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A SIMPLE MODEL WITH PEER PRESSURE: THE ANTISOCIAL BEHAVIOR CASE

UN MODELO SIMPLE CON PRESIÓN DE GRUPO: EL CASO DE LAS CONDUCTAS ANTISOCIALES

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Abstract

Antisocial behaviors such as consumption of addictive substances and eating disorders are modeled using a SIR model. We propose a function $l(y(t))$ that describes the relapse-recovery-recycling rate. $l(y(t))$ describes either strengthening or weakening of convictions of recovered individuals to relapse in antisocial behaviors. We show that a wide variety of functions can induce the existence of multiple equilibria points for $R_0 < 1$ which is a catastrophic scenario for the susceptible population. Finally, conditions for avoiding a sudden and catastrophic jump in the number of individuals with antisocial behaviors are given.

Keywords: peer presure; antisocial behaviors; backward bifurcation; forward bifurcation.

Resumen

Comportamientos antisociales tales como consumo de sustancias adictivas y desordenes de la alimentación son modelados usando un modelo SIR. Proponemos una función $l(y(t))$ que describe la tasa de reciclamiento-recuperación-recaída. $l(y(t))$ describe ya sea el fortalecimiento o debilitamiento de la convicción de los individuos a recaer en conductas antisociales. Mostramos que una amplia variedad de funciones puede inducir la existencia de múltiples puntos de equilibrio para $R_0 < 1$, lo que constituye un escenario catastrófico para la población susceptible. Finalmente, se dan condiciones para evitar un aumento repentino y catastrófico en el número de individuos con comportamientos antisociales.

Palabras clave: presión social; comportamientos antisociales; bifurcación hacia atrás; bifurcación hacia adelante.

Mathematics Subject Classification: 34A34, 34C40.

1 Introduction

Mathematical modeling of social processes has captured the interest of mathematicians and sociologists [26, 16, 23]. In particular, some results of the mathematical epidemiology are used to describe how antisocial behaviors can be spread in a population. It is known that there are differences between modeling social behavioral processes and infectious diseases; however, the number of new individuals who acquire one particular antisocial conduct may be modeled as the number of effective contacts of individuals in different behavioral groups. Peer pressure has been linked to behavioral changes on individuals. Peer pressure is defined as the social pressure to adopt certain behaviors in order to fit in with

others individuals [12, 20, 21, 24]. Marsden [18] organized six scenarios where behavioral contagion might occur; violence, deliberate self-harm, hysterical contagion, rule violation behavior, financial behavior and consumer behavior.
Freedman et al. [14] observed that there is behavioral contagion if there is a high population density in each group. Although this kind of contagion is still speculative, Goldstein et al. [15] analyzed the aggressive behavior in a children's day care centre in which was identified a statistically significant contagion. Peer pressure has been associated to antisocial behaviors which are actions that harm or lack of consideration about the well-being of others. Examples of antisocial behaviors are threats, vandalism, rudeness, anger issues, verbal abuse, fighting, littering, manipulating others, violence and criminal activities [10, 17, 19, 22].

In the epidemiological mathematical literature, there are models that describe the dynamics of certain social behaviors, for example: the consumption of addictive substances such as alcohol, ecstasy, tobacco; eating disorders such as anorexia and bulimia, or depression in young women [26, 2, 5, 6, 7, 9, 13, 16, 23, 25]. Also, the spreading of knowledge and the spreading of scientific ideas have been modeled using results of the mathematical epidemiology [3, 8]. That is, the peer pressure can be associated with both negative and positive behaviors.

