REDIMAT
Journal of Research in Mathematics Education

Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

Workplace Mathematics Research: Reflections on Personal Practical Experiences

Nirmala Naresh[1] & Iman Chahine[2]

1) Miami University. United States of America
2) Georgia State University. United States of America

Date of publication: October 24th, 2013
Edition period: October 2013-February 2014

To cite this article: Naresh, N., Chahine, I. (2013). Workplace Mathematics Research: Reflections on Personal Practical Experiences. *REDIMAT – Journal of Research in Mathematics Education*, *2*(3), 316-342. doi: 10.4471/redimat.2013.34

To link this article: http://dx.doi.org/10.4471/redimat.2013.34

REDIMAT – Journal of Research in Mathematics Education Vol. 2 No. 3 October 24th 2013 pp. 316-342

Workplace Mathematics Research: Reflections on Personal Practical Experiences

Nirmala Naresh
Miami University

Iman Chahine
Georgia State University

(*Received: September 3rd 2012; Accepted: July 7th 2013; Published: October 24th 2013)*

Abstract

This article describes our transitions through three phases of a reflective cycle as a journey from the past to the future. In the descriptive phase, we delve into our past research experiences and address questions such as: What is the role of mathematics at work? In doing so, we uncovered additional venues for exploration that called for a new mode of analysis. We transition into a theory-building phase where we share our learning experiences that occurred in-the-moment. We then shift to an action oriented (reflexive) phase during which we construe personal practical theories that enable us to negotiate broader understandings of the role of mathematics at work and identify areas for future inquiry. We document our lived experiences as informed and inspired by our work with two groups of workers – bus conductors and street vendors.

Keywords: workplace mathematics, ethnomathematics, everyday cognition.

2013 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2013.34

REDIMAT – Journal of Research in Mathematics Education Vol. 2 No. 3 October 24th 2013 pp. 316-342

Investigando las Matemáticas en el Lugar de Trabajo: Reflexiones sobre Experiencias Prácticas Personales

Nirmala Naresh
Miami University

Iman Chahine
Georgia State University

(*Recibido: 3 de Septiembre 2012; Aceptado: 7 de Julio 2013; Publicado: 24 de Octubre 2013*)

Resumen

Este artículo describe nuestras transiciones a través de tres fases del ciclo reflexivo de una jornada desde el pasado hasta el futuro. En la fase descriptiva ahondamos en nuestras experiencias pasadas de investigación y examinamos preguntas tales como: ¿cuál es el rol de las matemáticas en el lugar de trabajo? Exploramos una nueva fase de construcción teórica donde compartimos nuestras experiencias de aprendizaje que ocurren en-cada-momento, para cambiar a una fase de acción orientada (reflexivamente) en la que se construyen teorías prácticas personales que nos permiten negociar significados más amplios del papel que juegan las matemáticas en el lugar de trabajo e identificar áreas para futuras indagaciones. Documentamos nuestras experiencias vividas con dos grupos de trabajadores –conductores de autobuses y vendedores de la calle.

Palabras clave: matemáticas en el lugar de trabajo, etnomatemáticas, cognición cotidiana.

2013 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2013.34

A shared interest in the research and practice of ethnomathematics brought the authors together in the joint endeavor of creating this manuscript. We first met at an international ethnomathematics conference – an informal chat centered on each other's teaching and research interests quickly turned into a focused discussion of our individual research projects on workplace mathematics. Post conference, our discussions transformed into a purposeful reflection rooted in our lived experiences as researchers of ethnomathematics – in particular, workplace mathematics. Unbeknownst to each other we had worked with two different groups of workers, during different time frames, and across diverse cultural settings. Using ethnographic case studies (ECS) we had documented the workplace mathematical activities of two different groups of workers, bus conductors and street vendors (Naresh, 2008; Jurdak & Shahin, 1999). As we began describing our research experiences and research findings, we noticed commonalties related to the nature of the mathematical activities of workers across two varied social and trans-national settings. These descriptive sessions led into knowledge-building sessions where we drew upon our shared-descriptions to conceive new research goals and explore new venues for enquiry. We then transitioned into an action oriented (reflexive) mode during which we construed personal practical theories that enabled us to go beyond our immediate research goals and to identify areas of further enquiry. In this paper, we share our journey through these three phases by situating our reflections in the overall context of our research projects. This reflecting-sharing process required us to revisit and relive our research experiences and engage in a critical analysis of our role as researchers and our explorations of everyday mathematics. Consequently, we discerned broader understandings of workplace mathematics as inspired by our work with bus conductors and street vendors.

