LONG-TERM NUMERICAL PROPAGATION FOR EARTH ORBITING SATELLITES

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Numerical propagation techniques have been extensively studied and are routine for precise satellite operations. Most studies focus on time spans of a few days to several weeks, specific orbital classes, or interplanetary orbits. As long term numerical operations become more commonplace, it's useful to quantify accuracy performance for propagations of several months, to years. This talk performs long-term numerical propagation comparisons against reference orbits in a variety of orbital classes. Semianalytical techniques are also used in the comparisons including a general discussion of the initial osculating to mean element conversion. Simplified General Perturbations (SGP4) is discussed with some comments on the new SGP4-XP semianalytical routine. Finally, orbital size, shape, and orientation considerations are examined.

INTRODUCTION

Space Situational Awareness (SSA) implies the knowledge of how, what, where, when and why things are in orbit. I refine that definition so that SSA is the process by which an organization maintains orbital state and mission information on all objects in space with some level of accuracy, now, and at a future time. The general flow of events is to process observations with an Orbit Determination (OD) scheme (that uses a propagator with an integrator), and then to create an ephemeris with the same propagator and integrator. Propagators are either analytical and semianalytical (using mean elements), or numerical (using osculating elements). Mean elements are averaged based on separating secular, short and long periodic variations, and have many dependencies on the type of averaging. Osculating elements vary over time and should represent the best estimate of the satellite state at any given time. As such, precise SSA operations often use osculating elements, and mean element theories always have some way to extract osculating values at instants of time.

Researchers seek techniques to provide solutions to the various missions of SSA. One focus area is the rapid long-term propagation of satellite orbits. This is especially true for the analytical and semianalytical techniques. The SGP4 routine that has been used for many decades by the US Space Force is a prime example, and one that I'll focus on a little later. The initial development of a technique focuses less on the routine daily OD from observations to support SSA operations, but rather the speed and accuracy of the propagator. This often neglects the need to convert between elements, although it may be relevant in some cases, and is always considered as development matures. Accurately comparing semianalytical techniques to reference numerical ephemerides provides some interesting challenges as we'll see later.

There is interest in mega constellations today, mostly for Low Earth Orbit (LEO), and *if* all the projections come true, over 100,000 additional new payloads will reside in orbit – an increase of 3-4 times the current catalog! Additionally, as the new US Space Force S-band radar "fence" comes online, the space catalog may grow by many thousands of objects due to the improved observability of small debris objects.

Selecting which propagator is the most accurate approach always evokes a passionate discussion! Combined with trying to process a space catalog that's 5-10 times larger than it is today presents another challenge – not to mention how to transmit all the information to users! Numerical propagation is generally accepted as the most accurate for SSA mission operations (several weeks). But how does it perform 6 months out, a

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year, or more? Are analytical and semianalytical techniques perhaps better for some situations? How do I get the initial state vectors so I can accurately compare each of these approaches?

If all the observations used to create the Two Line Element (TLE) catalog were publically available (they're not), one *could* process them all (OD) with each specific propagation technique to create initial mean or osculating element state vectors (lots of computer processing). The largest [only comprehensive*] collection of public data for space objects is the TLE database produced over time by the U.S. Air Force Space Command. This data derives from an analytical propagator (Simplified General Perturbations SGP4, Vallado et al. 2006) that produces an osculating ephemeris of modest accuracy given a mean element initial state. Unfortunately, the accuracy of the TLE catalog is not sufficient to perform highly accurate studies and the errors of the observations and the mathematical theory are intertwined. Thus, I do not consider OD from observations in this talk.

I also do not address covariance matrices in this talk even though they could be useful in understanding the accuracy. There are many formulations and their public availability is limited for most applications. For example, no covariance information exists for the TLE catalog. Finally, recognize that maneuvering satellites change the nature of the problem because the propagation must be accurate only for the length of time between maneuvers. This is often quite short and may be a few days to a few weeks.

Two solutions are readily available to find accurate reference ephemerides. Several non-maneuvering Satellite Laser Ranging (SLR) satellites have long, well defined, independent ephemerides. These ephemerides are produced in the Consolidated Predicted File (CPF) formats that are available for many SLR equipped satellites†. These are predicted orbits, so the accuracy is generally less than rigorous OD solutions of the SLR data, but they do afford a comparison at about the meter-level. Several centers produce the ephemerides‡. The CPF ephemerides can be considered Precision Orbit Ephemerides (POE's), or "truth" orbits.

