Introducing STEM pedagogy in secondary school by means of Study and Research Path (SRP)

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Abstract

This work shows the results of a research carried out in compulsory secondary school developing a Study and Research Path (SRP) on questions linked to the STEM approach. We present preliminary results of the SRP_{STEM} developed in five courses at secondary school with N=116 students. We describe the development and scope of the SRP by using its components as defined by the Anthropological Theory of Didactics (ATD). Furthermore, we present a short description of the Epistemological Model of Reference. We discuss some general results and conclusions concerning the connections between the STEM approach and the pedagogy of research and questioning the world.

Keywords: STEM education, Study and Research Path; Modelling.

Resumen

Este trabajo presenta los resultados de una investigación llevada a cabo en la escuela secundaria obligatoria a partir de un Recorrido de Estudio e Investigación (REI) con preguntas relacionadas con el enfoque STEM. Presentamos los resultados preliminares del REI_{STEM} desarrollado en cinco cursos en la escuela secundaria con N=116 estudiantes. Describimos el desarrollo y alcance del REI_{STEM} utilizando sus componentes, según lo define la Teoría Antropológica de la Didáctica (TAD). Además, presentamos una breve descripción del Modelo Epistemológico de Referencia. Discutimos algunos resultados generales y conclusiones sobre las conexiones entre el enfoque STEM y la pedagogía de la investigación y el cuestionamiento del mundo.

Palabras clave: educación STEM, Recorrido de Estudio y de Investigación, Modelización.

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I.INTRODUCTION

The present work shows the results obtained from a research that aims to develop STEM pedagogy at the Compulsory Secondary Education (CSE). The STEM approach (Science, Technology, Engineering & Maths) proposes an interdisciplinary approach to real-world problem solving, modelling, engineering and technology utilization [1, 2, 3].

Despite its importance, there is still no unified pedagogical framework for the design and implementation of STEM programs in school institutions [4]. The Pedagogy of Questioning the World (PQW) [5] proposed by the Anthropological Theory of Didactics (ATD) [6,7] has diverse contact points with STEM approach allowing to be implemented in the CSE. The PQW advocates an epistemological and didactic revolution [5,8] of the teaching of mathematics and school disciplines and calls for the dropout of the traditional teaching paradigm, for which knowledge is taught as something important in itself and not because of its usefulness or its potential uses in life. The ATD describes the so-called monumentalism phenomenon of knowledge (Ibid.), referring that monuments are visited, admired, worshipped, without any consideration of the reasons of its existence. Thus, we witness paradoxical social events, such as the overvaluation of mathematics whereas the practical mathematics for day-to-day uses are ignored by teaching. On the other hand, the STEM approach highlights the different uses of mathematics in the scientific and technological domain. The Study and Research Path (SRP) [9, 10] are essential didactic devices proposed by the ATD.

The SRP allows the study of mathematics and other disciplines focusing on questioning and establishing that the starting points of knowledge are questions, (generating questions in the ATD framework), because its study should generate new derivating questions.

The aim of this research has been to design, develop, implement and evaluate a SRP involving STEM principles and disciplines, which involves the study of science (mainly physics in this case) mathematics and technology. We describe the SRP_{STEM} functioning and analyze the feasibility of teaching through this device at the CSE. The starting point is the question Q_0: Why did the Movediza Stone in Tandil fall down? [11, 12, 13] Which, in order to be answered, in a provisional and unfinished way, requires for the joint study of Physics, Mathematics and technology The SRP_{STEM} was developed in five groups of students in the secondary level whose teachers are also researchers. The ATD is applied to describe in general terms the study procedure in each course, as framed within the Research and Questioning the World Pedagogy [9]. In the following, the main ATD constructs used in the present work are underlined.

II. THEORETICAL FRAMEWORK

The ATD defines the SRP as devices that allow the study of mathematics by means of questions. The SRP establish that the starting points of mathematical knowledge are questions called generating questions, because its study should generate new questions called derivative. Teaching by means of RSC is complex and demands rootle changes in the roles of the teacher and students. The SRP's are defined by the *developed Herbartian model* [7, 10]:

$$\left[S(X;Y;Q) \rightarrow \left\{R_1^{\lozenge}, R_2^{\lozenge}, R_3^{\lozenge}, \ldots, R_n^{\lozenge}, Q_{n+1}, \ldots, Q_m, O_{m+1}, \ldots, O_p\right\}\right] \rightarrow R^{\blacktriangledown}$$

Here, Q is a certain generating question; S is a didactical system around of the study of Q. S is formed by a group of people trying to answer the question (X) and by people helping the study (Y). In classrooms of mathematics, X represent the students and Y represent the teacher and other instruments helping in the search of answers to Q. S has to build a didactic medium M to study Q, whereas M is composed by different knowledge, expressed by $R^{\lozenge}_i$, Q_j and O_k. The $R^{\lozenge}_i$ for i= 1,...,n are any existing answer or "socially accepted answer", the Q_j for j= n+1,...,m are derivative questions of Q, and the O_k for k= m+1,...,k are any other knowledge that must be studied developing the answers. Finally, $R^{\blacktriangledown}$ is some possible and partial response to Q given by S.