Conceptual Framework

For our conceptual framework, we draw upon the theoretical and empirical fields proposed by Brown and Dowling (1998). While the research domains of *cognition in practice* (Lave, 1988) and *ethnomathematics* (D'Ambrosio, 1985) contribute to our theoretical base, the three-component reflective model (Rolfe, Freshwater & Jasper, 2001) serves as a useful empirical lens to discuss our work.

Theoretical Considerations

Investigations that have focused on studying people's use of mathematics outside the classroom (Gay & Cole, 1967; Scribner, 1986; Lave, 1988; Saxe, 1991) is divided into two main groups namely, those interested in "everyday cognition" or "cognition in practice", where Lave is a prominent figure, and those interested in "ethnomathematics", where D'ambrosio is a key figure. Both groups of researchers call for a new conceptualization of mathematics that is rooted in nonacademic practices. For instance, some research investigated the mathematical problem solving of Kpelle farmers of Central Liberia (Gay & Cole, 1967), while others examined the Southern California house wives' strategies for finding best buys in the supermarket (Lave, 1988). A second study explored the dairy workers' strategies for assembling products, pricing delivery tickets, and taking inventory (Scribner, 1986), while another inspected the arithmetic practices of Brazilian candy sellers (Saxe, 1991), and another investigated the mathematical ideas of carpenters in South Africa (Millroy, 1992). The work of these groups focuses on three main issues: analysis of school practice, investigation of the transfer of school knowledge to out-of-school situations, and using the social theory of practice to challenge conventional cognitive theory.

Workplace mathematics, a subset of the research field of everyday cognition and ethnomathematics investigates the mathematical practices of children and adults in various workplaces. This line of research sheds insight into how people conceptualize the role of mathematics in their work. The essential principle guiding such studies is the acknowledgement of the fact that people in several walks of life perform mathematical activities out of school, at home, and at work. However, adults who perform these activities usually do not make the mathematics used at their workplace explicit. In particular, Vergnaud (2000) claims "most of the knowledge used at the workplace remains implicit, sometimes unconscious" (p. xvii). However, the main issue here is not whether people perform mathematical activities at their workplace, but to find out the nature of mathematical knowledge used at work and how it is similar to and different from mathematics learned in school (Carraher, 1991). Different workplaces demand different mathematics from workers and such demands provide "challenges and opportunities" for people to develop meaningful mathematical knowledge (Carraher, 1991, p. 195). Thus, researchers have called for theoretical and empirical research to aid the understanding of the role of mathematics in the workplace. Relations between theory and practice are necessary for developing research methodologies and also for curriculum development (Millroy, 1992; Zevenbergen, 2000).

Empirical Considerations

We chose Rolfe's (Rolfe, Freshwater & Jasper, 2001) three component reflective model to position our work (see Fig.1). This model allows practitioners to reflect on their experiential and practical knowledge and enables them to develop a body of "experiential theoretical knowledge" (Rolfe, Freshwater & Jasper, 2001, p. 22). The three phases of this reflection cycle are labelled as *What?*, *So what?* and *Now what?* respectively. During *what* phase, a practitioner reflects on a specific situation or a past experience in order to describe it. In the *so what* phase, the practitioner goes beyond providing simple descriptions to experience new learning and glean additional insights about the past experience. During the *now what* phase, a practitioner articulates about the new theories that have emerged as a result of his/her participatory experiences and discerns implications for future practice.

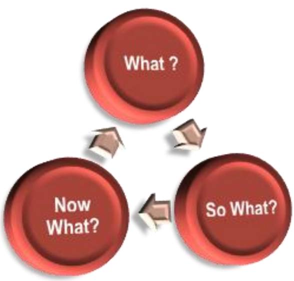

Figure 1. Rolfe's Reflective Cycle

We chose Rolfe's (Rolfe, Freshwater & Jasper, 2001) three component reflective model to position our work (see Fig.1). This model allows practitioners to reflect on their experiential and practical knowledge and enables them to develop a body of "experiential theoretical knowledge" (Rolfe, Freshwater & Jasper, 2001, p. 22). The three phases of this reflection cycle are labelled as What? So what? and Now what? respectively. During what phase, a practitioner reflects on a specific situation or a past experience in order to describe it. In the so what phase, the practitioner goes beyond providing simple descriptions to experience new learning and glean additional insights about the past experience. During the now what phase, a practitioner articulates about the new theories that have emerged as a result of his/her participatory experiences and discerns implications for future practice.

