



## TERM STRUCTURE IN THE EUROPEAN INTERBANK MARKET

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### ABSTRACT

The objective of this paper is to provide a monthly estimation of the interest rate term structure in the European interbank market since the beginning of the European Monetary Union. In order to do this, we apply the Fama-Bliss bootstrapping method with the approximating function of one of the methods most commonly applied by the central banks, the Nelson and Siegel method (1987).

### KEYWORDS

Interest Rate Term Structure - Interbank Interest Rates - Swap Interest Rates - Euribor - Interbank Market.

### 1. INTRODUCTION

The term structure of interest rates (IRTS from now on) is one of the main research topics regarding Financial Economy, because it can be used for the prediction of macroeconomic magnitudes such as inflation, for the verification of several explanatory theories of IRTS, as well as for pricing fixed income securities, swaps and other derivatives.

The primary objective of this paper is to provide a daily term structure of interest rate in the European interbank market since the establishment of the European Economic and Monetary Union (EMU) on January 1st 1999.

With this aim, this paper is organised as follows: In Section 2 the background is described. Section 3 develop the approximating function. Section 4 shows the data and the methodology. In Section 5 the spot and forward rates are estimated for the period January 1999-March 2006. In the last sections

### RESUMO

El objetivo de este artículo consiste en proporcionar una estimación mensual de la estructura temporal de tipos de interés en el mercado interbancario europeo desde el establecimiento de la Unión Económica y Monetaria. Para ello nosotros aplicamos el método de bootstrapping de Fama y Bliss y utilizamos como función de aproximación la utilizada por uno de los métodos más utilizados por los bancos centrales nacionales para estimar sus estructuras temporales de tipos de interés, la del método de Nelson y Siegel.

### PALABRAS-CLAVE

Estructura Temporal de Tipos de Interés - Tipos de Interés del Mercado Interbancario - Tipos de Interés Swap - Euribor - Mercado Interbancario

conclusions and consulted bibliography are given. In order to achieve this aim, we use daily euribor rates and euro vs euribor swap interest rates. The data are collected from DataStream. According to the nature of these data, we use the most frequent method, the bootstrapping method. Later, we apply one of the most commonly approximating function used by central banks to estimate IRTS, Nelson and Siegel's (1987) and one objective function: minimization of the sum of squared errors of interest rates.

### 2. BACKGROUND

In general, the complete spot interest rate curve is not directly observable, so it is necessary to estimate it. Several methods have been developed to obtain spot rates from par yields available in the market. Among them, the most simple is boot-



strapping, based on a recurrent procedure which yields spot rates in successive dates. In order to apply this technique, it is necessary to adjust yields to a smooth curve, and linear or cubic splines are generally used.

Most of central banks apply Nelson and Siegel's and Swenson's methods to estimate IRTS. With this aim, information about government debt instruments is used because it refers to riskless issues. However, some countries, for instance Italy, Switzerland and Norway, use data from the money market to estimate the short term curve, given the limited number of observations in this term.

To estimate IRTS from the information given by financial markets, it is usual practice to use the following tools: short term interbank deposit rates, forward rate agreements (FRAs) or interest rate futures contracts for the middle area of the curve (1-2 years), and swaps on interest rates for the long term, exactly as is indicated in Ron (2000) and Alexander and Lvov (2003) for Canada and the United Kingdom, respectively.

However, FRAs for most currencies are not observable or suffer from lack of liquidity and extracting forward rates from futures rates, requiring a convexity adjustment due to the difference in convexity characteristics of future contracts and forward rates. So as to avoid this restriction and use more homogeneous data, we also use swap rates for the period 1-2 years of the curve.

The most usual method to estimate IRTS by applying data from the money market is to use the so called bootstrapping method. Various researchers have used this method. Mansi and Phillips (2001) propose a new method to estimate the par yield curve using on-the-run Treasuries. This model is compared with the Nelson and Siegel (1987) and Diament (1993) functional forms, and they conclude that the proposed model offers the better adjustment to most of the maturities. Jordan and Mansi (2003) also use five yield curve smoothing methods to calculate spot rates using on-the-run Treasuries. The methods considered use bootstrapping in discrete time (linear and cubic splines) and in continuous time (Nelson and Siegel, Diament and Mansi and Phillips). This paper differs from previous research in that it uses simulated and actual bond data generated from the Longstaff and Schwartz two-factor model. They conclude that the best methods are those based on bootstrapping techniques in continuous time. Bliss (1996) tests and compares five distinct methods for

estimating the term structure: the Unsmoothed Fama-Bliss, the McCulloch, the Fisher-Nychka and Zervos cubic splines, an extended model of Nelson and Siegel and the Smoothed Fama-Bliss. It concludes that the Smoothed Fama-Bliss is better than the others for fitting long-maturity term structures. Rendleman (2004) introduces a model based on cubic splines to estimate the interest rate term structures using the US swap curve, and verifies that it coincides with results obtained from the application of Longstaff and Schwartzs two-factor model.