Monumentalism [5] is a metaphor proposed by the ATD, which describes a didactic phenomenon that consists in treating mathematical knowledge as a monument. In general someone is summoned to admire, visit, preserve, immortalize and even love those monuments, as if they had always been there. Consequently, within the monumental paradigm, knowledge is conceived and treated in that way.

Teachers naturally invite students to visit knowledge, without altering it, transforming it or deconstructing it. To encounter a monument, supposed to discover it, at most to live an aesthetic experience with it. Monuments are rigid and non-adaptable, remaining always at the same place. In a monumental epistemology, something similar happens with mathematical knowledge, it is considered immutable on time, it is enough to show it, hence the ostensive treatment of the mathematical objects. Teachers are not aware of the variety of monumental gestures they perform. According to the ATD, the SRP's allow to face the phenomenon of monumentalism, because:

· They are developed from a so-called generating question Q_0, because it does not admit an immediate response. That is, it will be necessary to formulate deriving questions, and de-label the available answers.
· The didactic medium M is not built a priori, but from elaborating answers. Resources are incorporated when they are needed, at any time, under the condition that they have to be validated by the study community.
· The teacher directs the study process, but he doesn't have a preponderant role constructing M, and their contributions may or not be incorporated into M. In a SRP, the principle of authority does not apply, there are no privileged media systems or more authorized than others, unlike what happens in the monumental paradigm.

The study group formulates and answers the questions, except the generating question, which is proposed by the teacher. The diffusion of the response has a strongly epistemic component, unlike the narrative character of diffusion in the monumental paradigm, where the teacher's role is more similar to that of a guide in the visit to a museum, than to the director of a study whose travel is not known in advance.

III. METHODOLOGY

This research is a qualitative and exploratory study aiming to carry out the STEM in the Compulsory Secondary Education (CSE). Being Q_0 genuinely co-disciplinary with STEM disciplines, the research team took on an SRP within which several discussions arose. The researchers were physics and mathematics specialist. At the beginning, they experience the SRP in person, since that they do not know the answer beforehand as there are no previous ready-made answers to Q_0. The team personally experienced the drawbacks when constructing a physical model to describe the real situation [11, 12]. The models were proposed in increasing order of complexity, culminating with an effective model that accounts for certain parameters included ad hoc in the first one, as we will detail later. Then, considering the role of the teacher and the didactics, the researchers analyze the scopes and limitations of the proposed SRP, to be implemented in the CSE. The Argentine secondary school syllabus does not foresee the conditions to develop the STEM pedagogy, thus we introduced it in normal math courses.

Five implementations were carried out in three secondary schools with different characteristics and contexts [13], where in total N = 116 students aged between 16 and 18 years participated, were organized into work groups with approximately 4 members each. The teacher integrated the research team. The implementations I_1 and I_2 were done in a state private management school, with N = 36 and N = 32 students respectively. Each lasted 7 weeks divided into 11 lessons. The I_3 and I_4 were carried out in a rural state school, with N = 13 and N = 19 students during 19 lessons (9 weeks) and 17 lessons (8 weeks), respectively. The I_5 was developed in a sub-urban state school, with N = 16 students and during 17 encounters displayed in 8 weeks. Researchers obtained class-by-class, written protocols of the students and the teacher's class diary; in addition, participant and non-participant observation was made.

IV. THE EPISTEMOLOGICAL MODEL OF REFERENCE (EMR) AND THE SRP

As we mentioned, the starting question Q_0 is: Why did the Movediza Stone in Tandil fall down? This enormous basalt stone has remained the city's landmark, providing it with a distinctive feature. Many local people and national celebrities visited the place to closely observe the natural monument. It was a 248-ton rock, sitting on the top of a 300-meter-high hill above sea level, as illustrated in the Figure 1.

