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SOME REMARKS ON THE HISTORY OF NUMERICAL ANALYSIS ESPECIALLY IN THE AREA OF PRAGUE*

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RESUMEN

Se confronta el concepto de análisis numérico *de Golstein con una visión más amplia del papel de los métodos de cómputo y los problemas prácticos en el desarrollo de las matemáticas hasta el siglo XIX. Se presta especial atención a la relación entre los cálculos matemáticos y el desarrollo de la matemática en el entorno científico de Praga en el tránsito de los siglos XVI al XVII. En esa época, con objeto de facilitar el cálculo astronómico, Brahe y Wittich utilizaron la llamada* prostpheresis *y Bürgi y Kepler calcularon sus tablas logaritmicas. El trazado de las órbitas planetarias exigía una creciente precisión en las observaciones y exactitud de cálculos y métodos de cálculo, esfuerzos que llevaron a la obtención de las Leyes de Kepler.*

ABSTRACT

Goldstein's concept of "numerical analysis" is confronted with a more broad apprehension of the role of mathematics up to the 19th c. The attention is especially devoted to the relation of the computing mathematics to the development of mathematics in the Prague scientific centre on the border of 16th and 17th c. At that time Brahe with Wittich used the so called prostphereses *for making the astronomical computing easier, and by the same reason Bürgi and Kepler computed themselves the logarithmical tables. The interest in drawing up the real planet orbits required a higher precision of observations and more exact calculations and methods: efforts in this direction led to Kepler laws.*

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Se muestra la conexión entre los problemas teóricos y los problemas de cálculo, y cómo estas cuèstiones intervinieron en la aparición de la matemática numérica, pese a que no están contempladas en el concepto de análisis numérico de Goldstein. En el subsiguiente desarrollo de las matemáticas en Bohemia se destaca la figura de Václav Láska, que ocupó la primera cátedra de matemática aplicada en la Charles University *(1911). Sus monografías sobre métodos numéricos (1915, 1923, 1934 etc.), fueron publicadas en una época de floreciente interés por el análisis numérico en todo el mundo.*

It is shown how the theoretical problems were connected with computing problems and how those questions became one of the sources in the formation the numerical mathematics although they are not included in the Golstein concept of numerical analysis. In the further development of mathematics in Bohemian Lands the first chair of applied mathematics on the Charles University, created by Václav Láska (1911), is mentioned as well as his monographs on numerical methods (1915, 1923, 1934 etc.), which came within a time of bright interest in numerical analysis in the world mathematics.

Palabras Clave: Historia del Análisis numérico, Goldstine, Métodos numéricos de la Astronomía en los siglos XVII y XVIII, Leyes de Kepler, Métodos numéricos y gráficos en el siglo XX, Barabashoff, Brahe, Kepler, Bürgi, V. Láska.

It is not unknown that there is only one large book on the history of numerical analysis, the one by Hermann H. Goldstine published in 1977. Going through it we see, with surprise, that its text as such begins in the 16th and early 17th century. Prague, with Bürgi's and Napier's tabulation of logarithms, is almost the very place where the text starts. This also indicates what Goldstine considers as the start of numerical analysis: it is not the calculation by the Archimedian geometrical methods with the determination of trigonometrical functions from the sides of polygons inscribed in a circle of a basic radius, but just calculations using comparisons of arithmetical and geometrical progressions and methods of interpolation of numerical values with a known accuracy of this process.

One question will crop out in this context: whether there was, or not, any numerical analysis before that time, and also another question follows immediately: Why did the history of numerical analysis receive attention so late into the 20th century?

It is clearly realized that both these questions are closely related to the definition of the subject, methods, and the very concept of what numerical analgsis is. Goldstine -though not saying that explicitly- considers numerical analysis as a set of calculation methods and tools used to speed up the calculations, with a knowledge of the exactness of these methods and the results achieved.

I do not think that these problems can be solved here: my intention, rather, is just to draw attention to them and thus, perhaps, arouse the interest of a broader audience of mathematicians.

Goldstine's exposition starts with the method of finite differences, a technique already used in the past but formulated rigorously by Newton, though Harriot and Gregory knew a number of general procedures later to be part of Newton's theory. This suggests that Goldstine still narrowed the subject of numerical analysis. He included in it only the theories of computational processes or, rather, only comprehensive theories. This conception can be the subject of discussion.

