REDIMAT
Journal of Research in Mathematics Education

Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

Examining the Impact of a Video case-Based Mathematics Methods Course on Secondary Pre-service Teachers' Skills at Analysing Students' Strategies

Mara Vanina Martinez[1], Alison Castro Superfine[1], Theresa Carlton[1] and Chandan Dasgupta[1]

1) University of Illinois-Chicago, United States of America.

Date of publication: February 24th, 2015
Edition period: February 2015-June 2015

To cite this article: Martinez, M.V., Castro Superfine, A., Carlton, T. & Dasgupta, C. (2015). Examining the Impact of a Video case-Based Mathematics Methods Course on Secondary Pre-service Teachers' Skills at Analysing Students' Strategies. *REDIMAT*, *Vol 4*(1), 52-79. doi: 10.4471/redimat.2015.59

To link this article: http://dx.doi.org/10.4471/redimat.2015.59

REDIMAT, Vol. 4 No. 1 February 2015 pp. 52-79

Examining the Impact of a Video case-Based Mathematics Methods Course on Secondary Pre-service Teachers' Skills at Analysing Students' Strategies

Mara Vanina Martinez
University of Illinois - Chicago

Alison Castro Superfine
University of Illinois - Chicago

Theresa Carlton
University of Illinois - Chicago

Chandan Dasgupta
University of Illinois - Chicago

(*Received: 30 July 2014; Accepted: 5 November 2014; Published: 24 February 2015)*

Abstract

This paper focuses on results from a study conducted with two cohorts of pre-service teachers (PSTs) in a video case-based mathematics methods course at a large Midwestern university in the US. The motivation for this study was to look beyond whether or not PSTs pay attention to mathematical thinking of students, as shown by previous studies when engaging with video, and, in turn, characterize at a more specific level areas in which PSTs' responses change. Our findings show that regarding PSTs anticipation of strategies, both cohorts showed a significant increase in the overall number of strategies PSTs were able to anticipate, and a significant increase in the mathematical depth of the anticipated strategies. However, there was no change in terms of PSTs identification and description of high school students' strategies as displayed in video given that both cohorts of PSTs performed equally proficient at both pre- and post- tests.

Keywords: Pre-service teachers, mathematics methods course, video case, students' strategies

2015 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2015.59

REDIMAT, Vol. 4 No. 1 February 2015 pp. 52-79

Examinando la Influencia de un Curso de Didáctica de las Matemáticas Centrado en el Uso de Video Casos en el Análisis de las Estrategias de Estudiantes

Mara Vanina Martinez
University of Illinois - Chicago

Alison Castro Superfine
University of Illinois - Chicago

Theresa Carlton
University of Illinois - Chicago

Chandan Dasgupta
University of Illinois - Chicago

(*Recibido: 30 Julio 2014; Aceptado: 5 Noviembre 2014; Publicado: 24 Febrero 2015)*

Abstract

Este artículo presenta los resultados de un estudio llevado a cabo con dos grupos de estudiantes del profesorado de matemática en una universidad del medio oeste en los Estados Unidos. Dado que estudios previos han demostrado que el uso de video casos en cursos de formación docente fomentan que los estudiantes de profesorado focalicen su atención en el pensamiento matemático de los escolares, este proyecto se centró en identificar y caracterizar a un nivel más específico el aprendizaje de los estudiantes de profesorado. Nuestro análisis muestra que después de participar en el curso de didáctica, los estudiantes de profesorado pueden anticipar más estrategias y pueden describirlas con mayor especificidad matemática. No hubo así cambio en cuanto a la identificación de estrategias en el video debido a que, tanto antes como después del curso, los estudiantes de profesorado identificaron correctamente todas las estrategias.

Keywords: Estudiantes de formación de profesorado, didáctica de las matemáticas, video casos, estrategias de estudiantes

2015 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2015.59

Hipatia Press
www.hipatiapress.com

Mathematics teachers have to know mathematics in ways that are unique to teaching. In addition to understanding the content of mathematical tasks they give to students, teachers also have to analyse unusual solution methods that students may pose, appraise students' explanations, and ask mathematical questions that further students' thinking (Ball, 1991; Lampert & Ball, 1998). However, mathematics methods courses may not provide adequate opportunities for high school pre-service teachers to develop such specialized knowledge of mathematics. Indeed, as reported by the California Association of Mathematics Teachers Educators (Lutz, 2008), which compiled information about mathematics methods courses in California, pre-service methods coursework primarily focuses on aspects of daily lesson planning, sometimes longer-term planning, and systematic study of current mathematics curricula. Teaching tasks such as analyses of teacher instructional moves and student thinking do not appear as explicit components of the syllabi of the surveyed methods courses (Lutz, 2008). Despite the need for high school pre-service teachers to know mathematics in ways needed for teaching, mathematics methods courses may not be designed to provide adequate opportunities for them to analyse teacher instructional practice and students' mathematical thinking.

Given the growing research base on the use of video in mathematics methods courses in particular (e.g., Alsawaie & Alghazo, 2010; Santagata & Angelici, 2010; Santagata, Zannoni, & Stigler, 2007), we developed a set of video cases depicting actual classroom practice designed to provide opportunities for pre-service high school teachers to learn about and analyse students' thinking in order to promote pre-service teacher learning through the analysis of issues related to mathematics, teaching practice, and student thinking. Based on previous research findings regarding the use and potential effectiveness of video- and text-based cases in teacher education (Kazemi & Franke, 2003; Seago & Goldsmith, 2006; Sherin & van Es, 2005, 2009), we hypothesized that the use of video cases would provide learning opportunities for pre-service high school mathematics teachers that more closely resemble the work of teaching.