The second source of reference ephemerides is to use a numerical technique to generate a reference orbit and then make the comparisons against this ephemeris. Rather than doing this without clarification, a section compares the numerical propagator and selected integrator to establish that the settings and approach are accurate compared to independent truth ephemerides.

The structure of the talk is as follows. First, I review some of the literature and error sources in integrators and generic propagation schemes. Next, the study methodology is setup including satellites, time spans, integrators, etc. The results lay out a sequenced approach to establishing accuracy. First, integrators are compared against each other to understand the specific implementations and any settings (ie step-size) that may need to be adjusted. After selecting a baseline integrator for the numerical propagation, comparisons are made to the CPF orbits to establish a baseline accuracy. To illustrate one analytical approach, comparisons are made to TLE vectors, both from a single state, to the regularly released TLE values. The comparisons to the CPF orbits reveal a need to obtain the proper initial state for all techniques (Vallado, 2007). Many comparisons are made for satellites in different orbital classes. Several semianalytical technique comparisons are shown against a GEO orbit. A discussion of SGP4 and the new SGP4-XP provides continuity on one of the most widely used analytical techniques available. Finally, there is a discussion of the accuracy from the orbital plane, size and shape characteristics. Essentially, the positional accuracy may degrade, but the orbital plane may be fairly accurate, and therefore sufficient for some applications.

PRIOR STUDIES

Many papers have examined aspects of this problem, or techniques used in this study. A few follow.

Montenbruck (1992) examines various numerical integrators for orbital propagation. He shows function calls vs digits accuracy for many Runge-Kutta methods, noting that many propagators exist that can fulfill modern mission analyses. He also remarks that traditional fixed size step workhorses like GEODYN, UTOPIA, and PEPSOC, etc still have a solid place in numerical propagation, and therefore SSA activities.

* ESA (2019) lists additional catalog sources as the JSC Vimpel Space Data, and the Keldysh Institute for Applied Mathematics (KIAM). Note that these sources do not exist for as long a historical interval as the TLE catalog.
† ftp://edc.dgfi.tum.de/pub/slr/cpf_predicts_v2 (accessed March 2022)
‡ https://ilrs.gsfc.nasa.gov/data_and_products/predictions/prediction_centers.html (Accessed June 2022).

Wilkins et al. (2000) examined creating a general perturbation (GP, analytical) catalog from the numerical catalog the Navy was producing at the time. This was the first large catalog conversion from osculating to mean elements, and it showed the benefits of increased density of observations (in the form of ephemerides instead of sparse observations). The paper also demonstrated the need to perform a PCE to properly find the best initial conditions for the mean element theory. I'll expand on this requirement later.

Bradley (2009) discusses symplectic propagators for long-period propagation (many years). He looks at several orbits. There is some focus on International Terrestrial Reference Frame (ITRF) propagation. Finally, he examines the number of function calls and EOP/ International Astronomical Union (IAU) transformations.

Hofsteenge (2013) examines long term debris propagations for hundreds of years including some symplectic integrators. He uses the TU Delft Astrodynamics Toolbox (Tudat) program. Hofsteenge concludes that symplectic methods are very efficient for Hamiltonian systems, but might be difficult to implement for other systems and applications. He noted the difficulty in determining the initial conditions, and the effect it had on the results.

Atallah (2019) examines several integrators, including the Adaptive Picard Chebyshev (Woollands and Junkins 2019). He describes several tests that can also be applied to integrator comparisons. Because my efforts were more general, I opted to use a simple positional comparison, but these additional tests could provide some rigor to future studies.

Vallado (2019) examined the long term propagation and this talk derives primarily from that paper.

METHODOLOGY

Several things were needed to accomplish this study. First, satellites were selected to represent various orbital classes that are relevant for SSA. Generic satellites were used for the initial integrator tests and for some semianalytical tests. Actual satellites were used in the comparison to independent orbits. Integrators and propagators were selected based on availability of code and programs. Reference orbits were generally selected from SLR satellites as they afford several orbital classes combined with very high accuracy.