We view our transition through the three phases of the reflective cycle as a journey from the past to the future. Our reflections related to the What phase require us to delve deeper into our past research experiences – we launch this phase of our discussion by providing an overview of our ECSs. In particular, we attend to and describe salient aspects such as: the context of our research projects, the participants, research purposes, and key findings. During the So what phase, we reflect and act in the moment to view our participants' mathematical actions using an alternate theoretical lens. Specifically, we use components of Saxe's four-parameter model (1991) to view and exemplify participants' situational mathematical activities. We transition into the Now what phase by constructing personal practical theories (PPTs) that are rooted in our participatory experiences. We use these newly constructed PPTs to negotiate a broader understanding of workplace mathematics.

Methods of Inquiry

The main purpose of this paper is to reflect on our work and draw parallels related to our characters' goal-driven mathematical activities. To this end, we revisited the data sources related to our individual ECS. These data sources underlie the heart of our reflections and guided our work through each phase of the reflective cycle. For each stage of reflection, we drew upon and reflected on specific sources to frame and describe our assertions (see Table 1).

Table 1
Modes of inquiry

Stages of the reflective cycle	Guiding Questions	Data Sources (specific to ECS)	Researchers' reflections (rooted in an analysis of ECS data sources)
What?	What social contexts frame the ECS? Who are the participants? What did they do? What mathematical actions were described? What were the findings?	ECS study designs Research goals Participants' information Computational tasks & participant strategies Key findings	Face-face discussions Journal entries Online communications
So what?	What personal, experiential and theoretical knowledge base can we draw from? What new knowledge can we bring to the situation?	Field notes Observations Informal interviews and semi structured interviews	Face-to-face discussions Online communications (centered on a new framework of analysis)

Stages of the reflective cycle	Guiding Questions	Data Sources (specific to ECS)	Researchers' reflections (rooted in an analysis of ECS data sources)
Now what?	What personal practical theories have we constructed? How do we use our theories to negotiate broader understandings? Where do we go from here?	Researchers' practical experiences; introspections	Personal practical theories (centered on research experiences & reflective learning experiences)

What?

What Social Context(s) Framed the Case Studies?

We present an overview of two workplace mathematical investigations that are central to our reflections: a) Workplace mathematics of bus conductors in Chennai, India and b) An Ethnographic Study of the Computational Strategies of a Group of Young Street Vendors in Beirut. These ethnographic case studies not only contributed to the broader research fields of ethnomathematics and everyday cognition but also helped us gain insights into the workplace mathematical activities of the two groups. The overall goals of the ECS were to unravel, analyse and describe the mathematical ideas and decisions employed by the participants to solve work-related mathematical tasks.

Bus conducting, Chennai, India. Chennai is a metropolitan city in Tamilnadu, a state in South India. The major form of transport for people in

Chennai is the public transportation system of buses provided by the Metropolitan Transport Corporation (MTC). Bus conductors are employees of this state government organization. The state government expects MTC officials to find ways and means to increase their daily revenue. In turn, the MTC officials set targets revenue sales for their employees. The revenue thus generated fills the state treasury. A driver and a conductor operate every MTC bus. Unforeseen encounters with accidents, traffic delays, and strikes may result in loss in their daily allowance as well as the overall revenue for the MTC. When an untoward incident happens in the bus (e.g., theft, encounters involving rowdy passengers) or if the bus is involved in an accident, conductors face the wrath of common folks in addition to being reprimanded by their supervisors. It is against this complex backdrop that bus conductors engage in rapid-fire acts of mental computation whilst managing the simultaneous acts of fulfilling their employer's economic goals and satisfying their customers' service goals.

Street Corners in Beirut, Lebanon. Political and religious clashes have been inescapably notorious for inflicting insurmountable damage on the educational systems and intellectual advancement of countries wedged in such conflicts. Lebanon is of no exception. Caught amidst regional and international crises, Lebanese territories became an open ground for thousands of refugees many settling on the outskirts of the Lebanese capital Beirut. Throughout the refugees' camps, more than 60% of both Lebanese and other refugees live below the poverty line. Children suffer greatly- born into camps as refugees; they have lived no other way. With no other source of support, students have to work to support themselves and their families thus work as mechanics in auto repair shops, vendors of food or handmade goods and vegetable or what has been called the 'informal sector of the economy'. In this context, the child vendor is involved in a plethora of encounters that require highly sophisticated cognitive processes and decision-making skills.

Who Are the Participants? What Mathematical Activities Did They Do?