In this paper, we report findings from a study involving the use of a set of video cases in a methods course for high school pre-service teachers. We focus on the following two research questions:

1. To what extent does the video case-based methods course promote pre-service teachers' anticipation of students' strategies for a given mathematical task?
2. To what extent does the video case-based methods course develop pre-service teachers' ability to attend to students' mathematical strategies as depicted in a video?

In the sections that follow, we first discuss research related to the use of video in pre-service teacher education. We then present and discuss the results of our study, and conclude with a discussion of the implications of this study for the design of mathematics methods courses for high school pre-service teachers.

The Use of Video in Pre-service Teacher Education

The use of video in teacher learning, in both in-service and pre-service contexts, has grown considerably in the field of teacher education. Video provides a medium in which high school pre-service teachers can critically analyse teaching practice in ways that are safely distanced from their own teaching experiences. In addition, such a medium affords more time for high school teachers to respond to and reflect on what they are reading or observing, and also provide a narrower view of classroom interaction and thus a more focused investigation of teaching practice and students' mathematical thinking (Ball & Cohen, 1999).

Current research demonstrates the potential of video cases to foster teachers' ability to attend to student thinking and explore mathematical concepts (Kazemi & Franke, 2003; Seago & Goldsmith, 2006; Sherin & van Es, 2005, 2009), and to develop teachers' *professional vision*, i.e. the ability to notice and interpret significant features of classroom interactions (Sherin & van Es, 2009). For example, in their research involving teacher video clubs, Sherin & van Es (2009) have shown that inservice teachers who initially gave little attention to students' thinking increased significantly in terms of attending to and analyzing students' mathematical thinking across the eight video club meetings during the school year. Similarly, Seago & Goldsmith (2006) found that the in-service teachers participating in their study learned to unpack the mathematics underlying students' thinking and to tie the students' explanations to fundamental mathematical ideas about linear relationships. These findings are aligned with what Sherin and van Es (2009) identify as a shift to a more interpretative stance in terms of students' thinking.

More specifically, studies centered on the use of video cases in methods courses (e.g., Santagata, Zannoni, & Stigler, 2007; Alsawaie & Alghazo, 2010; Santagata & Angelici, 2010) have found promising evidence regarding what pre-service teachers' learn about students' thinking in such courses. Santagata, Zannoni and Stigler's (2007) results show that pre-service teachers improved their analyses of teaching by moving from simple descriptions of what they observed to analyses focused on the effects of the teacher actions on student learning as displayed in the video. Studies (i.e., Alsawaie & Alghazo, 2010; Santagata & Angelici, 2010) that compared pre-service teachers' learning between an experimental group and a control group found promising evidence in the former group. For example, Alsawaie & Alghazo (2010) found that the use of video helped pre-service teachers improve their analysis of math teaching. Specifically, pre-service teachers learned to pay attention to noteworthy events in classroom interactions, and they developed the ability to pay attention to student learning when watching and analysing a lesson. The control group did not attend much to student learning. Santagata & Angelici (2010) found that pre-service teachers in the experimental group who were taught using the Lesson Analysis Framework, as opposed to the control group pre-service teachers who were taught using the Teaching Rating Framework, developed a more critical analysis of teachers' instructional decisions and the alternative strategies/activities were described in detail.

Taken together, the extant research demonstrates the potential for video-based curricula to impact pre-service teachers' ability to attend to substantive features of students' mathematical thinking. Building on this research, in this study our goal is to identify potential areas of impact of the video cases, specifically regarding students' strategies.

Video cases Overview

Generally, pre-service teachers' images of mathematics teaching often correspond to their experiences as high school and college students. As discussed previously, these images of teaching are insufficient and sometimes incongruent to current standards-based teaching and learning practices (National Council of Teachers of Mathematics, 2000). Indeed, pre-service teachers need a context to explore and analyse in a systematic way what the work of teaching high school mathematics entails since they need to understand the complexity of the work of teaching and how teachers' decision making process impacts students learning. Moreover, pre-service teachers need the opportunity to analyse students' mathematical thinking and to start developing potential ways of interacting with students' strategies.

In response to this issue, we developed a set of video cases focused specifically on high school mathematics (grades 9-12), which was designed to depict and provide opportunities for high school pre-service teachers to understand the complexities in teaching practice (see Table 1). Each video case includes several components representing different dimensions of the work of teaching high school mathematics (e.g., teacher interview before teaching the class, lesson plan, math tasks, students' written work, etc.). The set of video cases is comprised of seven video cases, and focuses on mathematics topics ranging from algebra (e.g., quadratics) to calculus (e.g., surface area and volume). Each video case portrays classroom episodes and artefacts from the same teacher and group of high school students.