Selected Satellites

The initial task was to select a number of satellites with existing POE information, or other high quality ephemerides that could be used for comparisons. The satellites selected for this study form a spectrum from LEO to Geosynchronous (GEO) satellites (Table 1). The epochs varied due to the presence of available information. A year or more of data was selected for as many satellites as possible, and periods of higher solar activity were preferred to highlight the effects of atmospheric drag on the lower altitude orbits. The additional satellites in the LEO category were designed to better determine the results in the drag regime, and at the lower end of the solar radiation pressure regime. The LEO satellites also account for repeat ground track, sun-synchronous and frozen orbit conditions which can exhibit interesting resonances. The study satellites in Table 1 permitted a quick look at various orbital classes, but the specific satellite parameters (coefficient of drag (c_D), coefficient of radiation pressure (c_{SR}), mass (m), area (A)) are often difficult to obtain.

The study interval was generally from 1 Jan 2018 00:00:00.000 to 1 Jul 2019 00:00:00.000, depending on the available data. The time frame for the analysis is important for LEO orbits. For atmospheric drag, solar maximum is desirable, but the last solar max was quite low.

Integrators and Propagators

All numerical propagations are performed with Ansys/AGI's Systems Tool Kit/High Precision Orbit Propagator (STK/HPOP). STK/HPOP lets you select the following types. I did not test other types of integrators (including symplectic, Picard Chebyshev, differential algebra, etc).

- RK4
- RK7/8
- RK8/9
- Burlisch-Stoer

- Gauss-Jackson*

Table 1: **Satellite Orbital Parameters:** This table lists the general orbital parameters for satellites used in the study. There are 2 overall categories. The simulated orbits were general orbits chosen for integrator performance evaluation. The SLR orbits contain orbital classes (Sun-synchronous, repeat ground-track, frozen orbit) that are useful in testing, and have accurate ephemerides for comparisons. The mass and area parameters were assumed, or derived from the Internet. The mass is often the initial mass – not intended to be definitive values! Acronyms: Highly Elliptical Orbit (HEO), Medium Earth Orbit (MEO), Global Positioning Satellite (GPS), Laser Relativity Satellite (LARES), Ajisai (the Japanese name for the Hydrangea plant, but also referred to as the Experimental Geodetic Satellite or Payload EGS/EGP), the Laser Geodynamics Satellite or Laser Geometric Environmental Observation Survey (LAGEOS 1 and 2), Etalon (1 and 2), the Chinese Beidou/Compass system and the Indian Regional Navigation Satellite System (IRNSS). The Japanese QZSS system provides maneuvers which may make some analysis possible (https://qzss.go.jp/en/technical/qzssinfo/).

Name	Category	Apogee Alt (km)	Perigee Alt (km)	*a*	*e*	*i* (°)	*n* (rev/day)	SSC #	Mass (kg)	Area (m2)
Simulated Propagation Orbits										
	LEO			7078.14	0.00100	28.50			100.00	20.000000
	HEO			26000.00	0.70000	63.40			200.00	20.000000
	GEO			42164.00	0.00100	0.02			3000.00	97.000000
SLR orbits										
Larets	LEO	691	674	7060.64	0.00120	98.10	14.633112	27944	23.82	0.0314159
Stella	LEO	806	795	7178.64	0.00077	98.90	14.273798	22824	48.00	0.0452389
LARES	LEO	1449	1439	7822.14	0.00064	69.50	12.549152	38077	386.80	0.1040621
Ajisai (EGS)	LEO	1497	1479	7866.14	0.00114	50.00	12.444007	16908	685.00	3.5968094
LAGEOS 1	MEO	5948	5838	12271.14	0.00448	109.90	6.386682	8820	406.97	0.2827433
COSMOS 1989 (Etalon 1)	MEO	19182	19069	25503.64	0.00222	64.20	2.131569	19751	1415.00	1.3150990
QZS-3 (MICHIBIKI-3)	GEO	35793	35780	42164.64	0.00015	0.10	1.002721	42917	2379.20	19.9200000
IRNSS-1B	GEO	35878	35691	42162.64	0.00222	29.30	1.002792	39635	1432.00	10.000
IRNSS-1D	GEO	35882	35690	42164.14	0.00228	29.20	1.002739	40547	1425.00	10.000
QZS-3	GEO	35793	35779	42164.14	0.00017	0.10	1.002739	42917		
COMPASS-I3	GEO	35957	35627	42170.14	0.00391	58.80	1.002525	37384		

I also chose a few analytical and semianalytical propagators to highlight important considerations for these techniques. Indeed, there are dozens and dozens of integrators and propagators (Vetter 2007, Table 5). The methods examined include:

- Chao and Hoots (Chao and Hoots, 2018). Note that this *method* is only equations for short period (SP) and long period (LP) variations and not a formal semianalytical approach per se.
- Draper Semianalytical Satellite Theory (DSST: McClain 1978, 1992 and Danielson 1994, 1995†)

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* Note that the GJ in STK does not have an error control to adjust the step size to maintain accuracy. The GJ is coded in the summed form, which is accurate and stable but changing the step size is not easy. The normal step size control involves either halving or doubling the step when needed. Krogh (1974) discusses changing the steps size for multi-step methods. Without automatic error control, the user's step size selection is critical to accurate integration results. Choosing the step size consists of reducing it until it no longer seems to affect the end result.