When social phenomena are studied through epidemiological models, the social dynamics can be described as a function of threshold parameters. In the mathematical epidemiology, the most important threshold parameter is the basic reproduction number, which is denoted by R_0. R_0 is a dimensionless quantity and usually is interpreted as the number of secondary infections that an infectious individual is able to produce when it is introduced in a totally susceptible population during its infectious period. In a classical way, R_0 is associated to a forward bifurcation. That is, if $R_0 < 1$, the number of infectious individuals decrease, and the disease disappears, meanwhile if $R_0 > 1$, the number of infectious individuals tends to one endemic equilibrium ($I^* > 0$) (see Figure 1 A)). However, epidemiological models may show multiple endemic states if $R_0 < 1$. That is, in this scenario can appear two nontrivial equilibria points (multiple equilibria) for $R_0 < 1$. In particular, an endemic state persists if the number of infectious individuals is big enough, even though R_0 is less than 1. In this case, for $R_0 < 1$, there will be two locally stable equilibria points, one with no disease and other with a positive endemic level. This scenario is associated to a backward bifurcation in $R_0 = 1$ (see Figure 1 B) shaded region).

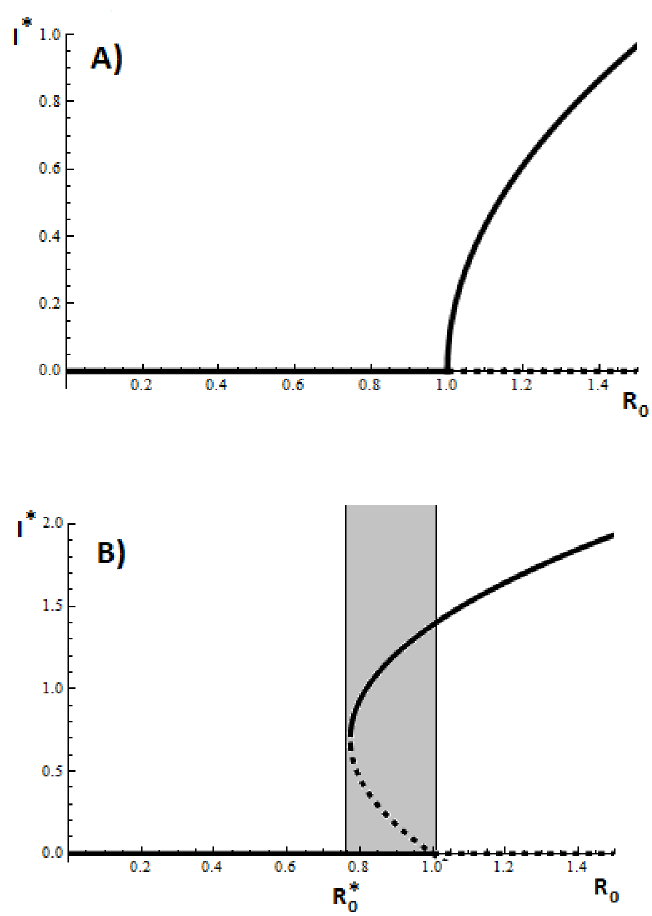

Figure 1: Case A) shows a forward bifurcation at $R_0 = 1$. In this case, a unique endemic equilibrium point appears for $R_0 > 1$, meanwhile, case B) shows a backward bifurcation at $R_0 = 1$. In this scenario, a bistability phenomenon occurs for values of R_0 below one.

In this work, we construct a simple model with terms describing the peer pressure exerted by individuals with antisocial behaviors to susceptible and recovered individuals. For this, a widely variety of nonlinear functional forms of relapse-recovery-recycling rate $l(y)$ are considered. We analyze the direction of the bifurcation in $R_0 = 1$ as a function of the parameters of the model. Also, we show numerical simulations of the solutions of the model. Finally, we discuss the obtained results.

2 The model and the bifurcation analysis

2.1 The model

Following the well-known SIR model structure, it is proposed three differential equations describing the interactions of individuals in different classes. We consider a population $N(t)$ that is divided in three classes. So, individuals who have never presented antisocial behavior, but are prone to present it, are in the class $x(t)$. Individuals who present some antisocial behavior such as abuse of substances like alcohol, tobacco, ecstasy are in the class $z(t)$. Also, $z(t)$ includes individuals with eating disorders like anorexia and bulimia. Finally, individuals who no longer display antisocial behaviors after a certain time are in the class $y(t)$. Then, $N = x(t) + y(t) + z(t)$.