Table 1
Video cases

Teacher	Course	Topic	Grade	Curriculum
Tanya	Algebra I	Quadratics	9th	Project-Based and Teacher Designed
Zara	Pre-calculus	Arithmetic and geometric sequences and series	12th	IMP and Functions, Modelling and Change.
Carla	Algebra and geometry	Reflections	10th	IMP
Frank	Algebra	System of equations	9th	Agile Mind
Tomas	Advanced Algebra with Trigonometry	Logarithms	11th–12th	Project Based and Teacher designed
Louis	Integrated Mathematics	Slope and equation of the line	10th–12th	Imp 3 & 4

(continued)

Table 1
Video cases (continued)

Teacher	Course	Topic	Grade	Curriculum
Tomas	AP Calculus	Surface Area and Volume	12th	Project Based and Teacher designed

Instructional Framework for Using the Video cases in the Methods Course

The overall goals of the methods course are to promote pre-service teachers' conceptualization of teaching in terms of student learning (Hiebert, Morris, Berk, & Jansen, 2007), learning of mathematical knowledge for teaching (e.g., knowledge of pedagogy and students), and of curricular knowledge (e.g., understanding of current math programs (similarities and differences as it relates to student learning)). As part of the methods course, pre-service teachers engage in activities such as lesson plan design, analysis of findings from articles in practitioner journals, enactment of teaching, analysing samples of student work, and interpreting students' mathematical ideas. Thus, we developed the *Learning to Teach Mathematics from Records of Practice* framework, based in part on the Cognitively Guided Instruction (CGI) professional development framework (e.g., Carpenter, Fennema, & Franke, 1996) and Thinking Through a Lesson Protocol (2008) to create an effective instructional model for video cases to be used with high school pre-service teachers. The aim of the proposed framework is to focus pre-service teachers' work around the need to provide evidence for their claims about student thinking. In what follows, we describe the goals of each component in terms of pre-service teacher education. The framework encompasses the various phases of structuring pre-service teachers' interactions with the video cases that were used in the methods course in this study.

Learning to Teach Mathematics from Records of Practice: A Framework

I. Mathematical Task Analysis for Teaching

The main goal of this stage is to engage pre-service teachers in doing the mathematics as needed for teaching prior to seeing a video of the problem in a classroom. It involves pre-service teachers conducting a Mathematical Task Analysis (i.e., pre-service teachers solve the math task) and Task Analysis for Teaching (i.e., what mathematics could be taught, anticipate high school students strategies and mistakes, and, devise teacher interventions).

II. Learning About "The Work of Teaching Before Teaching".

The main goal of this stage is to introduce pre-service teachers *by observing an experienced teacher* do the work of teaching before teaching (Lampert, 2001). Thus, pre-service teachers will watch the interview with the teacher before teaching explaining the considerations she/he took into account during planning the lesson.

III. Observing High School Mathematics Teaching and Learning.

The main goal of this stage is to offer pre-service teachers the opportunity to observe different aspects of teaching and students' learning as they happen during different moments of the lesson.

IV. Analysing High School Mathematics Teaching and Learning.

The main goal is to promote a shift in pre-service teachers' attention from *observing* to *analysing* different aspects depicted in the video clips. PSTs answer specific questions addressing issues that arose in the video clip.

V. Analysing Students' Written Work.

The focus is on analysing students' written work (i.e., correct, incorrect, incomplete, and alternative solutions) for purposes of fostering an *analytic* stance as opposed to a *judgmental* one (i.e., incorrect, correct). Other goals in this stage are to introduce pre-service teachers with a repertoire of students' potential strategies and to provide the opportunity for pre-service teachers to develop questions they would ask a specific student in order to better understand the work presented in written form.

VI. Learning About the Work of Teaching After Teaching.

The main goal of this stage is to show how the teacher's reflection may function as a bridge between the "just-taught-class" and "next-class".

Method

Research Site and Participants

The study was conducted at a large Mid-Western University in the US, a diverse higher education research institution that situates the work of preparing high school teachers in the mathematics department. Specifically, the study was conducted in one of two required methods course for high school pre-service mathematics teachers during the fall semester of Year 1 and again in the fall semester of Year 2. Cohort 1 in Year 1 included 19 research participants (4 males and 15 females) while Cohort 2 in Year 2 included 17 (10 males and 7 females). A typical semester includes 28 classes meeting twice a week for a total of 150 minutes. The first author of this paper was the instructor for the course.

The particular methods course that serves as the research site for the proposed project is designed to prepare high school pre-service teachers to learn about mathematics teaching and learning. In this course, many different aspects of the work of teaching are addressed such as lesson planning, analysis of students' thinking, analysis of teacher moves, and study of current curriculum materials. PSTs in both cohorts worked on video cases approximately half of all instructional time during the duration of the semester that would amount to about 19 hours or 22 modules of 50 minutes.

Data Collection

To address questions related to changes in pre-service teachers' conceptions of student learning, mathematical thinking and teachers' instructional strategies, a pre-post assessment was administered. This assessment required pre-service teachers to conduct a Mathematics Task Analysis for Teaching (Stage I), as described in the Learning to Teach Mathematics from Records of Practice Framework. Consequently, before watching the video case, pre-service teachers were asked to solve a mathematical task about linear functions (i.e. Question 1). After that, they were asked to analyse what mathematics could be taught using that specific task (i.e., Question 2), anticipate high school students' strategies (i.e., Question 3), anticipate possible students' mistakes (i.e., Question 4). After watching a video case on linear functions, pre-service teachers were asked to describe what they notice in terms of student thinking (i.e., Question 5), to describe what they notice in terms of what the teachers does (i.e. Question 6), to identify students' strategies when solving the task (i.e., Question 8), and to identify teachers' moves (i.e., Question 9). The video case involves the "Growing Dots" math task (Figure 1) on linear functions (Seago, Mumme, & Branca, 2004).