However, the question seems even wider. For instance, the finite differenes method, used long before Newton, was studied after Newton by Euler, Clairaut, Lagrange, Laplace, de Prony, Gauss, Cauchy, Abel, Hermite and others, actually, as Goldstine says, ... *we shall see that virtually all the great mathematicians of the seventeenth and eighteenth centuries had a hand in the subject*, and we can add that this is true not only of the 17th and 18th centuries. It could rather be said that computational mathematics was not on the margin of mathematicians' interest but constituted an integral part of all mathematics.

This situation, as viewed by todays' historians, began to change in the first half of the 19th century. Kolmogorov therefore rightly speaks of a change of the subject of mathematics in that time and of its prevailing orientation to its own problems and of a further degree of abstraction. The abstract logical structure of the mathematical theory of individual disciplines in the first place, in the 19th century, and later of all mathematics, as for instance in Bourbaki, including the various conceptional approaches to the principles of mathematics and the ensuing implications for the structure of the whole of mathematics, particularly in the first decade of the 20th century, began to prevail in the 19th and 20th century and began to be considered as the very centre of all mathematical research. Mathematicians generally involved in this trend seem to have forgotten that their science rose and was developed as a science studying the real world. This is by no means to say that other aspects of mathematical work were left aside or that no attention was paid to them any longer.

However, even the historiography of mathematics, which had not a long tradition before the 19th century, did fall for this trend and in studying the problems of the mathematics of the Classical Antiquity so interesting to all, concentrated mainly on the investigation of the abstract logical directions of its development. In the 2nd edition of *The exact sciences in antiquity,* Otto Neugebauer[1] said explicitly that *it would not even be correct to reduce the purely Greek contribution to the development of mathematics merely to the Euclidean-Archimedean direction best known to today's reader; it is necessary to add to it a number of methods concerning the numerical and graphical problems encountered in mathematical astronomy*[2]. Only the historiography of mathematics of the latter half of the 20th century with its systematic study of mathematical astronomy of antiquity, its mathematical methods, did begin to investigate systematically medieval mathematics, particularly the Arab literary circle, and gather enough material for a different view of the main motive forces in the development in mathematics, especially when the mathematical knowledge and methods began to be viewed not in isolation, but fully in the context of the development of human knowledge in all its social contingencies.

It is not surprising that Otto Neugebauer's initiative was involved in the birth of Golstine's book. Still last year (1987) at the international conference of historians of mathematics held at Oberwolfach, which dealt with the development of oriental mathematics, it was stressed again that the mathematical contents and methods of astronomical writings should be investigated more thoroughly and that these analyses should then be integrated in our conceptions of that time's mathematics[3].

Since the mid-fifties, much work in the investigation of the Arab mathematical writings of the Middle Ages has been done by Soviet historians of mathematics and therefore it is not surprising that an essay such as that by Alexei Georgievich Barabashov could appear in the Soviet literature.

Barabashov inferred from the knowledge of the actual historical material that either practical or theoretical postulates were preferred in the various epochs of the development of mathematics and that the basic conceptions of these epochs varied accordingly. The author's argument is that there are no traces to prove these conceptions' continuous transition to one another and that, on the contrary, in a new social situation the old conception completely breaks down to give way to a new one, which corresponds better to the status of mathematics and its position in society.

Four stages are recognized from this point of view: the practical mathematics of ancient Egypt and Mesopotamia, the classical theoretical mathematics of Antiquity and the Hellenic period, the synthetic practical mathematics of the Middle Ages and the theoretical mathematics of the practical mathematics of the modern age. Several questions help to separate these epochs:

a) what was the mechanism of finding mathematical knowledge in each period,

b) what was the social objective or social task of the people dealing with mathematics, that is to say, what was the purpose mathematical work was to serve.

c) which were the criteria for *correctness* of mathematical processes, that is, the necessary standards set for the verification of the results obtained.

d) what was the sense in which the totality of all mathematical knowledge of the given time was understood.

Let us see how Barabashov analyzed the oldest period in the development of mathematics:

> "It is widely believed that the mathematics of ancient Egypt and Mesopotamia only gathered empirical material and that it was only in the Classical period of Greece that the mathematical knowledge changed into a system of interconnected statements. It is considered as self-evident in this connection that the only way of arranging systematic findings and to unify the mathematical material, is the logical derivation of statements from one another"[4].