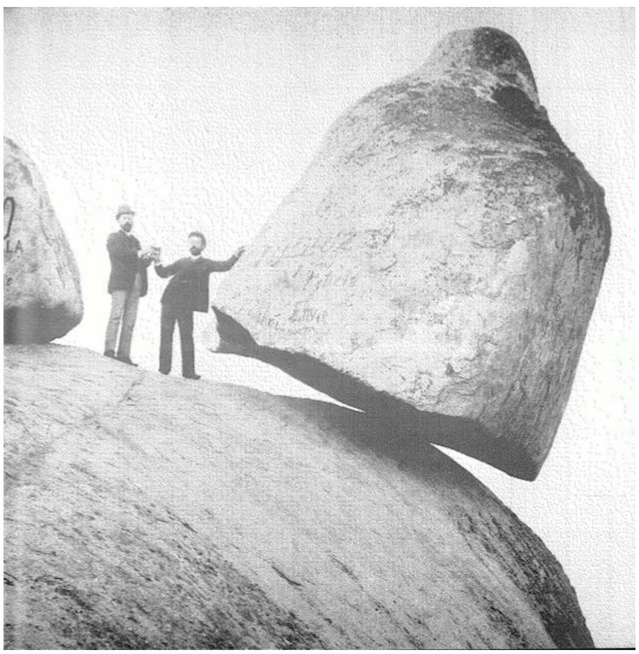

FIGURE 1. Photography of the Movediza Stone - north view (Photo Archivo General de la Nación Argentina, [14]).

This stone presented very small oscillations when disturbed in a specific spot. The locals knew this property [16], who came to the place to perturb the stone by themselves. There are many photographs showing this activity. But pictures as the Figure 2, are only illustrative images, because it was impossible to move the stone single, and less using the shoulder! Other pictures, as the Figure 3, present a more realistic action to moving the stone.

If we consider that the stone was an oscillating system, the study can be carried out within the Mechanic Oscillations topic, starting from the ideal spring or the pendulum. In this case, frictionless systems are used, in which the only force in action is the restoring force depending (for small amplitude oscillations) in a linear way on the deviation respect to the equilibrium position. This model is known as simple harmonic oscillator whose motion, via Newton equations, is

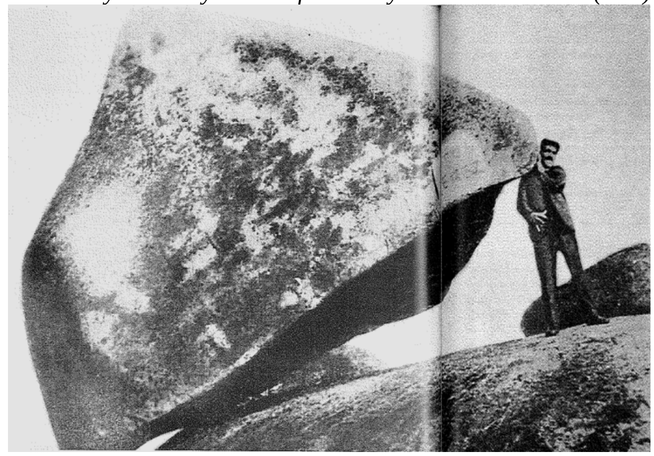

FIGURE 2. Photography of the Movediza Stone – South view (Photo Archivo General de la Nación Argentina, [14]).

FIGURE 3. Photography of the Movediza Stone. (Photo Archivo General de la Nación Argentina, [14]).

Unexpectedly, on February 28, 1912, the stone fell down the cliff and fractured into three pieces, filling the town with dismay by the loss of their symbol. For over 100 years, the event produced all kinds of conjectures and legends for the causes of the fall. Within the two groups where the SRP was performed, there existed a certain curiosity and interest in finding a scientific answer to this question. Once in contact with the available information, the question evolved into: What are the conjectures about the causes the Movediza Stone (MS) fall, and which is the most likely from a scientific viewpoint? Assuming that the fall can be explained by means of the Mechanical Resonance phenomenon, several questions Q_i emerged which are linked to the physical and mathematical knowledge necessary to answer Q_0. Described by a second-order linear differential equation. Progressively, the system becomes more complex. If friction-produced damping is considered, it provides a new term to the differential equation connected to the first derivative of the position (speed). Finally, it is possible to study systems that apart from being damped, are under the influence of an external force, and therefore called driven systems. In the case that the external force is periodic and its frequency is approximately equal, (the order of the approximation will be clarified later) to the natural (free of external forces) frequency of the oscillating system, a maximum in the oscillation amplitude is produced, generating the phenomenon known as mechanical resonance.

By increasing the complexity of the model, it is possible to consider a suspended rotating body, instead of a punctual mass. This leads to the study of the torque and the moment of inertia of an oscillating body. Here again, the linear system is for small amplitude oscillations and the damped and driven cases can be also considered, corresponding to the same mathematical model, but in which the parameters have a different physical interpretation.