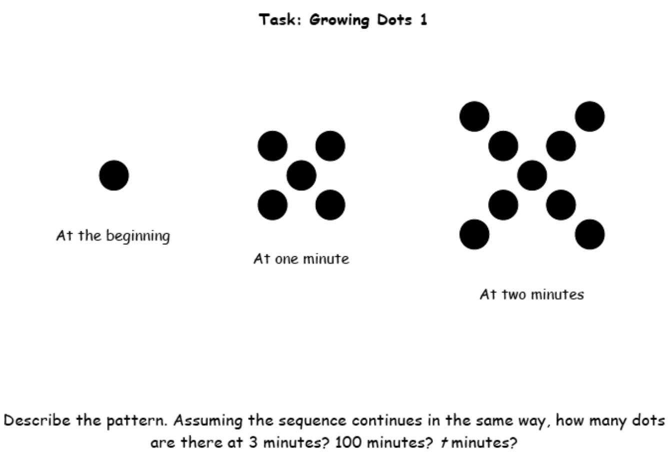

Figure 1. Mathematical task included in the pre-post assessment

Data Analysis

In this report, we focus on questions 1, 3, and 8. Question 1 was used as a measure of pre-service teachers' mathematical understanding. We anticipated that these two groups of pre-service teachers would perform proficiently given that they are high school mathematics majors and linear functions is an elementary topic for them. Question 3 was used as a measure of pre-service teachers' anticipation of students' strategies. Question 8 was used as a measure of pre-service teachers' identification of students' strategies in the video case.

For each question the research team developed successive coding schemes. Tasks from the pre-post test written assessment were coded blind as to pre- and post-course administration, using criteria developed by the research team, and testing for inter-rater reliability. Item responses were compared across each semester and between semesters.

Coding

Question 1 – The mathematics

Each subpart of the question was coded separately (Table 2). "Describe the pattern" was coded for "present" and "not present". The other three sub-parts of the first question (i.e., How many dots at 3 minutes? How many dots at 10 minutes? How many dots at t minutes?) were coded using the possible codes "incorrect", "correct", and "no answer."

Table 2
Question 1 Coding

Subpart of Question 1	Code Name	Code Value	Code Description
Describe the pattern	Presence	Present	An answer was coded as present whenever the PST provided a colloquial description of the pattern

(continued)

Table 2
Question 1 Coding (continued)

Subpart of Question 1	Code Name	Code Value	Code Description
		Not present	An answer was coded as present whenever the PST did not provide a colloquial description of the pattern
How many dots at 3 minutes?	Correctness	Incorrect	An answer was coded as incorrect when the provided numeric answer was different than 13.
		Correct	An answer was coded as correct when the provided numeric answer was equal 13.
		No Answer	There was no answer provided.
How many dots at 100 minutes?	Correctness	Incorrect	An answer was coded as incorrect when the provided numeric answer was different than 401.
		Correct	An answer was coded as correct when the provided numeric answer was equal 401.
		No Answer	There was no answer provided.
How many dots at t minutes?	Correctness	Incorrect	An answer was coded as incorrect when the provided expression was not “4x+1”, “d+4” or equivalent.
		Correct	An answer was coded as correct when the provided expression was “4x+1” or “d+4” or equivalent.
		No Answer	No answer was provided.

Question 3 – Anticipating students' potential strategies

First, let's recall question 3: *What are possible ways/strategies that high school students might solve this task?* We started by determining how many strategies were provided as part of the answer. Each strategy was then categorized according to two dimensions: mathematical depth and complexity (Table 3).

Table 3
Question 3 Coding

Code Name	Code Value	Code Description	Example
Mathematical Depth	General	The strategy is anchored on general mathematical features. However, nothing mathematically specific about the linear relation involved in the given task.	Students will generalize the pattern using algebra.
	Specific	The strategy is anchored on mathematical properties specific to the linear relation in the task.	<u>Adding four</u> to the <u>previous picture</u>. Four dots each minute plus the dot at the beginning.
Complexity	Simple	The strategy comprises one or two steps.	Guess and check
	Multiple steps	The strategy comprises more than two steps.	Drawing more pictures. Recognizing a pattern. Find a general equation that fits all the pictures.

Question 8 – Identifying students' strategies in video

First, let's recall question 8: *Identify different strategies used by students in the video*. We started by counting how many strategies students include in their answer (i.e., 0, 1, or 2). There is a possible maximum of two strategies given that in the video students model the problem using either a closed form and/or recursion. Each of the strategies was coded for correctness (i.e., correct, incorrect, and no answer). Each of the strategies was coded for mathematical depth, and for the representation included in the description of the strategy (Table 4)

Table 4
Question 8 Coding

Code Name	Code Value	Code Description	Example
Mathematical Depth	0: Non-Mathematical	The description of the strategy is not anchored in any specific mathematical features and /or concepts.	Students had multiple ways of solving the task. Students were thinking aloud and engaging in math talk. They were also prepared to defend their answers.
	1: Math - General	The description of the strategy is anchored on general mathematical features, terms, or properties. However, nothing mathematically specific about the linear relation is included.	They use different methods to approach identifying the <u>pattern.</u>
	2: Math – Specific-Implicit	The description of the strategy implicitly establishes the type of relation (e.g., recursive, linear, etc.) without	James thinks about the next picture in relation to the previous picture. (continued)

Table 4
Question 8 Coding (continued)

Code Name	Code Value	Code Description	Example
		describing the mathematical parameters. It doesn't characterize *exactly how,* a_n relates with a_{n+1} or, alternatively, how t [number of minutes] relates to d [number of dots].	
	3: Math – Specific-Explicit	Describes the relation by providing a characterization of the parameters of the relation in terms of the context and/or in mathematical terms.	See the pattern going around in the circle, so each time she adds 1 to each "arm" and goes around. Adding four to the previous picture. f(t)=4t+1, t minutes f(t)= [f(t-1)+4], t minutes

Results

Question 1 provides evidence that the mathematical task was well suited as part of the pre- and post- assessment video case since PSTs were able to solve it proficiently. Consequently, and given that all other questions in the assessment rested on PSTs' understanding of the mathematics of the task, we assumed that the task would not interfere with measuring performance in the rest of the questions.