He adds that long before the Classical times mathematical teaching had to arrange knowledge in systems and it would not be incorrect to say that Pre-Greek mathematics was a school subject, although the system there was entirely different from today's theoretical method of teaching. Mathematics was considered as practically oriented knowledge, and the social conditions which encouraged handing down knowledge included it in a canon of hermetic books as a comprehensive system of education on the basis of which the user was able to solve the problems which it was known to-be encountered. Thus in fact it was not necessary to know the formulae for the calculation of the area of simple geometrical configurations: what had to be known was how much grain can be harvested from an area in order to calculate -and the main question was how to determine how much tax in kind should be taken for the Farao's granaries. In such situation the area itself, precisely determined for the majority of the simple developed configurations of the time, was a secondary matter. It was important to know that the ratios of these areas corresponded to the taxes from the fields and to determine them.

A metallurgist today does in fact the same thing when estimating the volume of castings to be made from one tapping of a blast furnace, a procedure called a mathematical calculation, though it is just an inaccurate estimate sufficient for practical purposes.

And something like that is also done in the mathematical training of some practical technicians today: like the ancient Egyptian "practicians", they are given the basic mathematical knowledge as a kind of a canon and such knowledge is in connection to what they will later need in the special subjects of their field of study, preparing them in this way for their practical profession.

Thus conformity with the practical purpose for which problems had to be solved was the main criterion for the classification of mathematical problems. At its beginning, mathematics developed as a method of solving practical problems (mainly economic), it was method by which these problems could be solved non-empirically, a method giving sufficiently accurate results for practical activities. Mathematical calculations in fact used to be part of the methodology of economic management.

The first mathematical texts dated to the second millenium B. C. are not divided into the arithmetical and geometrical problems; the division is into the problems of calculation of the volume of granaries, calculation of the areas of fields, calculations associated with the construction of pyramids, calculation related to amounts of bread and beer and their subsecuent division among certain groups of people in given ratios and quality, calculation of the amount of food required for livestock, calculation of interest on debts, calculations on the division of inherited estates and the like.

What are the other characteristic features of this "first" stage of mathematics? The authoritativeness of conclusions. Dogmatic nature of computation prescriptions guaranteed by the authority of the scribe. Fixation of the algorithm of solution of problems by a verbal prescriptions without the possibility of its general recording (symbols did not exist). The numeric-calculatory nature of mathematics was generally based on examples. All this characterizes a *practical mathematics* while *theoretical mathematics* was still lacking. This practical mathematics, in summarizing texts, was already a generalization of the mathematical procedures used in the solution of practical problems. Generalization dictated by repeated problems and by the need to instruct a wider group of administrative clerks -scribes- in the solution of these problems through school teaching.

And school teaching is where the theoretical aspect is first introduced, owing to the need to arrange the exposition and selection of problems, the formulation of which suggests the impossibility of them assuming a practical character, while they are necessary for teaching purposes. Hence, it can be said that mathematics rose and developed for a long time mainly as a method of solving actual real situations where comparison of quantities or homogeneous structures was possible. It was in this direction that mathematics calculated, numerically solved, and also classified the various problems into groups and sought rules of solution for these groups.

Mathematics in fact has never lost this aspect of its character throughout its development. The practical problems, particularly those of mechanics, astronomy, were always rich sources encouraging mathematical efforts, sources of problems still lacking algorithmized solution procedures; in this way mathematics expanded as a subject, always finding new methods and at the same time drawing attention to all gaps in the existing structure of mathematics, to the drawbacks in its conception, and even to the inaccuracy of notions. Attempts were made to bridge the gaps and remove the drawbacks by various methods, perhaps less exact but practically sufficient, and at the same time to show the limits of their validity or to establish limits of tolerance for their solutions. Finally, theories explaining the inaccuracies and enabling to remove the errors were actually found.