However, as it refers to a suspended oscillating body, this is not a suitable physical model for the MS system. Since that, the base of the Stone was not flat, it is necessary to consider more precise models of the real situation. This leads to the mechanics of supported (and not hanging) oscillating rigid solids. In this case, we consider a rocker-like model in which the MS base is curved, and it lies on a flat surface, where the oscillation is related to a combined translational and rotational motion [11]. The application of Newton laws to the rocker model of the stone leads to a differential equation where the parameters are specific of the MS system: mass, geometry, inertia moments, friction at the base, external torque, etc., which is given by the following *effective* Harmonic oscillator mathematical model of the MS physical system:

$$\ddot{\varphi} + \gamma\dot{\varphi} + w_0 = \left(\frac{M}{I}\right)\cos(wt). \qquad (1)$$

The stationary solution to equation (1) is

$$\varphi(t) = \varphi_M \cos(wt - \psi).$$

being the amplitude φ_M and the phase ψ

$$\varphi_M = \frac{M_0/I}{\sqrt{(w_0^2 - w_0) + w_0^2\gamma^2}} \text{ and } \psi = tg^{-1}\left(\frac{\gamma w}{w_0^2 - w^2}\right). \qquad (2)$$

The maximum of φ_M is for

$$w_M = \sqrt{w_0^2 - \frac{\gamma^2}{2}}. \qquad (3)$$

The parameters: M_0 (external torque), I (inertia moment), w_0 (natural oscillation system frequency) and γ (damping coefficient), must be estimated. Detailed data about the shape, dimensions and center of mass position of the MS are available [15] after a replica construction and its relocation in 2007 on the original place (although fixed to the surface and without possibility to oscillate). These data bring us the possibility to estimate some parameters in our model, as e.g. mass, inertia moment, and the distance of 7.1 m, from which the external torque could be exerted efficiently by up to five people (according to historical chronicles) to start the small oscillation. By using these values, it is possible to study the behavior of the $\varphi_M(w)$ function for w_0 in a range of frequencies between 0,7 Hz and 1 Hz, historically recognized [16, 17] as the natural oscillation frequencies in the MS system and calculate for each case the maximum amplitude φ_M.

The Stone would fall if

$$\varphi_c \leq \varphi_M(w), \qquad (4)$$

being

$$\varphi_M(w_M) = \frac{M_0}{w_0 I\gamma}. \qquad (5)$$

If γ is very small (as is expected to be in this case) we can neglect it from $w_M = \sqrt{w_0^2 - \frac{\gamma^2}{2}}$, leading to $w_M \approx w_0$. By using this approximation in Eq. (2) (left) the falling condition becomes

$$\varphi_c \leq \frac{M_0}{w_0 I\gamma}. \qquad (6)$$

The value of φ_c can be determined by an elementary stability analysis, which per the dimensions of the base of the stone and the center of mass position is estimated to be approximately of 6° [11]. Note that in the present model γ is a free parameter, for which we set “ad doc” a magnitude order $\gamma \geq 10^{-2}$. This is justified in the frame of a more sophisticated model that we will comment briefly below. With this constraint, we find several situations, comprising different torques within the mentioned frequencies interval, supporting the overcoming of the critical angle, i.e., predicting the fall.

Finally, in search of a more appropriate approximation of the physics model for the damping that is clearly not due to air, we consider a more sophisticated model of the stone as a deformable solid, where the contact in the support is not a point but a finite extension, along which the normal force is distributed, being larger in the motion direction and generating a rolling resistance, manifested through a torque contrary to the motion. The rolling resistance depends on the speed stone, giving a physical interpretation to the damping term. Therefore, the physics behind the damping is the same that makes a tire wheel rolling horizontally on the road come to a stop, but in the case of the stone, the deformation is much smaller. Although the deformable rocker model has extra free parameters, tabulated values of rolling resistance coefficient for stone on stone, which are available in the specialized literature, allowed us to estimate and justify the damping values that we incorporate otherwise ad-hoc in the rigid rocket SM model. The Figure 4 shows the models performed.