We propose the model shown here.

$$
\begin{aligned}
\dot{x} &= f(x) - \beta x z, \\
\dot{y} &= -l(y) z - \mu y, \\
\dot{z} &= \beta x z - \mu z + l(y) z.
\end{aligned}
\tag{1}
$$

In model (1), all the parameters are nonnegative. In particular, β is the effective influence rate because of the peer pressure. So, in considering consumption of addictive substances, $\beta x(t)z(t)$ is the average number of susceptible individuals who become habitual users of addictive substances per unit of time due to peer pressure. μ is the natural death rate. Also, births and deaths in the susceptible class are described through $f(x)$. We assume that $f'(x^*) < 0$, where x^* is the total population in absence of antisocial behavior. This assumption will be used when the stability of the disease-free equilibrium is analyzed.

In model (1), $l(y(t))z(t)$ describes the interactions of individuals with antisocial behavior and the recovered individuals. In particular, $l(y(t))z(t)$ is the number of individuals that may be exchanged between the classes $y(t)$ and $z(t)$.

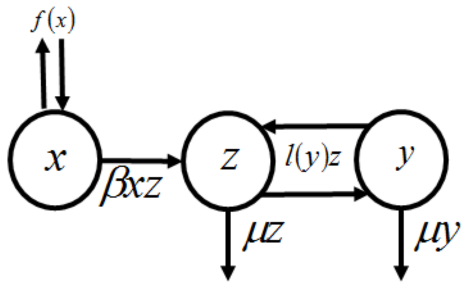

Figure 2: Compartamental model.

For model (1), the relapse-recovery-recycling rate $l(y(t))$ satisfies the following conditions; $l(y) \in \mathcal{C}^1(R)$, and $l(0) < 0$.

2.2 The basic reproduction number, R_0

In analogy with epidemiological models, we calculate the basic reproduction number, R_0, for model (1). Let $E_0 = (x^*, 0, 0)$ the trivial equilibrium for model (1). Then $f(x^*) = 0$. We define E_0 as the antisocial-behavior-free equilibrium. To analyze the stability of the equilibrium solutions, we calculate the Jacobian matrix of (1). This is shown here.

$$J((x,y,z)) = \begin{pmatrix} f'(x) - \beta z & 0 & -\beta x \\ 0 & -\mu - l'(y)z & -l(y) \\ \beta z & l'(y)z & \beta x - \mu + l(y) \end{pmatrix}. \tag{2}$$

The Jacobian matrix evaluated in the antisocial-behavior-free equilibrium is given by

$$J(E_0) = \begin{pmatrix} f'(x^*) & 0 & -\beta x^* \\ 0 & -\mu & -l(0) \\ 0 & 0 & \beta x^* - \mu + l(0) \end{pmatrix}. \tag{3}$$

The characteristic polynomial associated to (3) is

$$P(\lambda) = (\lambda + \mu)(\lambda - f'(x^*))(\lambda - \beta x^* + \mu - l(0)). \tag{4}$$

Then, the eigenvalues for matrix (3) are

$$\lambda_1 = -\mu, \quad \lambda_2 = f'(x^*), \quad \text{and} \quad \lambda_3 = \beta x^* - \mu + l(0). \tag{5}$$

Analyzing the eigenvalues showed above, the basic reproduction number can be defined as $R_0 = \frac{\beta x^*}{\mu - l(0)}$. Notice that, $\lambda_2 < 0$ because $f'(x^*) < 0$. The stability of the antisocial-behavior-free equilibrium, as a function of R_0, is described in the following result.

Lemma 1 *For model (1), the antisocial-behavior-free equilibrium E_0 is locally asymptotically stable if and only if $R_0 < 1$.*

To examine the sensitivity of R_0 to each of the parameters, we calculate the sensitivity indexes.
$S_\beta = \frac{\beta}{R_0}\frac{\partial R_0}{\partial \beta} = 1$, $S_\mu = \frac{\mu}{R_0}\frac{\partial R_0}{\partial \mu} = \left|\frac{-\mu}{\mu - l(0)}\right| < 1$ and $S_{l(0)} = \frac{l(0)}{R_0}\frac{\partial R_0}{\partial l(0)} = \left|\frac{l(0)}{\mu - l(0)}\right| < 1$.