The integrated conception of the *primary* practical mathematics did not involve any difficulties in calculation. What could be calculated was calculated, and nothing else was sought but the solution of a given practical of interest within the conditions of the time. Approximate expressions were necessary in some cases, for instance the Mesopotamian tables of inverse values. The approximations used then could sometimes be verified geometrically. An interesting example is problem 50 in Rhind's papyrus where it can be constructively estimated that the replacement of a circle by a square the side of which is equeal to 8/9 of the diameter will produce a value exceeding the area of the circle[5]. The accuracy of the approximations was often excessively high: in the Yale cuneiform table YBC 7289 (see [6], fig.16), $\sqrt{2}$ is given as having the value 1;24,61,10, which in the decimal system means 1.414213; this value was reached on the basis of a stepwise approximation of $1/2\ (a + 2/a)$ from below. Taking $a_1 = 1$, then a_4 is a very accurate value, however, the value $a_3 = 1.\dot{3}$ was often used.

Problems soon appeared with the conception of theoretical mathematics proposed in the Classical antiquity. The Pythagorean concept of number as the ratio of natural numbers, and immediately afterwards the discovery of the impossibility in this form shifted computation practices away from Greek mathematics. Attention was focused on geometry and on logically deductive construction of theoretical mathematics, as we know it today.

Even Archimedes considered it was his duty to reformulate the results he had obtained in a *physically empirical* way, as he mentions, for instance in his letter to Eratosthenes, and to demonstrate them by a method based on this Greek logico-deductive tradition (Socrates, Aristotheles, Euclides). This can undoubtedly be considered as a great contribution of the mathematics of the classical times which has permanently influenced all fields of mathematics, including practical calculations.

The calculation tradition began to play a more important role only in the Middle Ages, in both the oriental and European mathematics of the time. In the works by Al-Khvarismi of Baghdad (9th century), in the Chinese mathematical tracts as well as in the Indian writings and the *Liber abaci* (1202) by Leonardo of Pisa, emphasis is laid again in calculations and the algorithmic aspects of mathematics. As in the arithmetical books of antiquity, also in the works of the Middle Ages problems are concentrated in groups according to their practical orientation, for example its practical relevance to comerce, the calculation of taxes, problems of constructions or of military nature, the division of inherited estates, the measurement of distances to inaccessible points, religiously motivated determination of the direction of Mecca, calculation of elevations for shooting, etc. Unlike in the texts of antiquity, these works already contain a clear formulation of the general algorithms of calculation, as used for solving each type of problems.

Therefore, practical mathematics at that time is mainly characterized as an algorithmic-calculation mathematics. As Barabashov says, the algorithm of calculation becomes a central phenomenon in the systematized practical mathematics of that time. The algorithm is a sort of code enciphering the theoritical tradition of antiquity but optimally meeting, at the same time, the nonpractical purpose of mathematical knowledge ((2), 27). Traditions not directed to practice were simultaneously enhanced in the practically oriented mathematical disciplines, and practical mathematics found its direction in the trend towards the synthesis of its theoretical and the practical sides ((2), 29).

The requirements of astronomy became an increasingly important source of impulse to mathematics. Astronomical tables were calculated, for which the tables of trigonometrical functions had to be used. Scientists in antiquity had used the tables of chords (chrd), or *bowstrings*, of angles. For them it holds that chrd $\alpha = 2 \sin \alpha/2$ for $r = 1$. The method of calculation of these tables was geometrical -in fact the calculation of the sides of a regular polygon- a method used also by Archimedes for his approximation of the number π.

Since chrd 72^o is a side of a regular pentagon and chrd 60^o a side of a regular hexagon, it is possible to calculate from them also chrd 12^o, and then, using the formulae for chrd $(\alpha + \beta)$, chrd $(\alpha - \beta)$ and chrd $(\alpha/2)$, Ptolemaios calculated the chords of smaller angles only by approximations from chrd $0^o\,45'$ and chrd $1^o\,30'$, thus reaching the chrd 1^o. As Goldstine points out, the interest in the theory of the solution of equations, and in the method of iteration for the solution of algebraic equations, was motivated by efforts to calculate sin 1^o from the given sin 3^o; this interest was particularly high among the Arab mathematicians and astronomers.

As it is known, a systematic study of Arab astronomical texts finally resulted in the separation of trigonometry from astronomy in the works by Peurbach and Regiomontanus, and in the systematic calculation of trigonometrical functions. This happened at a time just before the work of Copernicus, *De revolutionibus orbium coelestium*, was published in 1543. Copernicus' book forced astronomers to seek an answer to the question of which of the two hypotheses was the correct one. The answer required a higher precision of observation -new instruments for observation and measurement and more exact calculations involving sums and multiples of numbers with many decimal places.