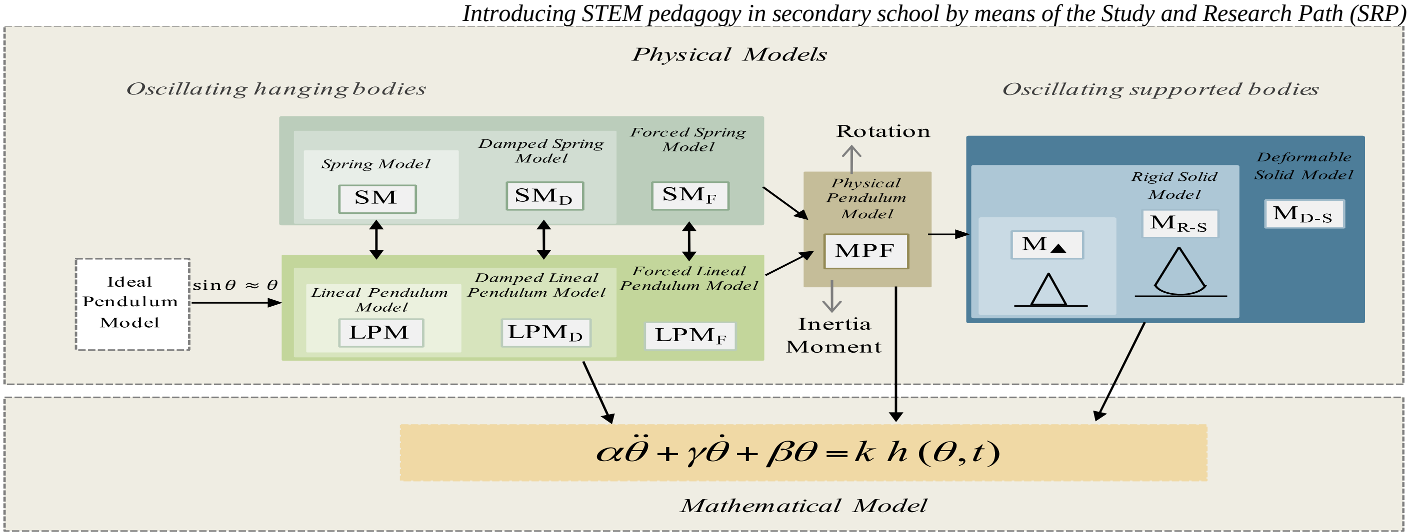

FIGURE 4. Models studied in the SRP_{STEM} in order of increasing mathematical and physical complexity.

V. DESCRIPTION OF EACH IMPLEMENTATION

A general way to describe the SRP_{STEM} is to consider its components. The starting point is the analysis and questioning the probable causes of the fall of the stone, finally, each group reformulates new questions according to its possibilities (second column on the table) and produces an answer under certain conditions. In the I_1 and I_2, firstly there was an attempt to understand the guesses on the fall by resonance and the connections with mathematics [18]. Initially, the study referred to the oscillations and the systems of the pendulum and spring type, the AMS and the harmonic functions that describe it. The Geogebra and Graphmatica software allowed analyzing, representing and interpreting motion equations of oscillating systems and its solutions. Students considered damped and driven systems of the previously mentioned models. They also analyzed damped systems, both driven and resonant and their differences from energetic and vector-related considerations, using representations. Energy time and the modules of the position, speed, acceleration and force vectors in the different sections of the motion (see the third column, second row of Table I). Applets and physlets off-line improve the interpretation of these systems. Some simple but amazing experiments performed in the classroom enabled the proprioceptive experiencing of the resonance phenomenon.

TABLE I. Summary of the SRP_{STEM} developed in each implementation.

S(X,Y,Q)	Q_i	R_i	Knowledge STEM
Urban State School with private administration	Q_1: What are the conjectures about the fall? What would be the most appropriate explanation?	$R^{\lozenge}_{1}$: Analysis and discussion of the conjectures about the fall. $R^{\lozenge}_{1}$: Selection of the conjecture that would explain the fall through oscillations and mechanical resonance.	
Q: Why did the MS fall? a)N=36 students Professor-Investigator (P-I).	Q_2: How do oscillating systems work and how are they described? $Q_{2.2}$: What are the functions that represent the Simple Harmonic Oscillator (SHO)? What characteristics do they have? $Q_{2.3}$: How are damped, forced and resonant systems described? $Q_{2.4}$: How does resonance occur?	$R^{\lozenge}_{2}$: Analysis and description of the SHO and the associated magnitudes. $R^{\lozenge}_{2.2}$: Study of the harmonic functions sine and cosine. $R^{\lozenge}_{2.3}$: Description of damped and forced oscillating systems, from energetic and vector-related considerations $R^{\lozenge}_{2.4}$: Description of Resonance. Experiences with pendulums in class.	Oscillation Harmonic Functions Resonance Applets, physlets Geogebra Experimentation
b)N=32 students (P-I)	Q_3: How was it produced and what were the characteristics of the MS motion?	$R^{\lozenge}_{3}$: Description and analysis of the MS as a forced oscillating system when properly disturbed by people (discontinuously).	Oscillation Resonance

	Q_4: Is it possible that the stone has fallen by Resonance? How did it happen?	$R^{\Diamond}_4$: Analysis of the resonance conditions: frequency of the external torque equal to or close to the natural frequency.	Resonance
	$R^{\heartsuit}$: *The Movediza Stone fell because it came into resonance. It could have happened that two or more-people pushed it in the right place with the right frequency and so the system's energy and amplitude increased so much that it caused the fall.*		
Rural State School Q: Why did the MS fall? a)N=13 students P-I. b)N=19 students P-I	(+) Q_5: How are damped, forced and resonant systems described?	$R^{\Diamond}_5$: Analysis of damped and forced oscillating systems.	Oscillations Exponential e Irrational Functions Geogebra, Applets. physlet
	Q_6: What is the equation that describes the motion of the MS?	$R^{\Diamond}_6$: Solution of the equation of motion of a damped and forced system.	Oscillations Harmonic Functions
	Q_7: What parameters are possible to calculate or estimate to validate the conjecture of the fall by resonance?	$R^{\Diamond}$: Morphological characteristics of the MS. $R^{\Diamond}_7$: Approximate estimate of the damping coefficient, natural frequency. $R^{\Diamond}_7$: Analysis of the function that describes the motion of a damped and forced system: calculation of the critical angle. Estimation of the external torque.	Functional modelling, software, spreadsheets.
	$R^{\heartsuit}$: *The functional modelling supports the conjecture of the fall by resonance. Analysis, calculation and estimation of the parameters of the equation of motion solution.*		
Urban state school Q: Why did the MS fall? N=16 students P-I	(+) Q_8: How was the support base of the MS and what features did its motion?	$R^{\Diamond}_8$: *In situ* observation. Physic modelling of the base and the motion of the MS.	Physic modelling
	$R^{\heartsuit}$: *The observations made in situ, allow the physical modelling of the phenomenon and together with the data obtained from the functional modelling, the conjecture of the resonance fall is validated*		