Both cohorts of PSTs demonstrated a significant gain in anticipating high school students' strategies, not only in terms of number of strategies they were able to anticipate but also in the depth of the mathematical description of the strategies. Both cohorts performed equally well at identifying strategies from pre- to post-test. Thus, there was no significant change in this skill. Thus, it seems that the use of the video cases in the course had a differentiated effect in terms of what knowledge it affected. We explain these results in further detail in the sections that follow.

Results for Question 1 – The Mathematics

Question 1 required four distinct answers: a colloquial description of the pattern and three predictions for the pattern at various points in time. Colloquial language was quantified as either “present” = 1, or “not present” = 2. A mean value close to one would indicate that more participants included a colloquial language description than not; while a mean value close to two would indicate that the majority of participants did not include this description.

Cohort 1 – Year 1

Nearly an equal number of participants either included or did not include a colloquial description on the pre-test (i.e., 47% present, 53% not present); while a majority of participants did not include the description on the post-test (i.e., 26% present, 74% not present). A matched-pairs t-test was conducted to test the significance of this change. With a t-statistic of -1.29, difference in means -0.21, degrees of freedom 18, critical value 2.10 (two-tailed), the difference in mean number of colloquial descriptions from pre- to post-test was found to be statistically insignificant.

Most participants were able to provide correct solutions to the pattern at various points in time. All participants (i.e., n = 19) provided the correct solution for what the pattern would be at time = 3 minutes on the pre-test, while all but one participant provided the correct solution on the post-test. At time = 100 minutes, all but one participant provided a correct solution on both the pre-test and the post-test. For time = t minutes, all but one participant provided correct solutions on the pre-test, while all participants provided the correct solution on the post-test. Further statistics were not conducted since there was no expectation of significance.

Cohort 2 – Year 2

This second cohort revealed nearly identical results as the first cohort. Approximately an equal number of participants either included or did not include a colloquial description on the pre-test (i.e., 47% present, 53% not present); while a majority of participants did not include the description on the post-test (i.e., 27% present, 73% not present). The results of a matched-pairs *t*-test found the difference in mean number of colloquial descriptions provided from pre- to post-test is not statistically significant, with a *t*-statistic of -1.38, n=15, difference in means -0.2, degrees of freedom 14, critical value (two-tail) 2.145.

Participants in this second cohort were able to provide correct solutions to the pattern at various points in time. All but one participant (i.e., n = 14) provided the correct solution for what the pattern would be at time = 3 minutes on the pre-test, while all participants (i.e., n = 15) provided the correct solution on the post-test. The same is true at time = 100 minutes; all but one participant provided a correct solution on the pre-test, but all participants answered correctly on the post-test. For time = *t* minutes, all but one participant provided correct solutions on the pre-test and on the post-test. Further statistics were not conducted since there was no expectation of significance.

Comparison of Results for Cohorts 1 and 2 on Q1- the Mathematics

In sum, participants in both cohorts were generally able to correctly answer the question. As we were expecting given the math content, these results indicate that the task was accessible for PSTs to engage with and obtain a correct answer. Therefore, this task did not work as an obstacle to answer the following questions in the assessment.

Results on Question 3 – Anticipating Students' Potential Strategies

Question 3 asks participants to come up possible strategies they think high school students might use in solving the dots task. As mentioned before, the number of strategies provided by each participant was counted and coded for mathematical depth and complexity. Mathematical depth was quantitatively coded as general = 1 and specific = 2. Complexity was quantitatively coded as multiple = 1 and simple = 2. A summary of results is displayed in Table 5.

The number of strategies provided by each participant on the pre-test ranges from 1 to 4, and has a mean of 2.11. On the post-test, the number of strategies ranges from 1 to 5 with a mean of 2.68. To determine whether this change in mean number of strategies is significant, a matched-pairs *t*-test was conducted. The results of the *t*-test, indicate that, with a *t*-statistic of -2.16, the change in means is significant at the 5% level, with an actual significance level of 4%.

Table 5

Question 3 - Results Anticipating students' potential strategies

	Change in number of potential strategies (pre to post)	Change in Mathematical Depth (pre to post)	Change in Complexity (pre to post)
Cohort 1	Statistically significant increase	Statistically significant increase in the number of specific strategies	No change
Cohort 2	Statistically significant increase	Statistically significant increase in the number of specific strategies	Statistically significant increase in the number of complex strategies

Cohort 1 – Year 1

This provides sufficient evidence to conclude that the average number of strategies provided on the post-test is larger than the average number of strategies on the pre-test. Regarding the proportion of general and specific strategies, on the pre-test 10% of strategies were specific; while on the post-test, the same type of strategy increased to 37%.

In order to account for the change in number of strategies provided, the mean mathematical depth was calculated for each participant. A mean mathematical depth of one indicates that a participant only provided general strategies; while a mean of two indicates that all the strategies provided by that participant were specific strategies. Any mean value falling between one and two indicates that a participant provided both general and specific strategies; a value of 1.5 indicates that the participant provided an equal number of both general and specific strategies.

Looking at complexity of the strategies in terms of the proportion of multiple steps and simple strategies, our analysis shows that on the pre-test, 50% of the provided strategies had multiple steps; while only 37% of the strategies on the post-test were multiple step strategies.