Furthermore, the basic reproductive number R_0 is more sensitive to changes in β than μ and $l(0)$.

2.3 Direction of the bifurcation at $R_0 = 1$

As a consequence of the lemma 1, we define $R_0 = 1$ as one bifurcation point for model (1). Let $\beta^* = \frac{\mu - l(0)}{x^*}$ the bifurcation value such that $R_0 = 1$.

The direction of the bifurcation in the critical value $R_0 = 1$ will be determined using results of the central manifold theory and the normal forms theory. We will describe the behavior of the system solutions around of the bifurcation point. To do this, we analyze the signs of the coefficients of the normal form on the centre manifold associated to model (1). The coefficients of the normal form will be calculated in an analogous way to the coefficients calculated in the Theorem 4.1 in [4].

In this case, matrix (3) evaluated in E_0 for β^* is given by

$$J(E_0, R_0^*) = \begin{pmatrix} f'(x^*) & 0 & -\beta^* x^* \\ 0 & -\mu & -l(0) \\ 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

The eigenvalues associated to (6) are given by $\lambda_1 = -\mu$, $\lambda_2 = f'(x^*)$ and $\lambda_3 = 0$.

Hence, E_0 is a non hyperbolic equilibrium point in the bifurcation value R_0^*. The vectors,

$$w = \begin{pmatrix} \frac{\beta^* x^*}{f'(x^*)} \\ \frac{-l(0)}{\mu} \\ 1 \end{pmatrix} \qquad \text{and} \qquad v = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

are the right and left eigenvectors for $\lambda_3 = 0$. Observe that, these eigenvectors satisfy the condition $v \cdot w = 1$.

According to Theorem 4.1 in [4], the coefficients of the normal form associated to model (1) are given by

$$\begin{array}{rcl} a & = & 2\left(\frac{(\beta^*)^2 x^*}{f'(x^*)} + \frac{-l(0)}{\mu} l'(0)\right), \\ b & = & x^*. \end{array} \tag{7}$$

Notice that, the coefficient b is always positive. Then, the direction of the bifurcation depends only on the sign of the coefficient a.

We show the following result.

Theorem 1 *For model (1),*
if $\frac{(\beta^*)^2 x^*}{f'(x^*)} > \frac{l(0)}{\mu} l'(0)$*, there is a backward bifurcation at* $R_0 = 1$*,*
if $\frac{(\beta^*)^2 x^*}{f'(x^*)} < \frac{l(0)}{\mu} l'(0)$*, there is a forward bifurcation at* $R_0 = 1$*.*

An analysis of the condition in Theorem 1 leads to the following result.

Lemma 2 *If* $l(y)$ *is a non-increasing function in a neighborhood of* $y = 0$*, then system (1) shows a forward bifurcation at* $R_0 = 1$*.*

In the next section, we show that a bistability phenomenon may appear for model (1) if a backward bifurcation exists. This result is shown using numerical simulations.

3 Numerical simulations

We will use a variety of functions $l(y)$ to show different scenarios for the interactions between individuals in the classes $y(t)$ and $z(t)$. For the numerical simulations, some values of the parameters were taken from a previous model for alcohol consumption analyzed by Sanchez et al. [23]. Also, $f(x) = \mu - \mu x$ will be used to show the results. That is, the susceptible class increases with a constant rate μ. Also, μx susceptible individuals leave the $x(t)$ class per unit time. Then, the antisocial-behavior-free equilibrium is given by $x^* = 1$, $y = 0, z = 0$. A backward bifurcation occurs at $R_0 = 1$, if $\beta < \sqrt{-l(0)l'(0)}$. On the other hand, a forward bifurcation exists if $\beta > \sqrt{-l(0)l'(0)}$.