According to the students, the most satisfactory answer was understanding that the stone oscillated whenever it was disturbed in an appropriate way by people (not spontaneously and constantly as they had assumed at first).

Moreover, if such disturbance met certain periodicity and placement requirements "in the right place, with the right frequency", depending on the system, the oscillation amplitude could have been increased to the point of causing the fall. This can be seen in the protocol in the Figure 5. In these implementations, only the SM and LMP linearized models were analysed (see figure 4). That is, the physical spring and pendulum models. The study group developed a written response to Q_0, that is, a verbal explanation not strictly founded on the mathematical and physical knowledge studied in the class. This could due to that, epistemic validation is not part of the school culture, being the SRP_{STEM} a powerful device to promote it.

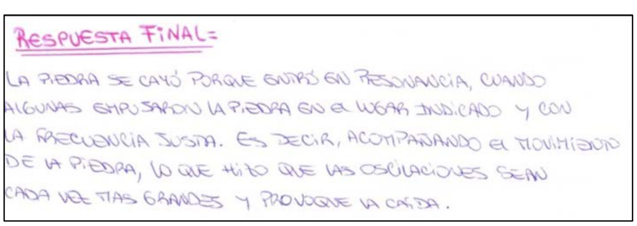
RESPUESTA FINAL=
LA PIEDRA SE CAYÓ PORQUE ENTRÓ EN RESONANCIA, CUANDO ALGUNAS EMPUJARON LA PIEDRA EN EL LUGAR INDICADO Y CON LA FRECUENCIA JUSTA. ES DECIR, ACOMPAÑANDO EL MOVIMIENTO DE LA PIEDRA, LO QUE HIZO QUE LAS OSCILACIONES SEAN CADA VEZ MÁS GRANDES Y PROVOQUE LA CAÍDA.

FIGURE 5. Protocol of the student A57.

In the other implementations, our objective was to validate the conjecture of the fall by resonance, based on science and mathematics. In I_3 and I_4, the damped and forced oscillatory motion, together with its mathematical description were studied. The solution of the equation of motion in damped and forced cases was analysed. The teacher proposed this mathematical model, because differential equations calculation is not available at this level. The class developed a dimensional analysis, and the mathematical and physical meaning of the solutions together with their parameters and measurement units. The parameters considered were M_0 (external torque), I (moment of inertia), w_0 (natural oscillation frequency of the system) and γ (damping coefficient). The researchers calculated the values of I and the damping coefficient, giving them to the class. The critical angle was estimated based on the available data and an elementary stability analysis (Holmberg, 1912). The Figure 6 shows how the critical angle of oscillation $\varphi = 6°$ (0,11 rad.) could be obtained from a geometric calculation, considering an elementary model proposing a supported base plane.

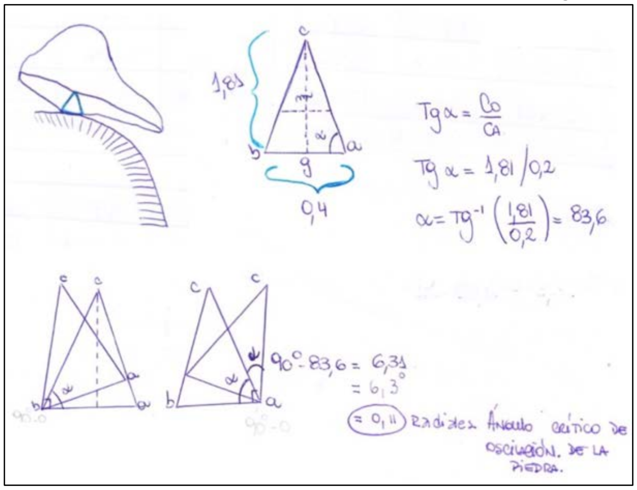

FIGURE 6. Protocol of the student A93.