While the actual number of multiple step strategies did not vary greatly from pre-test to post-test (n_1=20, n_2=19), the number of single step strategies did increase from 20 on the pre-test to 32 on the post-test. In order to account for the change in the number of strategies, a mean complexity score was calculated for each participant. A mean score of one indicates that participants only provided multiple step strategies; while a mean score of two indicates that a participant only provided simple strategies. Any mean score between one and two indicates that a participant provided both multiple step and simple strategies, with a mean score of 1.5 indicating that a participant provided an equal number of multiple step and simple strategies.

A matched-pairs *t*-test was conducted to determine if there is any difference in the mean complexity scores from the pre-test to the post-test. With a *t*-statistic of -1.29, the results show that there is no significant difference between the mean complexity scores from the pre-test to the post-test. There is not sufficient evidence to conclude that the complexity of the strategies changed from pre- to post-test.

Cohort 2 – Year 2

The number of strategies provided by each participant on the pre-test ranges from 2 to 4, and has a mean of 2.73. On the post-test, the number of strategies ranges from 1 to 7 with a mean of 4.07. To determine whether this change in mean number of strategies is significant, a matched-pairs *t*-test was conducted. The results of the *t*-test, indicate that, with a *t*-statistic of -3.16, the change in means is significant at the 1% level, with an actual significance level of 0.6%. This provides sufficient evidence to conclude that the average number of strategies provided on the post-test is larger than the average number of strategies on the pre-test.

Regarding the mathematical depth of strategies provided by pre-service teachers in terms of the proportion of general and specific, results show that on the pre-test, only 29% of strategies were specific; while on the post-test, this increased to 57%.

In order to determine whether the change in mathematical depth of strategies provided, the mean mathematical depth was calculated for each participant. A mean mathematical depth of one indicates that a participant only provided general strategies; while a mean of two indicates that a participant only provided specific strategies. Any mean value falling between one and two indicates that a participant provided both general and specific strategies; a value of 1.5 indicates that the participant provided an equal number of both general and specific strategies.

A matched-pairs *t*-test was conducted to test whether there was any difference between the mean value of specific versus general strategies provided from the pre-test to the post-test. With a *t*-statistic of -2.29, it was found that the difference between the mean number of general and specific strategies is significant at the 5% level, with an actual significance level of 3.8%. This provides sufficient evidence to conclude that the mean number of problem specific strategies increased from the pre-test to the post-test.

Our analysis indicates that there is also a change in the complexity of strategies provided by pre-service teachers in terms of the proportion of multiple steps and simple strategies. On the pre-test, only 32% of the provided strategies had multiple steps; while 61% of the strategies on the post-test were multiple step strategies.

While the actual number of simple strategies did not vary greatly from pre-test to post-test (n_1=28, n_2=24), the number of multiple step strategies did increase from 13 on the pre-test to 37 on the post-test. In order to account for the change in the number of strategies, a mean complexity score was calculated for each participant. A mean score of one indicates that participants only provided multiple step strategies; while a mean score of two indicates that a participant only provided simple strategies. Any mean score between one and two indicates that a participant provided both multiple step and simple strategies, with a mean score of 1.5 indicating that a participant provided an equal number of multiple step and simple strategies.

Comparison of cohorts' results on anticipating high school students' potential strategies

Both cohorts showed significant increase in the overall number of strategies participants were able to anticipate, and the number of problem specific strategies from the pre-test to the post-test. Cohort 1 showed no significant change in the complexity of the strategies, with an actual decrease in the proportion of multiple step strategies. However, Cohort 2 did show a significant change in the complexity of the strategies, with an increase in both number and proportion of multiple step strategies provided.

Results on Question 8 – Identifying Students' Strategies in a Video

Question 8 asked participants to identify and describe the strategies high school students used to solve the problem as depicted in the video. Only two types of strategies were used in the video – either closed (i.e., $f(x)=4x+1$) or recursive (i.e., $a_n=a_{n-1}+4$ and $a_0=1$).

Table 6
Results on Question 8 – Identifying Students' Strategies in a Video

	Closed Form Strategy		Recursive Form	
	Correct Identification	Mathematical Depth	Correct Identification	Mathematical Depth
Cohort 1	Almost all PSTs both at pre- and post-assessments	Large majority was specific at both pre- and post-assessments	Almost all PSTs both at pre- and post-assessments	Significant increase
Cohort 2	Almost all PSTs both at pre- and post-assessments	Large majority was specific at both pre- and post-assessments	Almost all PSTs both at pre- and post-assessments	Large majority was specific at both pre- and post-assessments

The type of strategy identified by the participants was noted, and each strategy was coded for correctness and mathematical depth. Correctness was coded as 0 = incorrect, 1 = correct, and 2 = not present. Mathematical depth was coded as 0 = Non-mathematical, 1 = Mathematical – General, 2 = Mathematical – Specific - Implicit, 3 = Mathematical – Specific – Explicit. A summary of results is displayed in Table 6.

Cohort 1 – Year 1

Closed Form Strategy

Overall, 17 out of 19 participants were able to identify the closed strategy used in the video on the pre-test, while 18 out of 19 were able to identify it on the post- test. Sixteen participants correctly identified the closed strategy on the pre-test, while seventeen participants correctly identified the closed strategy on the post-test. One participant incorrectly identified the closed strategy on the pre-test and the post-test. Overall, participants were generally able to identify and correctly explain the closed form strategy on both the pre-test and the post-test. No further statistics were conducted since there was no expectation of significance.