We carry out the estimation of the interest rates in the European interbank market using the Smoothed model of Fama-Bliss (1996) that attempts to smooth out the discount rates by fitting an approximating function through them. With this purpose, we utilize the Nelson and Siegel (1987) exponential function. The description of this model is given in the following section.

Nelson and Siegel (1987) use data from Federal Reserve Bank of New York on every fourth Thursday from January 22, 1981, through October 27, 1983 (37 samples). They only take into account short term maturities (30 maturities for 34 dates and 31 maturities for 3 dates) and they make a prediction for a long term bond. They find a high correlation between the present-value of a long-term implied by the fitted curves and the actual reported price of the bond.

It is possible to highlight the following researches carried out by central banks to estimate the term structure of interest rates using the Nelson and Siegel (1987) method: Deacon and Derry (1994), Brooke et al. (2000) and Anderson and Sleath (2001) for the Bank of England; Núñez (1995) for the Bank of Spain; Ricart and Sicsic (1995) for the Bank of France; Seppälä and Viertiö (1996) for the Bank of Finland; Schich (1997) for the Deutsche Bundesbank; Bolder and Stréliski (1999) for the Bank of Canada; Csajbók (1999) for the National Bank of Hungary; Geyer and Mader (1999) for the Oesterreichische National Bank; Meier (1999) for the Swiss National Bank; Kloster (2000) for the Norges Bank and Arango et al. (2001) for the Bank of Colombia.

### 3. DESCRIPTION OF APPROXIMATING FUNCTION

Approximating function which is used is the Nelson and Siegel (1987) functional form because it is one of the methods most used by central banks nowadays to estimate the term structure of



interest rates. It is a static model which attempt to estimate the interest rate term structure at a specific time. It is also defined a parsimonious model because they lead to soft and flexible curves. With this aim, they consider that the instantaneous forward interest rate tends towards a constant value in the future, and assume that the instantaneous forward rate at any time  $t$  is given by the following functional form:

$$f(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1} \quad (1)$$

where:

$m$ : time to maturity.

Parameters to estimate have the following interpretation:

$\beta_0$  is a positive constant which reflects the value towards which instantaneous forward interest rate tends.

$\beta_1$  can be positive or negative and indicates that the first exponential term  $\beta_1 \cdot e^{-m/\tau_1}$  is monotonically decreasing if  $\beta_1$  is positive or increasing if  $\beta_1$  is negative.

$\beta_2$  can be positive or negative and shows that the second exponential  $\beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1}$  produces a hump if  $\beta_2$  is positive or a trough if  $\beta_2$  is negative.

$\tau_1$  are time constants associated with trend changes in the evolution of real interest rates. Small values of  $\tau_1$  correspond to rapid decay of the hump or trough towards the limiting value of  $\beta_0$ .

Moreover it works:

$$\lim_{m \rightarrow 0} f(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1} = \beta_0 + \beta_1 \quad (2)$$

$$\lim_{m \rightarrow \infty} f(m) = \beta_0 + \beta_1 \cdot e^{-m/\tau_1} + \beta_2 \cdot \frac{m}{\tau_1} \cdot e^{-m/\tau_1} = \beta_0 \quad (3)$$

so if the time to maturity converges to infinity the limiting of the instantaneous forward interest rate functional form is  $\beta_0$  and if the time to maturity approaches zero the limiting of the previous functional form is  $\beta_0 + \beta_1$ .

Spot interest rates can be represented by an average, as an average of forward interest rates. In continuous time this turned out to be the definite integral of the instantaneous forward interest rates:

$$r(m) = \int_{t=0}^{t=m} f(t) \cdot dt = \beta_0 + \beta_1 \cdot \left[ \frac{1 - e^{-m/\tau_1}}{m/\tau_1} \right] + \beta_2 \cdot \left[ \frac{1 - e^{-m/\tau_1}}{m/\tau_1} - e^{-m/\tau_1} \right] \quad (4)$$

$$r(m) = \beta_0 + (\beta_1 + \beta_2) \cdot \left[ \frac{1 - e^{-m/\tau_1}}{m/\tau_1} \right] - \beta_2 \cdot e^{-m/\tau_1} \quad (5)$$

Nelson and Siegel (1987) demonstrate that this functional form allows the representation of available shapes of the term structure. Parameters have the same interpretation as forward interest rates. It can be also verified that the curve can adopt several shapes depending on the relationship among parameters, as it is shown in Table 1.