The students analysed Photographs and images of the period (similar to figures 2 and 3). The abundant material available allowed discussions about the real number of people who could have forced the stone, and the characteristics of that force. Considering that people who could force the stone were from 2 to 5 persons and that the individual average strength that each one could exercise varies between 40 and 70 kg, a range of possible torques was established between [11000; 19000] Nm.

The teacher proposed to analyse the equation (2) by means of spreadsheets and GeoGebra software, varying the different parameters. The students mainly used GeoGebra and evaluated several torques and conditions.

For example, the Figure 7 shows the analysis carried out by a group of students, taking the case $\gamma = 0.015$ and $w_0 = 6.28$ hz. For certain torques, the students founded that the curve exceeds the critical angle (represented in the figure red line), i.e., thus, the stone fall down.

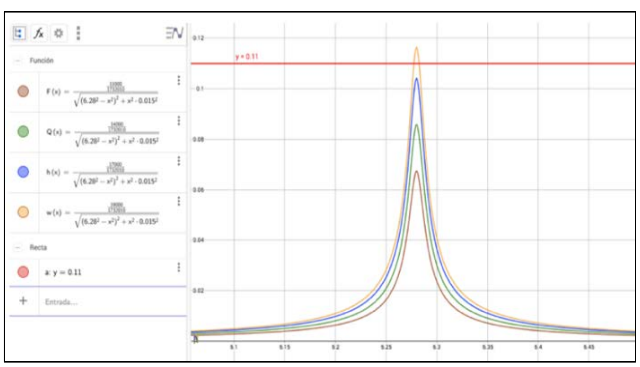

FIGURE 7. Protocol of the student A72.

Some students used spreadsheets and considered the variance of the damping (Figure 8). Some possible torques M_0 were fixed, and taking into account the resonance condition, the damping was varied. The rows painted in orange indicate the conditions in which the amplitude exceeds the critical angle, for instance, the stone falls.

Caso 1			Caso 3		
Frecuencia propia	Amortiguación	Amplitud Maxima		Amortiguación	Amplitud Máxima
6,28	0,01	0,091936922	Frecuancia Propia	0,01	0,137905383
	0,011	0,083579020	6,28	0,011	0,12536853
Inercia	0,012	0,076614102		0,012	0,114921153
1732010	0,013	0,070720709	Inercia	0,013	0,106081064
	0,014	0,065669230	1732010	0,014	0,098503845
Torque	0,015	0,061291282		0,015	0,091936922
10000	0,016	0,057460576	Torque	0,016	0,086190865
	0,017	0,054080543	15000	0,017	0,081120814
	0,018	0,051076068		0,018	0,076614102
	0,019	0,048387854		0,019	0,072581781
	0,02	0,045968461		0,02	0,068952692
	0,021	0,043779487		0,021	0,06566923
	0,022	0,041789510		0,022	0,062684265

Caso2	Amortiguación	Amplitud Máxima	Caso 4	Amortiguación	Amplitud Máxima
	0,01	0,110324307		0,01	0,183873845
Frecuencia propia	0,011	0,100294824	Frecuancia Propia	0,011	0,167158041
6,28	0,012	0,091936922	6,28	0,012	0,153228204
	0,013	0,084864851		0,013	0,141441419
Inercia	0,014	0,078803076	Inercia	0,014	0,13133846
1732010	0,015	0,073549538	1732010	0,015	0,122582563
	0,016	0,068952692		0,016	0,114921153
Torque	0,017	0,064896651	Torque	0,017	0,108161085
12000	0,018	0,061291282	20000	0,018	0,102152136
	0,019	0,058065425		0,019	0,096775708
	0,02	0,055162153		0,02	0,091936922
	0,021	0,052535384		0,021	0,087558974
	0,022	0,050147412		0,022	0,08357902

FIGURE 8. Protocol of the student A93.

This tasks allowed them to evaluate the hypothesis of the fall due to human action and formulate the answer R♥, as illustrate in the Figure 9.

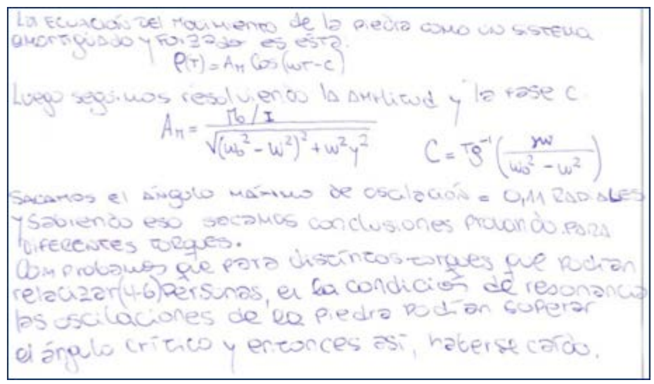

FIGURE 9. Protocol of the student A93.