Mathematical depth of the responses involving the closed form strategy did not change significantly from the pre-test to the post-test. One participant provided a non-mathematical explanation on each of the pre-test and post-test. The number of general mathematical strategies increased from one to two from the pre-test to the post-test; the number of specific mathematical strategies with an implicit relation increased from three to four from pre-test to post-test. The number of specific mathematical strategies with an explicit relation decreased from the pre-test to the post-test by one from 12 to 11. No further statistics were conducted since there was no expectation of significance.

Recursive Form Strategy

Overall, 17 out of 19 participants were able to identify the recursive strategy used in the video on the pre-test, while all participants were able to identify it on the post- test. Sixteen participants correctly identified the recursive strategy on the pre-test, while all nineteen participants correctly identified the recursive strategy on the post-test. One participant incorrectly identified the recursive strategy on the pre-test but not on the post-test.

Overall, participants were able to identify and correctly explain the recursive form strategy on both the pre-test and the post-test. No further statistics were conducted since there was no expectation of significance.

Regarding the mathematical depth of the responses involving the recursive form strategy on both the pre-test and the post-test, one participant provided a non-mathematical explanation on the pre-test, but all participants provided a mathematical explanation on the post-test. One response was coded as a general mathematical strategy on each of the pre-test and post-test; the number of specific mathematical strategies with an implicit relation decreased from five to three from pre-test to post-test. The number of explicit specific mathematical strategies increased from the pre-test to the post-test by five.

Since the number of responses falling under the mathematical-specific level 3 category increased by 50%, it seemed likely that this increase would be statistically significant. To determine this, participant responses were re-coded as level 3 = 1, not level 3 = 0. A matched-pairs *t*-test was conducted to test whether there was a difference in the mean scores for the level 3 category. With a *t*-statistic of -1.76, the difference in average number of level 3 responses is significant at the 5% level, with an actual significance of 4.8%. There is sufficient evidence to conclude that the number of mathematical-specific responses placed in context increased from pre-test to post-test.

Cohort 2 – Year 2

Closed Form

All participants were able to identify the closed form strategy used in the video on the pre-test, while 14 out of 15 were able to identify it on the post-test. All participants who identified the closed form strategy did so correctly on both the pre-test and the post-test. No participants incorrectly identified the closed strategy on either the pre-test or the post-test, but one participant did not include it on the post-test. No further statistics were conducted since there was no expectation of significance.

Regarding the mathematical depth of the responses involving the closed form strategy on both the pre-test and the post-test, one participant provided a non-mathematical explanation on each of the pre-test and post-test. The number of general mathematical strategies decreased from three to zero from the pre-test to the post-test; the number of specific mathematical strategies with an implicit relation remained the same from pre-test to post-test. The number of explicit specific mathematical strategies increased from the pre-test to the post-test by two. No further statistics were conducted since there was no expectation of significance.

Recursive Form

All participants were able to identify the recursive strategy used in the video on the pre-test, while all but one participant identified it on the post-test. All participants correctly identified the recursive strategy on the pre-test, and all participants who identified the recursive strategy on the post-test did so correctly. No participant incorrectly identified the recursive strategy on the pre-test or the post-test, but one participant failed to identify it on the post-test. No further statistics were conducted since there was no expectation of significance.

Regarding the mathematical depth of the responses involving the recursive form strategy on both the pre-test and the post-test, no participant provided a non-mathematical explanation on the pre-test or the post-test. One response was coded as a general mathematical strategy on the pre-test, but no response was coded as a general mathematical strategy on the post-test. The number of specific mathematical strategies with an implicit relation remained the same from pre-test to post-test, as did the number of explicit specific mathematical strategies. Overall, 14 out of 15 participants provided a specific mathematical strategy on the pre-test and the post-test. No further statistics were conducted since there was no expectation of significance.

Comparison of both cohorts' results on identification of students' strategies in video

For both cohorts, participants were generally able to notice and correctly explain both the closed form and recursive form strategies that were provided in the video. There was little change in the mathematical depth of the explanations provided for the closed form strategy in each cohort; though all students in Cohort 2 who provided a mathematical explanation on the post-test gave explanations that were specific to the problem being solved.

Cohort 2 showed a significant increase in the number of specific mathematical responses placed in context for the recursive form strategy; while 14 out of the 15 responses from the same cohort were mathematical specific, with the exact same number of responses coded as implied or in context on both the pre- and post-tests.

Discussion

As previously mentioned, this paper focuses on results from a study conducted with two cohorts of pre-service teachers (PSTs) in a video case-based mathematics methods course at a large Midwestern university in the US. Since previous studies found that after engaging with video teachers pay more attention to the mathematical thinking of students, the motivation for this study was to look beyond whether or not PSTs pay attention to mathematical thinking of students and, in turn, characterize at a more specific level areas in which PSTs skills change.

As previously mentioned, regarding PSTs' anticipation of strategies, both cohorts showed significant increase in the overall number of strategies PSTs were able to anticipate, and the mathematical depth of those strategies from the pre-test to the post-test. Results were not consistent in terms of complexity of strategy. While Cohort 1 showed no significant change in the complexity of the strategies, Cohort 2 did show a significant change, with an increase in both number and proportion of multiple step strategies provided. Anticipating students' strategies, particularly those strategies that involve multiple steps, is a central component in the set of desirable skills needed for the work of teaching. It is in this sense that being able to provide the opportunity for PSTs to develop this skill in a classroom environment seems very fruitful. Finally, for both cohorts, participants were generally able to notice and correctly explain both the closed form and recursive form strategies that were provided in the video at both instances pre- and post-assessment. A large majority of PSTs from both cohorts provided a mathematical specific description of the strategies.