It was for the simplification of these calculation, that Tycho Brahe with his assistant Paul Wittich re-discovered in the island of Hven the method of *prostpheresis* used in the Arab writings by which the multiplication and division are replaced by the addition and subtraction of trigonometrical functions, by using the well-known formulae[6]:

$$\sin \alpha \,.\, \sin \beta = 1/2\, [\, \cos (\alpha - \beta) - \cos (\alpha + \beta)\,]$$

$$\cos \alpha \,.\, \cos \beta = 1/2\, [\, \cos (\alpha - \beta) + \cos (\alpha + \beta)\,]$$

It is obvious that the use of such a method depends on the accuracy of the tables used. For several centuries mathematicians have been trying, for various reasons, to find methods to calculate more accurate tables and to make these calculations easier.

Efforts in this direction led to Kepler laws, which were published in Prague in 1609, in his *Astronomia nova*. However, Kepler could not escape from the physical essence of the phenomena. The only thing he could rely on was he accuracy of both the observed data and calculation. Accuracy came to Prague from Hven and Kassel through the persons of Bürgi, Brahe and Raymarus Ursus.

Kassel was the working place of Paul Wittich, who took there from Hven the method of *prostpheresis*, and also of Bürgi who lived in that city in the years 1579 to 1604. Nicolas Raymarus Ursus went to Aerius with these two scholars in 1586. However, in 1587 Ursus was already in Prague, at the court of Rudolf II, having left his book *Fundamentum astronomicum* in Strasbourg to be published there. This book was also the first publication containing *prostpheresis*[7]. Ursus become Rudolf's court mathematician and Professor of the Charles University. Bürgi also started to have contacts with the Prague Rudolfian court in that time; he already intended to publish his *Coss* (that is, algebra), as a means for finding algebraic solutions of calculations required for the sinus tables *Canon sinum*, which he finished in about 1598. The reason why the new tables were compiled is obvious. The most accurate and largest tables available then were those by Rhäticus who was stimulated to compile them by Copernicus. In 1551 the tables were published in Leipzig as *Canon doctrinae triangulorum*. Rhäticus intended to expand substantially the sinus and cosinus tables. He started work on the tables and published them in 1596 under the title of *Opus Palatinam* with the cooperation of V. Otho. However, it soon became obvious that they were imperfect. This is why the "eccentric" Bürgi took the effort to calculate more accurate tables, which would have intervals of 2" and an accuracy of eight decimal places. His intention was to produce tables applicable to *postpheresis*. This means that he did not yet intended then to compile logarithmic tables. Why I say "eccentric" Bürgi? In 1623 Kepler wrote to Philip, Count of Hessen, informing him about these now lost tables of Bürgi's: *Er hatt gleichwol das geschribne Werk nie von Händen gegeben, noch druckhen lassen* (Bürgi never let the written work out of his hands, nor did he have it printed). This was the case although Kepler wrote an introduction to these tables. After all, the same even happened with Bürgi's logarithmic tables which were compiled about the year 1610 (comp. [8], 208). Was this a reaction to Brahe's conflict with Ursus? It is a fact that Bürgi published his *Arithmetische und geometrische Progress-Tabullen* in Prague only in 1620 and that some historians have stated that it is not true that Kepler encouraged Bürgi to compile the tables. When the logaritmic tables were issued, Kepler had already been outside Bohemia (since 1612), although during one of his visits to Prague (in 1617) he saw Napier's tables and wrote about them to Schickardt to Tübingen in December 1618. He began to write his own tables in 1619, which were finished as a manuscript entitled *Chilias Logarithmorum* in the winter of 1621-22; thus it looks improbable that he might hardly have known about Bürgi's work before the end of the year 1623. On the other hand, he could use Napier's tables already in 1619. Nevertheless, he used his own tables for the calculation of the Rudolfinian astronomical talbes (published 1627). His own tables were first issued in 1624[8].

Kepler's name is also associated with further efforts for overcoming the technical difficulties and lengthiness of astronomical calculations. From 1617 on Kepler was in contact with Professor Wilhelm Schickardt of Tübingen. In 1623 their correspondence concerned mainly with Schickardt's computing machine which made possible to perform mechanically all four arithmetical operations and with Napier's rods which were used for multiplication. Perhaps Kepler even wanted to get one such machine for his work on the Rudolfinian tables. However, the already finished machines were destroyed by the great city's fire and a drawing of the mechanism was discovered only in 1958.