† There are several versions of DSST. The original starting point is the F-77 standalone DSST baseline of 1984 (Early, 1986). The top-level architecture of the DSST Standalone was modified by Carter in the early 1990s to support the maneuver planning functions of the Radarsat-1 Flight Dynamics System (FDS). The new architectural approach is described in the Orbit Propagator Services section of the Radarsat FDS Software Design Document (SDD) (Cefola et al, 1994). Further significant additions were made to the F-77 DSST Standalone in 1997 (Neelon et al, 1997). These included the 50 × 50 geopotential, solid Earth tides models, and the J2000 coordinate system. The DSST Tesseral Linear Combination model (Proulx et al, 1981) and the Lunar-Solar short-periodic model (Slutsky, 1983) were included in the Standalone to improve the short-periodic model. The 1997 version of the Standalone included structured modules to ensure that physical quantities had the same names throughout the software. Cefola et al. (2009) discusses further testing of the DSST. In 2011 to 2013, the DSST algorithms were programmed in Java for the open source Orekit Flight Dynamics library, https://www.orekit.org/. Comparisons of long-term orbit propagations made with the Orekit java DSST and with the F77 DSST Standalone led to further improvements in the F77 DSST Standalone (Cefola et al, 2014). San-Juan et al. (2017, 2019, and 2020) document the development of a C/C++ DSST version of the F77 DSST Standalone. The C/C++ DSST started from the F77 version that existed at the start of 2014. Setty et al. (2016) has developed a further improved

- Semi analytical Tool for End of Life Analysis (STELA: Morand et al. 2013 and Deleflie 2011). https://logiciels.cnes.fr/en/content/stela
- Tool for High-Accuracy, Long-term Analyses for SSA (THALASSA: Amato, et al. 2019[*])
- Simplified General Perturbations (SGP4, Vallado et al. 2006). SGP4 propagations are from STK and ODTK. This is an analytical technique.

Other semianalytical routines were envisioned for the study, but time didn't permit their inclusion.

- Long Term Orbit Propagator (LOP: Kwok, 1986, 1987)
- Highly Elliptic Orbit Semi-Analytical Theory (HEOSAT: Lara, San-Juan, and Hautesserres 2016 and Gondelach 2017)
- Planetary Orbital Dynamics (PLANODYN: Frey and Columbo 2018 and Wittig et al. 2014)

"Full" force models are generally used throughout the tests (defined as a 50 × 50 EGM-08 gravity field, Jacchia Roberts atmosphere, 3-body gravity, solar radiation pressure). For solar radiation pressure, the entry/exit conditions can occur at different times/steps with different interpolators. This requires some form of boundary mitigation, or smoothing to get all the approaches to treat the shadow discontinuity equally. Similarly, splining SPW indices produces more consistent results (Vallado and Finkleman 2014).

Reference orbit selection

Selecting the reference orbits consisted of satellites that had either independent high precision ephemerides, or observations from which precise orbits could be determined. In most cases, satellites without maneuvers were selected to permit long-term propagation effects to be understood. This proved challenging especially for GEO satellites. As mentioned earlier, the CPF orbits fulfilled this role. Files were combined to form 1-year ephemerides for each satellite.

Study Process

Several steps were necessary to conduct the study. First was to establish the accuracy for the baseline integrators in the numerical propagator.

- Compare integrators with the same force model, EOP and SPW data, and satellite configurations for generic LEO, HEO and GEO satellites.

Using the selected baseline integrator (RK78) and numerical propagator, compare differences (RSW) to the CPF orbits.

- Numerically propagate the first state vector in the ephemeris with complete force models and the actual observed EOP and SPW values.
- Perform a precise conversion of elements (PCE – use a differential correction to estimate the initial elements) on the ephemerides to better determine the initial state. Compare the PCE result and subsequent propagation to the CPF ephemerides.
- Examine the orbital plane characteristics of the PCE results to the CPF orbits.

