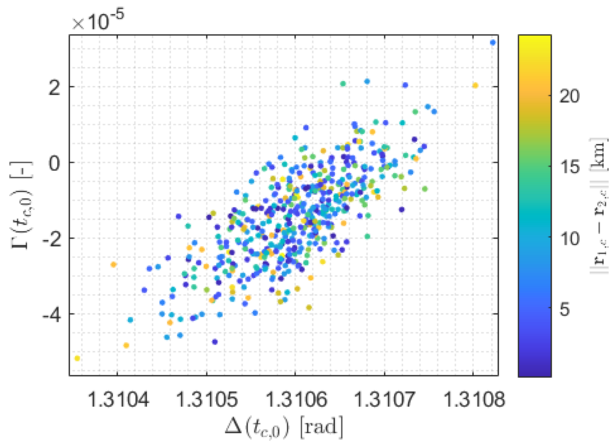

Figure 2: Conjunction geometry at first node passage.

requires analyzing the complete dimensionality of the state, thus the mapping $\mathcal{T} : \mathbb{R}^{12} \to \mathbb{R}^2$. As a consistency check, if the projection onto the conjunction plane is performed for the particles propagated using the high-fidelity dynamical model, the resulting distribution is very similar to the one predicted by the Keplerian model (see Fig 4).

4 Preliminary conclusions

Statistical numerical integration has shown that there is a class of two-dimensional mappings that naturally captures the dynamical evolution of a conjunction in orbit. Moreover, this mapping is not referred to a specific conjunction geometry but simply the one that leads to the closest approach, thus being free from any temporal or spatial limitation. Based on these preliminary results, it is possible to determine the domain of attraction of a collision, as suggested by the extreme value theory. The authors propose to 1) derive a low-order expansion of the effect of orbital perturbations in the conjunction geometry or 2) utilize sequential sampling methods to approximate the domain of attraction of the collision in lieu of developing efficient methods for computing the probability of collision.

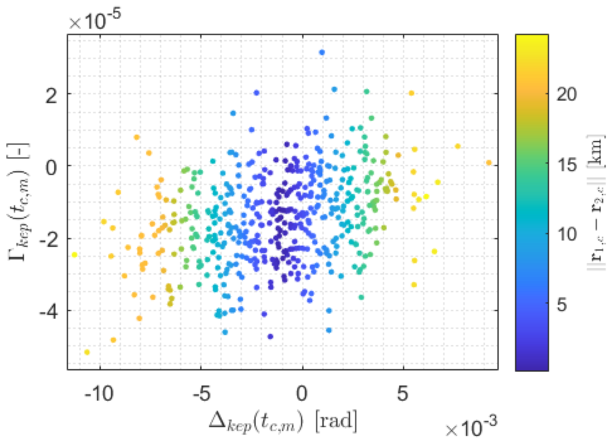

Figure 3: Conjunction geometry at closest approach under Keplerian dynamics.

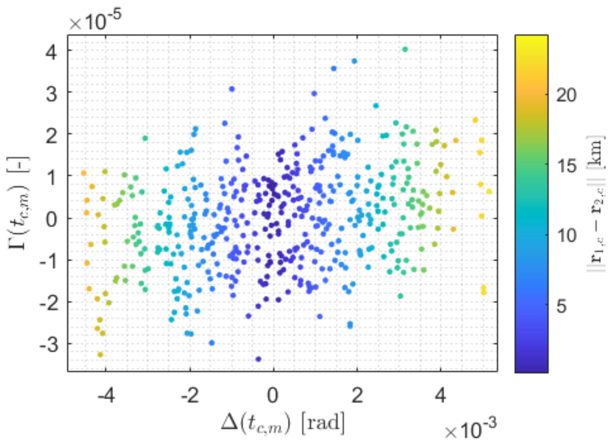

Figure 4: Conjunction geometry at closest approach with high-fidelity propagation.

References

[1] NASA (Oct. 2009). The Collision of Iridium 33 and Cosmos 2251: The Shape of Things to Come. *60th International Astronautical Congress*.

[2] Russell P. Patera (2007). Satellite Collision Probability for Nonlinear Relative Motion. *Journal of Guidance, Control and Dynamics*, Vol. 26, No. 5.

[3] Jorge Martinez, Guillermo Escribano & Manuel Sanjurjo-Rivo (2021). Orbital congestion assessment based on dimensionality reduction.

[4] Bruce Bowman et al. (Aug. 2008). A new empirical thermospheric density model JB2008 using new solar and geomagnetic indices. *AIAA/AAS astrodynamics specialist conference and exhibit (p. 6438).*