One should not forget the most important scientific personality of that time, with whom Kepler colaborated in Prague just for one year. Tycho Brahe was a systematic and accurate observer whereas Kepler was short-sighted. However, Brahe recognized Kepler's mathematical talent and the latter felt how highly valuable Brahe's observation material was. It was already in February of 1599 that he wrote to Mästlin to Tübingen ([9], 199):

> "Let all... respect Tycho who devoted 35 years of his life to his observatory... What I need is just Tycho. He has made a mess of my order and the location of the orbits. I believe, therefore, that if God lets me live long enough I shall once be able to build a remarkable edifice"

A year later Kepler started working for Brahe. His task was to investigate the movement of Mars, but he was not given all the material. The Emperor ordered that Kepler should take care of this material after Brahe's death (24th October 1601), but Brahe's heirs did not want to pass the material over to Kepler. Later with Brahe's observations, Kepler was to confirm or reject all a priori ideas and to create a model corresponding best to the empirically obtained values. This way of his is described in detail in the abovementioned book *Astronomia nova*. His calculations took more than 5 years and the saved calculation notes alone cover 900 sheets of very small handwriting. It should be noted again that he could not yet rely on logarithmic tables in these calculations[9]. On the basis of Brahe's empirical facts, Kepler first discovered the excentricity of the orbits, then he rejected the Aristotelian regularity of circular movements and arrived at the irregularity of the movement of planets along the orbit; on the basis of an analogy with the intensity of light, with the suspected properties of gravitation as the main force of movement in space, and still without the knowledge of the shape of the orbit, he reached the formulation of the "law of areas", the so-called 2nd Kepler law (at the turn of 1601/2). He them considered various orbits and compared the hypothetical position of Mars with the measured values. He assumed the orbit to be an oval or an ovate curve, and only in December 1604 did he mention in his letter to Fabricius that the truth is somewhere between circle and oval, as if the orbit of Mars were an exact ellipse. It was as late as in the early 1605 that Kepler found what is now called Kepler's equation, $x = e \sin x + M$, where e and M are constants, together with the fact that the Sun is not in the ellipse's centre but in one of its foci. Thus the so-called first Kepler law was found at the end of the first series of his investigations.

It must be added that these primary efforts in applied mathematics, at the onset of the modern development of mathematics, were concentrated in the Rudolfinian scientific center. This was enabled (1) by a significant but undecided problem of application (an astromical one in this case); (2) by a preparatory stage in which the methods of observation as well as the empirical data had been improved; (3) by the higher accuracy required and by the acceleration in perfoming the calculations[10] which led to a wider application of *prostpheresis* together with a wider use of the more exact tables of trigonometrical functions (efforts in the direction of the mechanization of elementary computing operations must also be considered); (4) rejection of the unverified speculative models which were found to be speculative when confronted with reality; (5) by the thorough search for a maximum conformity between the measured data and the possible mathematical models; and (6), last but not least, by the favourable social situation in which enough means were provided for the desired concentration of outstanding scientists and for their cooperation in an environment which constituted a good background for their activities.

It seems possible that such process can be understood as one aspect in the formation of numerical mathematics, although it does not fully falls within Goldstine's conception.

In a very long period of further development in the Bohemian Countries we could hardly find an epoch of a similar such upsurge of calculation methods and the ensuing mathematical problems. Reactions to the works by outstanding mathematicians were then sporadic.

Nevertheless, towards the end of the 19th century, another outstanding scientist in the field of applied mathematics -Václav Láska- appeared in Prague. By education he was a mathematician and physicist and by profession he had ties with astronomy. He obtained his first university degree in higher geodesy at the Czech Technical University in Prague (1890) where he read cartography, the calculation of trigonometric networks, photogrammetry. He was interested in seismology[11], and after a short stay at the University of Lvov he retorned to Prague, where the Charles University established for him in 1911 the first (and for a long time the only) chair of applied mathematics. He was interested in the numerical and graphical methods which so frequently occured in the application areas in which he worked. He was perhaps the first lecturer of the Charles University who began to read the theory of interpolation, numerical integration of differential equations, methods of graphical calculus, and nomography. In this context, he wrote in 1915 the first Czech textbook *Introduction to Nomography* which, however, failed to appear owing to the difficult conditions prevalent during the first World War. Later it was included in a wider work written in collaboration with Václav Hruska, lecturer of the Technical University of Prague,*The Graphical and Graphico-mechanical Calculus* (Praha, 1923) and in another book, *The Theory and Practice of Numerical Calculation* (1934). A third work, unpublished, was *The Theory of Processing the Observed Values*, kept now in the Library of the Geophysical Institute of the Czechoslovak Academy of Sciences. The work developed by Láska was further continued by some mathematicians of the Technical University of Prague (CVUT) and then in much more favourable conditions at the Central Mathematical Institute (1951), which was a direct predecessor of today's Mathematical Institute of the Czechoslovak Academy of Sciences.