During the I_5 implementation, it was also attempted that the students analyse other physical models (horizontal and vertical springs, pendulums, "rocking chairs") comparing them and analysing their applicability to the MS. The students considered the MS as a damped and forced oscillating system. The Figure 10 shows the answer of a student, who drew a picture of the stone on the top of the hill.

He identified where people should be located and where should exert a torque to move the stone (as it shows above in the Figure 2 and 3). In addition, he plotted the variation of the position as a function of time, representing a sub-damped system that is set in motion when forced it.

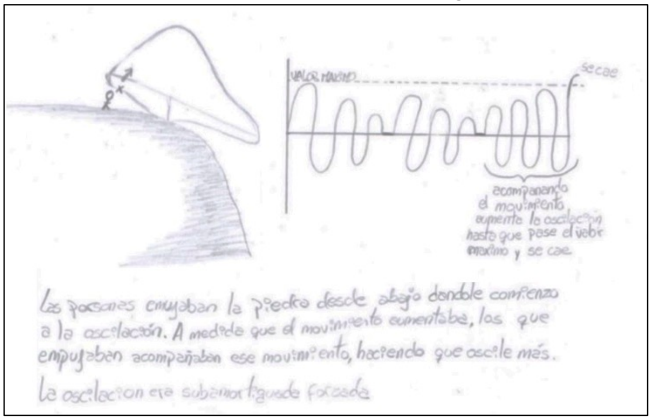

FIGURE 10. Protocol of the student A113.

Finally, the class adopted a rocking model proposed by a student. This idea is compatible with the MR-S proposed in Figure 4, but was not strictly treated in class. Instead, we carried out a simple analysis without addressing the problem of the roto-translation, which greatly exceeds the possibilities at this level (Figure 11). The joint treatment of the physical and mathematical models enriched the interdisciplinary study involving science, mathematics, modelling, as shown at the end of Table 1 in the R♥ of I_5.

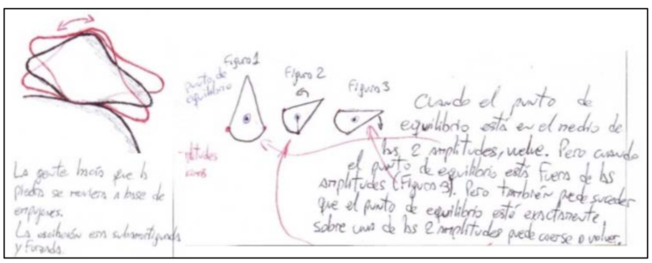

FIGURE 11. Protocol of the student A102.

VI. DISCUSSION

The groups experienced a SRP_{STEM} studying science and mathematics and using technology. The students showed a willingness to deal with questions they had never considered before. Between the first and the last implementation, we identified significant differences and improvements. In I_1 and I_2, oscillations and harmonic functions were studied, by means of useful physlets and applets and the mathematical software Geogebra, which were not known by the students. Nevertheless, only a verbal response to the problem could be elaborated. This result could be due to an insufficient treatment of the models and the lack of experience of the teacher with the SRP_{STEM}.

On the other hand, in I_3 and I_4, the students realized a detailed analysis of the available documents about the MS. The chronicles, photographs and historical images were considered and revised, obtaining relevant information about the main characteristics of the MS and the possible causes of fall. New physilet and applet based on Geogebra were used to improve the performance with the models. The students interpreted the mathematical model proposed by the teacher-researchers who also estimated the parameters of the equation. In addition, functional modelling tasks were carried out, which allowed to validate based on calculations the conjecture of the fall by resonance. However, in these implementations the students did not question the physical models from the mathematics. The relationships between both models and also with the PM system was insufficient.

Finally, in I_5, the study of more and different physical models allowed to deepen in the analysis of a more appropriate model to describe the MS, from a mathematical and physical point of view.

In each implementation, the incorporation of new activities that require an increasing use and interpretation of models in mathematics and physics, produced an extension of the research path improving the integration of the STEM disciplines.

The results show that experimentation allows both the adjustment of the device and the improvement of the performance of the teacher and the students. It is important to notice that this didactic device requires radically different roles for both the teacher and the students, who are alien to traditional pedagogy.

VII. CONCLUSIONS

In the present work, we have described in general terms, by making use of the components of an SRP as has been proposed by ATD, the distinctive characteristics that such a device displays when it is implemented in CSE, from a STEM generating question. In addition, we have provided a detailed description of the Epistemological Reference Model (ERM), its scope and didactic STEM implications.

The results evidence that SRP_{STEM} are useful devices to introduce the STEM approach in secondary school, whenever the generating question calls for the disciplines proper to this approach. Although the preliminary results seem promising, we consider necessary to enlarge our SRP_{STEM} involving students modelling activities by themselves. We are currently development this aspect.

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