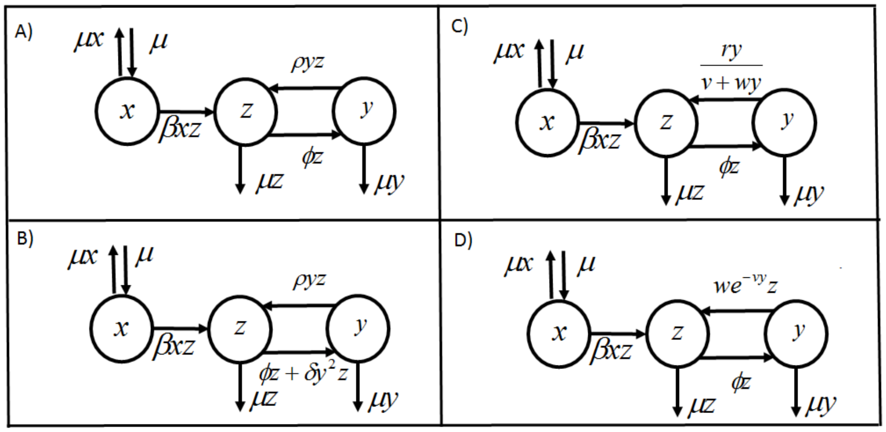

Figure 3: Compartmental models for the relapse-recovery recycling rate proposed, which show the interactions between individuals in different class.

3.1 Negative peer pressure

Compartmental models associated to system (1) with different functions $l(y)$ are shown in Figure 3. We analyze each case in the following. In particular, if $\beta < \sqrt{\phi\rho}$, model (1) has associated a backward bifurcation in $R_0 = 1$ for all the proposed functions. In other words, if the rate in which susceptible individuals acquire antisocial behaviors is less than the geometric mean between the rate ϕ in which individuals are recovered and the rate in which the individuals relapse ρ, then the number of individuals with antisocial conducts can increase even though $R < 1$.

3.1.1 Linear relapse-recovery recycling rate

We choose the linear function $l(y) = -\phi + \rho y$. In this relapse-recovery recycling function, $\frac{1}{\phi}$ is the mean time that an individual in the class $z(t)$ has antisocial conducts. After this time, the individual with antisocial behavior goes to the recovered class. ρ is the effective contact rate between individuals with antisocial conducts and recovered individuals. Then, $\rho y(t)z(t)$ is the number of recovered individuals per unit time that relapse in an antisocial behavior by encounters with individuals with antisocial behavior. This function was used to explain antisocial behaviors as alcohol, tobacco, and ecstasy consumption, bulimia and anorexia [26, 3, 5, 7, 16, 23]. The compartmental model, the bifurcation diagram and the numerical solutions of the model are shown in the Figure 3 A), the Figure 4 A) and the Figure 5, respectively.

3.1.2 Convex relapse-recovery recycling rate

We consider the function $l(y) = -\phi + \rho y - dy^2$. In this case, the first two terms of $l(y)$ have the same interpretation that the case previously analyzed. In this example, we assume that the intentions of belong to the class of individuals with antisocial behavior can be weakened by double exposures to recovered individuals. That is, new individuals with antisocial behavior arise from double exposure at a rate $dy^2 z$ (see Figures 3 B), 4 B) and 5).

3.1.3 Saturing relapse-recovery-recycling rate

In this scenario, we consider the functions $l(y) = -\phi + \frac{ry}{v+wy}$ and $l(y) = \phi - we^{-vy}$ with $w > \phi$ and $\rho = wv$. These functions describe a bounded number of contacts between individuals with antisocial behavior and recovered individuals. These functions may describe that the number of visits of recovered individuals to niches of individuals with antisocial behavior, for examples, raves and clubs is limited (see Figures 3 and 4, cases D) and E)). This kinds of functions have been used in epidemic models [27, 11, 29].

3.2 Positive peer pressure

We now consider the function $l(y) = -\phi - \rho y$. In this case, $\rho y(t)z(t)$ individuals with antisocial behavior are recovered. That is, the intentions of belong to the class of individuals with antisocial behavior is weakened by one single exposure to recovery individuals. In this scenario, the condition $\beta < \sqrt{-l(0)l'(0)}$ is not satisfied. Then there is a forward bifurcation associated to model (1) at $R_0 = 1$; see Figure 1 A). Then, the number of individuals with antisocial conducts decrease until 0 for all initial condition. The compartmental model for this case is shown in Figure 6.