It should be noted that Láska's work started in a period when the interest in the problems of numerical analysis was increasing in the world mathematical literature. It is seen from the diagram of percent proportions of mathematic papers roughly divided into major groups on the basis of analyses performed by the abstract journals *Jahrbuch über die Fortschritte der Mathematik* and *Mathematical Reviews*, that this field began to be recorded as a separate discipline only in 1925 (until that time such papers were included in other disciplines) and that the interest increased from 3-4% in the nineteen-twenties to as much as 10% of all recorded mathematical literature thirty years later ([11], 22).

When establishing the academic mathematical research institution in Czechoslovakia, much preference was given to the question of developing the interest of young scientists towards applied mathematics. This, undoubtedly, was influenced to a considerable extent by the mutually exchanged considerations of Academician Cech and the Polish mathematicians who keenly supported this orientation since the very beginning of the post-war development of Polish mathematics.

It can be said that in this way a group of experts was prepared in Czechoslovakia step by step who were later given great social stimuli in solving the questions associated with the important and great projects of socialist industrialization of the country in the nineteen-fifties and sixties (comp. [12]), which again confronted them with deep problems posed by numerical methods.

The development of practical problems in the 20th century from the building industry, through nuclear power engineering up to mathematical problems related to the social sciences, has given rise to numerous stimuli to mathematics. Work on these problems helped, on the one hand, to the solution of problems arising in these fields and, on the other, developed and enhanced the mathematical theory of these fields. These branches began to prevail over the classical mathematical disciplines dominant in the 19th century. This is also the reason why numerical analysis began to receive a wider attention.

NOTES

1 NEUGEBAUER, O. (1957)

2 See the russian translation, 1968, p. 251.

3 Particularly by D. King, E.S. Kennedy and others.

4 See BARABASHOV, A.G. (1983, 6).

5 For us now this is the use of the approximation $\pi \approx 3.16049$.

6 If the number A = 50.8791 and B = 207.343 are to be multipied, the multiplication will be $A \times B = 10^5 \times 0.508791 \times 0.207393$. We set a = 0.508791 = sin α and b = 0.207343 = sin ß. It follows that $\alpha = 30^{o}35'$ and ß = $11^{o}58'$. Then α - ß = $11^{o}58'$, α + ß = $30^{o}35'$ and from tables cos (α - ß) = 0.947676 , cos (α + ß) = 0.736687. Subtracting cos (α - ß) - cos (α + ß) = 0.210988 = 2 sin α sin ß. Therefore A x B = 10549.425 if a 6D table is used. The exact value is 10549.42523 (See [7], 78).

7 The book also contained other ideas which the author could have taken from Brahe's results during his short stay in Hven in 1584. This gave rise to Brahe's anger and even to action at the court of law in Prague which culminated only after Ursus' death and short before the death of Brahe.

8 It is not my intention here to deal with the method of calculation of the table; for Napier and Bürgi such method is described in [9] and for Kepler in [8].

9 Jean Delambre, who verified Kepler's calculations in the latter half of the 18th century, demonstrated that a remarkable compensation of errors had occurred in Kepler's calculations so that the results were correct; see comp. (8), 99 f.

10 The reverse influence was also involved: stimuli were provided for further acceleration and improvement of the calculations.

11 Later in 1920 he founded in Prague a geophysical institute.

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FIGURES DESCRIPTION

1 Constructive demonstration of the evaluation of the best (and arithmetically very simple) quadrature of circle, given in the example 50 of the Rhind Papyrus.