**Table 1: Possible term structure shapes**

| Curve shape                 | $\hat{a}_0$ | $\hat{a}_1$ | $\hat{a}_2$ | $\hat{o}_1$ | Restriction                |
|-----------------------------|-------------|-------------|-------------|-------------|----------------------------|
| Increasing and concave      | +           | -           | +           | +           | $ \beta_1  \geq  \beta_2 $ |
| Increasing                  | +           | -           | -           | +           | $ \beta_1  \geq  \beta_2 $ |
| Decreasing and convex       | +           | +           | -           | +           | $ \beta_1  \geq  \beta_2 $ |
| Decreasing                  | +           | +           | +           | +           | $ \beta_1  \geq  \beta_2 $ |
| Hump, above $\hat{a}_0$     | +           | +           | +           | +           | $ \beta_1  <  \beta_2 $    |
| Hump, crosses $\hat{a}_0$   | +           | -           | +           | +           | $ \beta_1  <  \beta_2 $    |
| Trough, below $\hat{a}_0$   | +           | -           | -           | +           | $ \beta_1  <  \beta_2 $    |
| Trough, crosses $\hat{a}_0$ | +           | +           | -           | +           | $ \beta_1  <  \beta_2 $    |

#### 4. DATA AND METHODOLOGY

To estimate the IRTS in the European interbank market since the establishment of the European Economic and Monetary Union daily data relative to the most liquid issues on every last Friday (day of close of business) from January 1999 through March 2006 were taken, making 87 monthly samples in all. Data are collected from DataStream and are the following:

- ❖ Eonia.
- ❖ Euribor for maturities of three, six and nine months and one year.
- ❖ Euro vs Euribor swap middle rates for maturities of two, ten, fifteen, twenty, twenty-four and thirty years.

Later we have proceeded to homogenize data:

- ❖ Eonia and euribor are quoted on an actual/360 day count convention. However, DataStream provides the 1 year-euribor on an actual/365 day count convention. All these interest rates are converted to the 30/360 day count convention of euro vs euribor swap interest rate, according to ISDA (International Swaps and Derivatives Association, Inc).



$$i_{(360)} = \frac{360}{365} \cdot i_{1\text{year}(365)} \quad (6)$$

where:

$i_{1\text{year}(360)}$  : euribor-1 year on an actual/360 day convention.

$i_{1\text{year}(365)}$  : euribor-1 year on an actual/365 day convention.

- ❖ Eonia and euribor represent the simple interest rate on a short term loan without coupon, i.e. spot interest rates. Then we calculate the annually-compounded interest rate and the continuously-compounded interest rate with the following expressions:

$$i_c = \left(1 + i_s \cdot \frac{t}{360}\right)^{360/t} - 1 \quad (7)$$

$$r(t) = \ln(1 + i) \quad (8)$$

where

$i_c$ : annually-compounded interest rate.

$i_s$ : simple interest rate.

$t$ : time to maturity (in days).

$r(t)$ : continuously-compounded interest rate or instantaneous interest rate.

- ❖ The most liquid interest rates swaps are available only for two, five, ten fifteen, twenty, twenty-five and thirty years. Thus, we utilize an exponential interpolation to achieve an estimate of the missing swap rates for the maturities of three, four, six, seven, eight, nine, eleven, twelve, thirteen, fourteen, twenty-one, twenty-two, twenty-three, twenty-four, twenty-six, twenty-seven, twenty-eight and twenty-nine years. With these data the spot interest rates are calculated using the bootstrap method. With this aim, we apply the following expression:

$$i_t = \frac{C_t}{100 - \sum_{j=1}^{n-1} \frac{C_j}{(1+i_j)}} - 1 \quad (9)$$

where:

$i_t$  : spot interest rate at time  $t$ .

$C_t$  : coupon at time  $t$ .

$$C_t = 100 \cdot \text{YTM}_t$$

$\text{YTM}_t$  = Yield to maturity at time  $t$ .

Thus, we obtain the instantaneous spot interest rates with (11).

- ❖ Finally, we apply the Nelson and Siegel approximating function to estimate parameters by minimizing an objective function, the sum of squared deviations between estimated and

observed spot interest rates:

$$\text{Min} \sum_{j=1}^{35} \left[ (r_j - \hat{r}_j(\beta_0, \beta_1, \beta_2, \tau_1)) \right]^2 \quad (\text{Nelson and Siegel}). \quad (10)$$

where:

$r_j$  : observed instantaneous spot interest rate.