4 Discussion

Consumption of addictive substances (alcohol, tobacco, ecstasy, cocaine) and eating disorders (bulimia and anorexia) bring significant economics losses to individuals and society at large [1, 28]. It is known that antisocial behaviors may be associated to peer pressure, which can be exerted consciously or unconsciously by individuals with some kind of antisocial behavior to susceptible individuals. The peer pressure may occur in specific environments such as schools, amusement/entertainment centers. In particular, reunion places of young people are niches where interactions of individuals with antisocial behavior and susceptible individuals occur in a natural manner.

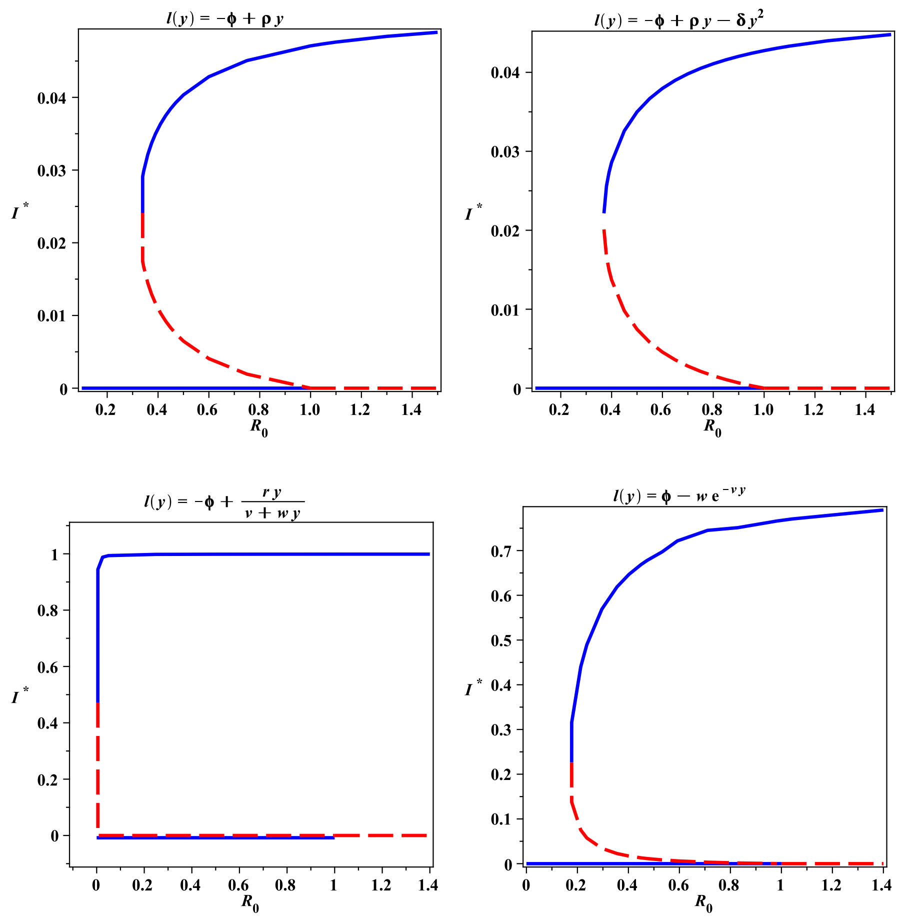

Figure 4: Bifurcation diagrams for model (1).
For $l(y) = -\phi + \rho y$ the values of the parameters are given by $\mu = 0.0000548, \phi = 0.2$ and $\rho = 0.21$. For $l(y) = -\phi + ry - \delta y^2$ the values of the parameters are given by $\mu = 0.0000548, \phi = 0.2, \rho = 0.21$ and $\delta = 0.001$. For $l(y) = -\phi + \frac{ry}{v+wy}$ the values of the parameters are $\mu = 0.0000548, \phi = 0.2, r = 0.4, v = 0.001$ and $w = 0.99$. Finally, for $l(y) = \phi - we^{-vy}$ and $w > \phi$ the values of the parameters are given by $\mu = 0.0548\phi = 0.2, w = 0.99$ and $v = 10$.