2 Cuneiform table YBC 7289 from yale Babylonian Collection. Side of the square is 30 (see on the left above). Diagonal is 42;25,30 long; the ratio of both lengths is written on the diagonals above: 1;24,51,10 (all values in the sexagesimal system) -this value is very accurate, in decimals: $\sqrt{2} \approx 1.414213$ (cf. [6]).

3 Percentage of yearly numbers of published reviews in the *Jahrbuch über die Fortschritte der Mathematik* (FdM: 1870-1940); *Mathematical Reviews* (MR: 1940-1960) and *Referativniy Zhurnal-Matematika* (RZ: 1954-1960) divided into special branches of mathematics: An-Analysis, G-Geometry, Al-Algebra, P-

Probability, MNG-Numerical and Graphical Methods, H-History and Didactics, MM-Mathematical Computers (cf. [11]).

Fig. 1

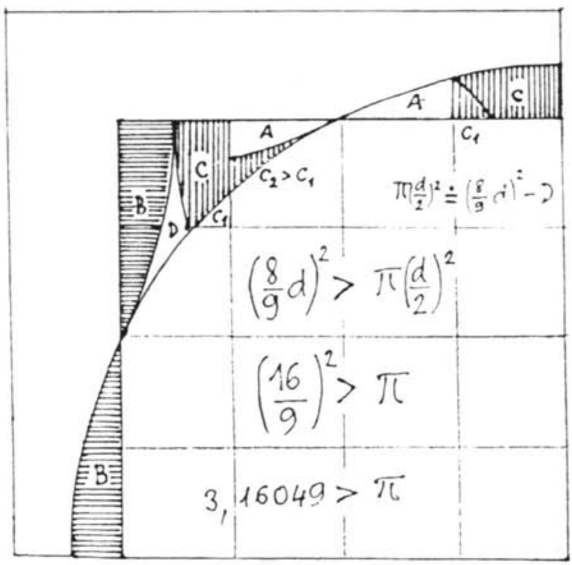

Fig.2

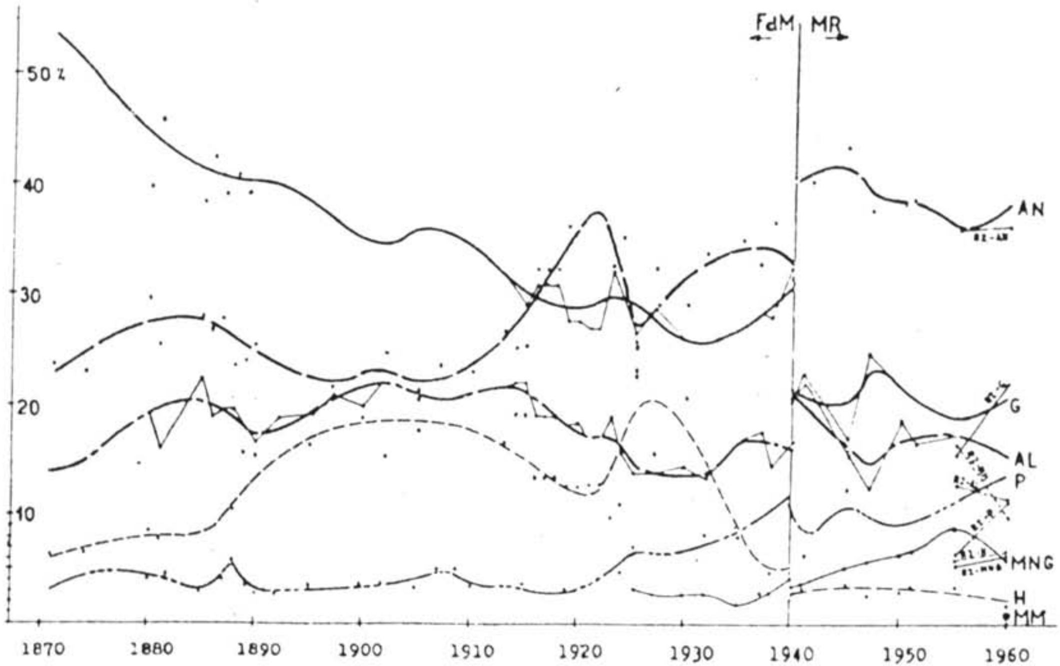
FdM
MR
50 %
40
30
20
10
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
AN
G
AL
P
MNG
H
MM

Fig. 3