Overall, with findings from this study involving a video case-based methods course, we were able to identify at a more detailed level whether or not PSTs pay attention to students' strategies. Specifically, findings from this study demonstrate a differentiated impact that is, participating in the video case-based methods course had a positive impact on PSTs' anticipating students' potential strategies. It was not the case, however, that the video case-based methods course had an impact in PSTs' skills at identifying students' strategies as displayed in video. This might be because PSTs performed equally proficiently both at the pre- and post-assessments. Nevertheless, our findings show that regarding PSTs anticipation of strategies, both cohorts showed a significant increase in the overall number of strategies PSTs were able to anticipate, and a significant increase in the mathematical depth of the anticipated strategies.

Implications for Secondary Methods Course Design

The results afore discussed join the growing body of evidence supporting the use of video in methods course (e.g., Alsawaie & Alghazo, 2010; Santagata & Angelici, 2010; Santagata, Zannoni, & Stigler, 2007). In fact, studies have been documenting in increasing detail how video impact pre-service teachers' learning spanning from a focus on what they notice (e.g., students' mathematical thinking) (Kazemi & Franke, 2003; Seago & Goldsmith, 2006; Sherin & van Es, 2005, 2009), to specific areas of impact such as anticipating students strategies for a given problem which is beyond the realm of the video itself as documented in this paper. It is important to point out that, as indicated by much of the extant research and this study, the use of video should be accompanied with a specific framework targeting desired learning goals for pre-service teachers.

The major implication of this study is that video cases can be effectively used to support PSTs' skills at anticipating students' mathematical strategies for a given task. Participants in both cohorts demonstrated significant increases both in the number of anticipated strategies and the mathematical depth. Thus, the integration of a video case-based curriculum into methods courses has strong potential for affecting this important skill needed for teaching. By providing opportunities to develop specialized content knowledge in their coursework, PSTs can begin engaging in the kind of teaching work in which they will soon be called upon to do.

References

Alsawaie, O, & Alghazo, I. (2010). The effect of video-based approach on prospective teachers' ability to analyze mathematics teaching. *Journal of Mathematics Teacher Education*, *13*(223), 241. doi: 10.1007/s10857-009-9138-8

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Vol 2. Teacher's subject matter knowledge and classroom instruction*. Greenwich, CT: JAI Press.

Carpenter, T., Fennema, E., & Franke, Megan. (1996). Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction. *The Elementary School Journal*, *97*(1), 3-20.

Hiebert, J., Morris, A., Berk, D., & Jansen, A. (2007). Preparing Teachers to Learn from Teaching. *Journal of Teacher Education*, *58*(1), 47-61. doi: 10.1177/0022487106295726

Kazemi, E, & Franke, Megan. (2003). *Using Student Work To Support Professional Development in Elementary Mathematics. A CTP Working Paper*. Center for the Study of Teaching and Policy, University of Washington.

Lampert, M. (2001). Teaching While Preparing for a Lesson. In M. Lampert, *Teaching Problems and the Problems of Teaching* (pp. 121-142). New Haven: Yale University Press.

Lampert, Magdalene, & Ball, Deborah Loewenberg. (1998). *Teaching, multimedia, and mathematics : investigations of real practice*. New York: Teachers College Press.

Lutz, P. M. (Ed.). (2008). *Secondary Mathematics Methods Courses in California* (Vol. 1). Bakersfield, CA: California Association of Mathematics Teachers Education.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Santagata, R, & Angelici, G. (2010). Studying the impact of the lesson analysis framework on pre-service teachers' abilities to reflect on videos of classroom teaching. *Journal of Teacher Education*, *61*, 339-350. doi: 0.1177/0022487110369555

Santagata, R, Zannoni, C, & Stigler, J. W. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*, 123-140. doi: 10.1007/s10857-007-9029-9

Seago, N., & Goldsmith, L. (2006). Learning Mathematics for Teaching. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 73-80). Prague: PME.

Seago, N., Mumme, J., & Branca, N. (2004). *Learning and Teaching Linear Functions: Video Cases for Mathematics Professional Development, 6-10. Facilitators' Guide.* Portsmouth, NH: Heinemann.

Sherin, M, & van Es, E. (2005). Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education*, *13*, 475-491.

Sherin, M, & van Es, E. (2009). Effects of Video Club Participation on Teachers' Professional Vision. *Journal of Teacher Education, 60*(1), 20-37. doi: 10.1177/0022487108328155

Smith, M, Bill, V, & Hughes, E. (2008). Thinking through a Lesson: Successfully Implementing High-Level Tasks. *Mathematics Teaching in the Middle School, 14*(3), 132-138.

Mara V. Martinez is an Assistant Professor of Mathematics Education in the Mathematics, Statistics and Computer Science Department, University of Illinois-Chicago, United States of America.

Alison Castro Superine is professor of mathematics education, in the Department of Mathematics, University of Illinois-Chicago, United States of America.

Theresa Carlton is a PhD student in Mathematics Education, College of Education, University of Illinois-Chicago, United States of America.

Chandan Dasgupta is a PhD student in Learning Sciences, Learning Sciences Research Institute, University of Illinois-Chicago, United States of America.

Contact Address: Direct correspondence concerning this article, should be addressed to the author. Postal address: SEO610 (MC249), UIC; 851 S. Morgan St. Chicago, IL 60607, USA. **Email**: mara.martinez@mac.com