Examine the performance of analytical and semianalytical techniques.

- Compare various analytical propagators (J2, J4, etc) to the LEO orbit.
- Compare a propagated single initial TLE to the CPF orbits.
- Compare the set of TLEs throughout the time interval to the CPF orbits. While this isn't really a long term propagation, it serves to show general performance over time.
- Compare the single point conversion from each technique from the osculating ephemeris to the CPF orbits.

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version of the DSST Standalone at the German Space Operations Center (GSOC). This version has expanded functionality with respect to the short periodic model and in dynamic parameter estimation. Setty also developed an Orbit Determination Services program. Both the propagator and OD programs employ the same DSST source code library. Folcik (2019) maintains the GTDS baseline including GTDS/DSST at the MIT/Lincoln Laboratory.

[*] https://gitlab.com/souvlaki/thalassa Note that THALASSA has no averaged formulation, so it isn't a true semianalytical technique in the usual sense. Further improvements to the code may include these effects.

- Develop a differential correction process to perform a PCE process to determine a better initial state and compare to the CPF orbits.

CONCLUSIONS

Several aspects of long-term numerical propagation have been examined. The goal was to present information about how long a numerical propagator could produce reliable information, from an operational perspective, with which to feed a particular SSA mission area. While many results were surprisingly accurate, others were not. Attention to detail is important as many things must be set and default program values are often incorrect. At the macro level, it seems that about 1-10 km error after a year propagation is achievable from both numerical and semianalytical methods. The results showed that the time of processing is quite different, but the overall accuracy is about the same.

First, several integrators were compared to determine any differences. Step sizes became important for the RK4 integrator, and regularized time was critical to all orbits with eccentricity larger than about 0.3. For the RK4 integrator, the default value time step needs to be adjusted smaller to match the step-size controlled integrators. For a LEO orbit, about 5 secs (with a 50 × 50 gravity field, drag, solar radiation pressure, srp, and 3rd-body) seemed to work well for the RK4. Make sure splining for atmospheric and srp eclipse boundary mitigation is turned on. For the HEO orbits, "all" integrators needed time regularization to accurately propagate. The RK4 also needed time regularization with 840 steps per orbit. A RK7/8 was chosen as the common integrator for the remainder of the talk as representing a middle position of the perfomance of each integrator tested. After about 18 months, the results generally were about 1 km, and this seems reasonable.

Next, comparisions were made with the RK78 integrator and various real-world Consolidated Predicted Format (CPF) orbits. The single state vector propagtaion did ok in some cases, but very poorly in others. A short PCE was conducted using the reference ephemeris as observations, and this improved most of the tests to about 1 km uncertainty. However, the lower altitude orbits lacked sufficient observability in the PCE to refine the initial estimate. This step re-confirmed the necessity of using a PCE to refine the initial state.

A few analytical propagators were examined. The force models must match – huge differences were noted in just a couple weeks due to the presence of un-modeled forces. The TLE propagation was sometimes good, sometimes very poor. Unfortunately, there's no indicator as to whether or not the TLE is bad. The ensemble over time generally shows an average accuracy, but determining if a current change is the result of a maneuver, or an erroneous TLE is unknown in advance. Be especially cautious with any maneuvering satellite!

Semianalytical propagators were investigated. The usual comparison method assumes an initial mean element state is known, or that one is found through a PCE from observations (except for THALASSA). For this talk, neither were true so the focus was on the osculating to mean conversion, subsequent propagation, and formation of the propagated osculating ephemeris. The results showed that particular attention to detail is needed in the osculating to mean conversion. Single point conversions were expected to be approximate, and generally performed that way. A simple PCE was programmed using finite differencing for some of the semianalytical techniques. This improved the performance, but not in all cases. When developer (not open source) PCE tests were performed, the results improved dramatically. Additional tests were conducted for DSST and alternate versions of the method, and the WTD seemed to be a large contributor to the differences for 3-body perturbations. The source of the Sun and Moon state vectors seems to be a large influencer as well. Using analytical, interpolated, and full JPL DE models for the 3rd body positions over a period of a few months seem to produce very large variations in the resulting propagations.

Finally, the best positional accuracy isn't always needed. All the techniques showed remarkable ability to preserve the orbital plane even as the positional differences grew large. Depending on the mission requirements, it may be feasible to use techniques that introduce positional errors, but maintain the orbital plane orientation. Maneuver planning for a complete mission is one example.

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