$\hat{r}_j$  : estimated instantaneous spot interest rate.

This model coincide with Smoothed Fama-Bliss (1996) method, which applies Nelson and Siegel function to estimate spot and forward interest rates using government bond data. However, we utilize money market interest rates. Parameters are estimated by an iterative extraction method and not directly from bond prices, as happens in Nelson and Siegel model. This is due to the fact that these models cannot be applied to swap interest rates because of the nature and features of data.

In order to obtain estimated spot interest rates we have used Visual Basic and a non-linear least squares optimization method, the algorithm quasi-Newton of Excel Solver Program.

It is necessary to start from an initial parameter vector in these two optimization criteria. We have assigned them the following values:

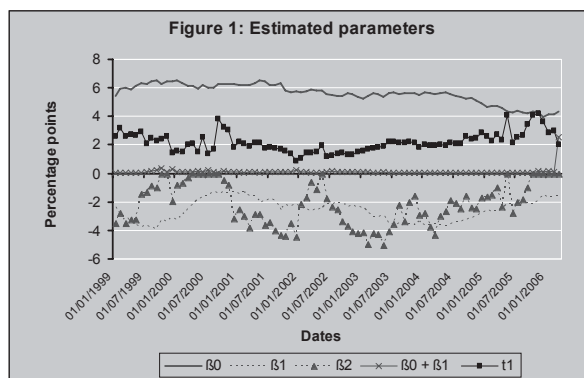
$\beta_0$  = 30 year-swap interest rate.

$\beta_1$  = Difference between the 30 year-swap interest rate minus the overnight rate. So as to fix the initial value of  $\beta_2$ , relationships established in table 1 are considered.

$\tau_1$  = Time when a trend change is observed in the spot observed interest rate.

## 5. RESULTS OF ESTIMATION OF THE INTEREST RATE TERM STRUCTURE IN THE EUROPEAN INTERBANK MARKET.

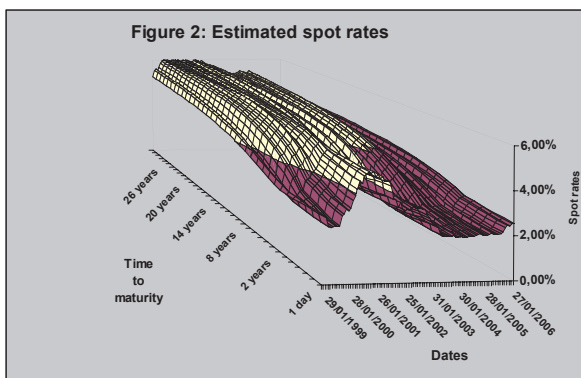
Fig. 1 shows the parameters estimated using the Fama-Bliss method. The advantage of this method is mainly the fact that, once parameters have been estimated, it is possible to analyse the evolution of the interest rate for any maturity. In general, in the long term the fit is rather good, the function is asintotic and changes in curve shape and slope are basically caused in short and medium term. The parameter  $\beta_0$ , which provides the long term interest rate is very stable. The variability logically appears in the short term ( $\beta_0 + \beta_1$ ). Also the spot and forward curves converge to parameter  $\beta_0$ .


**Figure 1: Estimated parameters**


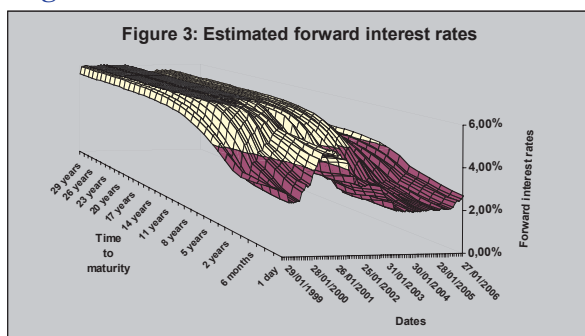
Source: Personal Research based on DataStream data.

The representation of the whole curves of interest rates allows the establishment of a monthly series composed of 87 samples of the period being studied.

Finally, Fig. 2 and 3 show selected term structures of interest rates in the European interbank market.

**Figure 2: Estimated spot rates**


Source: Personal Research based on DataStream data.

**Figure 3: Estimated forward interest rates**


Source: Personal Research based on DataStream data.

## 6. CONCLUSIONS

In this paper a monthly estimation of the interest rate term structure in the European interbank market since the establishment of the European Economic and Monetary Union is provided. With this purpose, the Smoothed Fama-Bliss method (1997) is applied, which attempts to smooth out the spot rates obtained by applying bootstrapping method. With this aim, we apply approximating function of Nelson and Siegel (1987) and an objective function.

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