# Apply the hybrid orbit propagators to the association problem in the GEO region

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## Abstract

Maintaining an accurate and updated catalogue of resident space objects (RSO) is of paramount importance for Space Surveillance and Tracking (SST). The success of catalogue maintenance is mainly driven by the timely availability of observations and the correct association of these observations to the catalogued objects. The latter task can significantly contribute to the computational effort needed in the whole cataloguing chain. The inputs for the association problem are the catalogue of objects at a given time –including extended state vector, associated uncertainty and object characteristics– and a set of observations at a later date. One possibility is to numerically propagate the whole RSO population with the highest accuracy to the observation date to apply the association algorithms (global nearest neighbor, for instance) to the objects that are likely to be within the field-of-view of the sensor. Another possibility, more efficient in time, is to pre-filter a reduced sub-population, applying the previous procedure with an analytical or semi-analytical solution and then using the time-consuming and highly accurate numerical propagator only to that sub-population. The drawback of this alternative is that the accuracy of the analytical or semi-analytical solution can lead to filter out potential candidates and miss the correct global association. Therefore, the use of hybrid propagators, combining the rapidity of analytical or semi-analytical propagators and the accuracy of numerical ones, is a promising alternative.

In many applications, a compromise between accuracy and efficiency must be established, based on a variety of criteria. High-fidelity propagation models usually require step-by-step propagation by using numerical methods, which are computationally intensive because they rely on small step sizes. However, simplified models may admit analytical solutions, which alleviate the computational burden. In either case, the orbit propagation program relies uniquely on the initial conditions, as well as on the propagation model, to make its predictions. On the other hand, the collection of past ephemerides can be used to improve orbit predictions by taking non-modeled effects into account.

Indeed, developing a new hybrid modeling approach to address the problem of accuracy inspired us to apply non-invasive techniques, such as machine learning or statistical time series techniques, to forecasting methods. This approach, proposed by San-Juan et al. [1, 2, 3, 4], asses that the hybrid modelling approach for orbit propagation is feasible and comparable to traditional models, improving their accuracy in most cases. Basically, this methodology investigates the main dynamical effects provided by any orbit propagator and make important contributions modelling its error and emulating other non-modelled dynamics.

The first step to develop a hybrid version of SGP4 is to understand the behavior of this propagator during the considered interval of time. To assess this, data from a space catalogue with 510 TLEs downloaded from Space Track[1] from different GEO orbits have been propagated using the analytical propagator SGP4 and PSIMU[2], a high-accuracy orbital propagator developed by the Centre National d’ÃĽtudes Spatiales (CNES).

PSIMU is implemented in Java and includes the following perturbation forces: geopotential acceleration computed up to an arbitrary degree and order for the harmonics, atmospheric drag, solar radiation pressure, rediffused solar radiation pressure, third body perturbations from Sun and Moon, and ocean and terrestrial tides. PSIMU implements a Dormand-Prince of 8th order with variable step-size and uses PATRIUS (PATrimoine de base siRIUS)[3]. It is the reference low level library used for mathematical and flight dynamics functions, as well as other supplementary libraries. The perturbation model taking into account in PSIMU includes Earth's gravitational field (up to $8 \times 8$), solar radiation pressure models with A/m = 1, 0.5, 0.1, 0.05, 0.01, 0.001 $m^2/kg$, and third

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[1] `https://www.space-track.org`

[2] `https://logiciels.cnes.fr/en/content/psimu`

[3] `https://logiciels.cnes.fr/en/content/patrius`

body point mass force models.

On the other hand, SGP4 is based on the analytical theories of the artificial satellite of [5] and [6]. Initially, the perturbations modelled by SGP4 consisted of only zonal gravitational terms (up to J4) and drag based on the work of [7]. When Molniya and geostationary orbits became more common, deep space modelling (Simplified Deep-space Perturbation-4, SDP4) was integrated into SGP4 [8, 9]. This included the lunisolar effects, and resonance effects of tesseral harmonics developed by [10].

In this study, for each TLE of our sample and different values of A/m, twenty days of accuracy numerical pseudo observations were generated with PSIMU, using as initial conditions the osculating elements provided by SGP4 at the same epoch $t_{\rm TLE}$ for each previous defined TLE. After that, a new TLE is calculated using a differential corrector method from the arc of $t_{\rm TLE}$ $+10$ days given by PSIMU. Finally, ten backward and forward propagation days are done using SGP4 from the new TLE. From now, the set of TLE given at the epoch $t_{\rm TLE}$ $+10$ will be considered as our new TLE sample, one for each value of A/m making a total of 3060 TLEs. It is worth noting that this scenario intends to simulate the same scenario given in the association problem in which during the process a high accuracy propagation can be obtained.