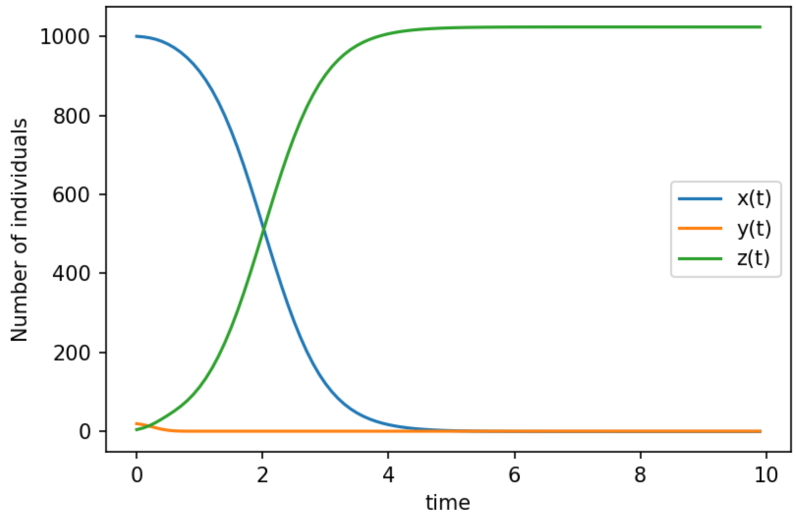

Figure 5: Numerical simulation of the solutions of model (1) with $l(y) = -\phi + \rho y$ for the parameters values $N = 1, \mu = 0.0000548, \phi = 0.2, \rho = 0.21$ and $\beta = 0.002$. The initial conditions are given by $x(0) = 1000, y(0) = 20$ and $z(0) = 5$. Notice that, even though $R_0 = 0.01$, the number of individuals with antisocial behavior increases.

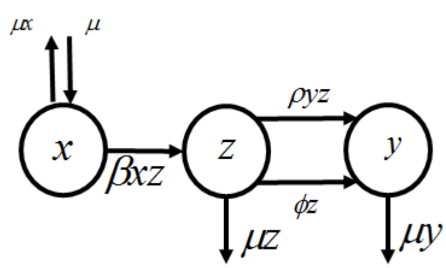

Figure 6: Compartamental model associated to the model with a positive peer pressure and a linear relapse-recovery recycling rate.

We construct a simple model to describe the dynamics of a population. We assume that a proportion of individuals have antisocial behaviors and they exert peer pressure to both susceptible and recovered individuals. We proved that a widely variety of functions describing the relapse-recovery-recycling rate induce the existence of multiple endemic equilibrium points for $R_0 < 1$. Numerical simulations suggest that the model can present a bistability phenomenon. So, the initial number of individuals with antisocial behavior, at the beginning of the epidemic, determine if the number of individuals with antisocial behavior goes to one endemic equilibrium or goes to the antisocial-behavior-free equilibrium. On the other hand, the positive peer pressure avoids that a backward bifurcation appear. In this case, a forward bifurcation is associated to model (1), and the classical strategy of bringing R_0 below 1 is enough to control the epidemic outbreak.

Even though the framework of the mathematical model is very simple and effects such as counsel or treatment against antisocial behavior do not appear in the model, the model analysis allows to make predictions about behavior of the solutions for long times. For the functions analyzed here, we show that the basic reproduction number is not a function of the parameters involved in the relapse of recovered individuals due to social pressure.

In summary, individuals with antisocial behaviors can induce the existence of multiple endemic equilibrium even though R_0 is less than 1. Notice that, nonlinear effects are not measured by R_0 because R_0 is associated with the linearization of the model in the trivial equilibrium point. In this scenario, the effects of the relapsed individuals can drastically increase the number of individuals with antisocial behavior. Then, public health strategies must involve to the relapse individuals too.

In ongoing work, the effect of peer pressure and negative parenting in the existence of high level of individuals with antisocial conducts will be analyzed.

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