Then, we calculate the time series of the error $\varepsilon_t^{\mathbf{x}} = \mathbf{x}_t^{\rm PSIMU} - \mathbf{x}_t^{\rm SGP4}$, where $\mathbf{x}$ represents any set of variables, such as cartesian, Delaunay, polar-nodal, or equinoctial elements, $\mathbf{x}_t^{\rm PSIMU}$ is the pseudo-observation given by PSIMU at epoch $t$, and $\mathbf{x}_t^{\rm SGP4}$ is the data obtained from SGP4 at the same epoch. The six time series $\varepsilon_t^{\mathbf{x}}$ contain the complete information related to SGP4 errors, which are caused by the perturbation forces not taken into account by the SPG4 algorithm and by the integration method used with this analytical propagator.

In order to understand the real influence that each variable, as well as some of their combinations, may have on the accuracy of the SGP4 propagation for GEO orbits, an Exploratory Data Analysis (EDA) is performed for the TLEs considered in this study. The variables and their combinations are ranked in terms of their capability to reduce the distance errors of SGP4 propagations. The main conclusion of this analysis is that the argument of the latitude $\theta$ and the argument of the node $\nu$ using the polar-nodal set allow reducing the distance error of SGP4 for all considered TLEs by modeling the evolution of their error.

An EDA of the distance error between PSIMU and SGP4, and PSIMU and the optimum hybrid SGP4 propagator (OptHSGP4) for a given predictive horizon $t$ is performed as a second step. The OptHSGP4 propagator is obtained when the time series of the errors are zero, $\varepsilon^\theta = \varepsilon^\nu = 0$, that is, $\theta^{\rm PSIMU} = \theta^{\rm SGP4}$ and $\nu^{\rm PSIMU} = \nu^{\rm SGP4}$.

In a third step, a preliminary analysis of the time series of error $\varepsilon^\theta$ and $\varepsilon^\nu$ is done. The analysis examines the approximate entropy (ApEn) during the training period. The training period is given in satellite revolutions and it varies between 3 and 10 revolutions. Approximate entropy was introduced to quantify the amount of regularity and the unpredictability of fluctuations in a time series. A low entropy value indicates that the time series is deterministic; a high value indicates randomness.

The four step applies the seasonal Autoregressive Integrated Moving Average (**A**RIMA) and Exponential Smoothing (**E**) forecasting methods to each time series $\varepsilon^\theta$ and $\varepsilon^\nu$. The number of points per revolution considered is 12, whereas the trained period varies between 3 and 10 revolutions. In this report, the total number of models ($\mathcal{M}$) evaluated has been 48960 (510 TLEs × 2 time series × 8 trained periods × 6 A/m values). From the combination of the two forecasting methods, four hybrid propagators have been developed: HSGP4(A,A), HSGP4(A,E), HSGP4(E,E) and HSGP4(E,A). In the cases of distance error improvement respect to SGP4, the EDA analysis includes box-and-whisker plots, outliers analysis, the relationship between the TLEs and the magnitude of their errors These models, automatically generated, can be used as a first attempt at classifications in function of the TLE. Finally, from these four hybrid propagators we derive the best hybrid combination (BestHSGP4) and try to identify what conditions allow to select the best combination. In the cases that there is no improvement in the distance error of SGP4, the time series are identified and their shapes characterized.

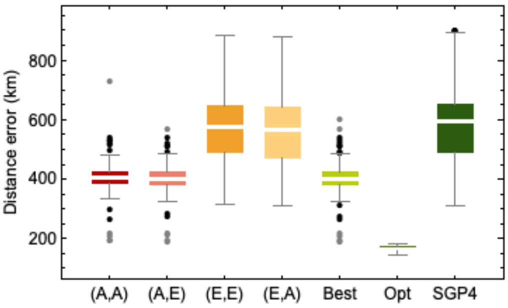

Figure 1: Box-and-whisker plots of the distance error (km) between PSIMU and the four hybrid SGP4, the BestHSGP4 and OptHSGP4 propagatosr for a time span of 10 days and A/m $= 1 m^2/kg$.

Figure 1 and 2 show the box-and-whisker plots of the distance error (km) between PSIMU and the four hybrid SGP4, the BestHSGP4 and OptHSGP4 propagators for a time span of 10 days and A/m $= 1,\ 0.001 m^2/kg$. The forecasting model have been training using 10 satellite revolutions. The distance

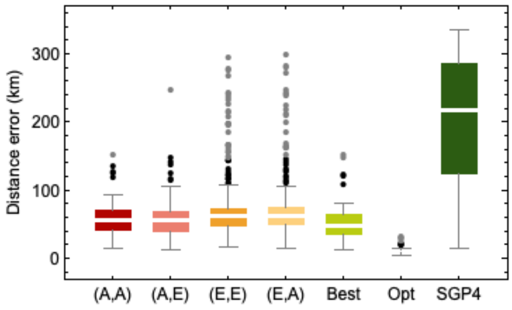

Figure 2: Box-and-whisker plots of the distance error (km) between PSIMU and the four hybrid SGP4, the BestHSGP4 and OptHSGP4 propagators for a time span of 10 days and A/m = $0.001 m^2/kg$.

error can be reduced from about 80% in the case of low A/m value to 35% in the case of high A/m value at 10 propagation days for the 75% of the cases.

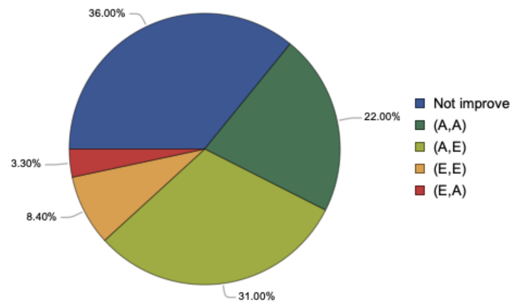

Figure 3: Percentage of forecasting models of the BestHSGP4 propagator for a time span of 10 days and A/m = $1 m^2/kg$.

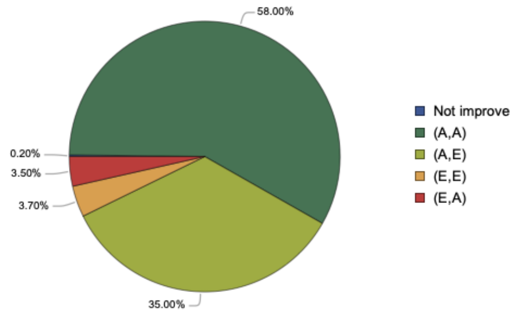

Figure 4: Percentage of forecasting models of the BestHSGP4 propagator for a time span of 10 days and A/m = $0.001 m^2/kg$.

Figure 3 and 4 plot the percentage of forecasting models of the BestHSGP4 propagators with the previous assumptions. The BestHSGP4 propagator improves from almost 100% in the best case (A/m = $0.001 m^2/kg$) to 65% in the worse case (A/m = $1 m^2/kg$).

## References

[1] J. F. San-Juan and M. San-Martín, "New families of hybrid orbit propagators based on analytical theories and time series models," *Advances in the Astronautical Sciences*, vol. 136, pp. 547–565, 2010, paper AAS 10-136.

[2] J. F. San-Juan, M. San-Martín, and D. Ortigosa, "Hybrid analytical-statistical models," *Lecture Notes in Computer Science*, vol. 6783, no. 2, pp. 450–462, 2011.

[3] M. San-Martín, "Métodos de propagación híbridos aplicados al problema del satélite artificial. Técnicas de suavizado exponencial," Ph.D. dissertation, University of La Rioja, Spain, 2014.

[4] J. F. San-Juan, M. San-Martín, and I. Pérez, "Application of the hybrid methodology to SGP4," *Advances in the Astronautical Sciences*, vol. 158, pp. 685–696, 2016, paper AAS 16-311.

[5] D. Brouwer, "Solution of the problem of artificial satellite theory without drag," *The Astronomical Journal*, vol. 64, no. 1274, pp. 378–397, November 1959.

[6] Y. Kozai, "Second-order solution of artificial satellite theory without air drag," *The Astronomical Journal*, vol. 67, no. 7, pp. 446–461, September 1962.

[7] M. H. Lane and K. H. Cranford, "An improved analytical drag theory for the artificial satellite problem," in *Proceedings of the AIAA/AAS Astrodynamics Specialist Conference*. Princeton, NJ, USA: American Institute of Aeronautics and Astronautics, August 1969, paper AIAA 69-925.

[8] F. R. Hoots, P. W. Schumacher, Jr., and R. A. Glover, "History of analytical orbit modeling in the U.S. space surveillance system," *Journal of Guidance, Control, and Dynamics*, vol. 27, no. 2, pp. 174–185, March-April 2004.

[9] D. A. Vallado, P. Crawford, R. Hujsak, and T. S. Kelso, "Revisiting spacetrack report #3," in *Proceedings 2006 AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, vol. 3. Keystone, CO, USA: American Institute of Aeronautics and Astronautics, August 2006, pp. 1984–2071, paper AIAA 2006-6753.

[10] R. S. Hujsak, "A restricted four body solution for resonating satellites without drag," U.S. Air Force Aerospace Defense Command, Colorado Springs, CO, USA, Spacetrack Report #1, 1979.