Chennai bus conductors are adults, who have at least completed their high school education, and are employed by the MTC, a government agency. Though bus conductors' official working hours are set at nine hours a day, often they end up working overtime (without pay) due to the uncertainty associated with the bus scheduling and completion of bus trips. Bus conductors' mathematical activities are intertwined with the following work-related activities: a) ticket transaction executions, b) completion of official records (e.g., enter and update ticket sales and revenue information) and c) determination of their daily allowance. To execute ticket transactions, conductors first determined the number of bus stops between the entry point and exit point (related to a commuter's trip), calculated ticket fares for single and multiple passengers, collected ticket fare from the passenger (s) and determined and issued the balance amount due to passengers, if any. In the official record, they calculated and entered the ticket collection amount for each ticket denomination and also determined the overall collection amount. The overall ticket collection amount was then used to determine the daily allowance based on the MTC guidelines.

Lebanese street vendors are young children who have experienced few years of school education and are self-employed. Some vendors worked alone while others worked with adult vendors or worked with their parents. Full time vendors worked on almost all days of a week averaging 10 hours per day. Depending on their years of experience in the vending business, vendors were responsible for either purchasing produce or pricing it for selling or both. As part of their daily encounters when street vending, children were involved in transactions that required them to make immediate decisions regarding the selling price of the produce. In order to sustain and survive the high competition prevalent in the open markets, vendors revised the predetermined selling price as and when needed during any given day. To accomplish this, young vendors relied on executing best strategies that will serve their potential goal at the end of the day, namely, having a significant profit to allow them to purchase a batch of produce the next day. Additionally, street vendors had to continuously engage in price negotiations with the customers as well as with other merchants when they buy wholesale produce to exhibit for vending.

Goals, Methods and Findings: A Snapshot

In line with qualitative methods of enquiry related to everyday mathematical investigations, we had employed a case study methodology to explore our research purposes. We discuss the specifics here. First, we negotiated entry into the work settings by working closely with gatekeepers (MTC officials, experienced senior street vendors). A purposive sampling was used to identify participants for the ECS. Over a period of three to six months, we used ethnographic methods to better understand the workplace dynamics. During this period, we shadowed our participants several times and identified those aspects of their work that required them to engage in mathematical activities. Specific work-related scenarios such as buying and selling produce, executing ticket transactions were carefully observed. Our observations helped us highlight several mathematical tasks that emerged in situ. The computational tasks that emerged related to the participants' acts of buying and or selling related artefacts (e.g. tickets, produce). Participants used language specific to their work to describe the tasks while we used mathematical language to interpret these tasks. During the informal and semi-structured interviews, we used a stimulated recall method to help participants reflect on their mental computational acts. They were encouraged to explain the strategies they employed to solve the computational tasks.

The overall goal was to uncover the mental mathematical components involved in the participants' work-related activities. To this end, we adopted a cognitive perspective and employed two analytical frameworks: (1) the Reformulation, Translation, and Compensation (RTC) model (Reys, Reys, Rybolt & Wyatt, 1982), and (2) theorems-in-action (Vergnaud, 1988). In Table 2, we provide a sampler of the tasks that emerged in context and the mental mathematical strategies that the participants used to solve these tasks. Note the usage of metric units and local currency (Indian rupee, Lebanese lira) in the tasks. We offer readers a glimpse of data here, as the main focus of this manuscript is to describe our reflective learning experiences discerned from these case studies. For a more detailed and thorough descriptions of the tasks and the mental strategies employed by participants we refer interested readers to the case studies (Naresh, 2008; Jurdak & Shahin, 1999).

Table 2
A snapshot of data

Context	Computational tasks	Participants' strategies (Researchers' interpretations)
Bus Conducting (Updating an official record)	Using ticket sales information for the 5.00 ticket bundles, determine the revenue earned. 838973 – 838999 845300 – 845320	First find how many tickets were sold: Bundle 1: 100 – 73 = 27; Bundle 2: 320 -300 = 20; 47 total tickets – Multiply this by 5. To find 47 x 5, first find 50 x 5 as 250; (Reformulation of the problem) Subtract 3 x 5 =15 from 250; (Compensate for the excess) 47 x 5 = (50 x 5) – (3 x 5) = 235.
Street Vending (Selling produce)	Ali was selling garlic, 4000 lira/kilo. How much would 5 ½ kilos cost?	Decompose 5 kilos into the sum ([(1 + 1) + (1 + 1)] + 1 + 1/2} kilos. Compute price through the same structural decomposition

Context	Computational tasks	Participants' strategies (Researchers' interpretations)
		{[(4000 + 4000) + (4000 + 4000)] + 4000 + 2000} = 22000 lira.

What Were the Key Findings?

Participants' computational activities were work-related and task-oriented. Their immediate work setting required them to avail and use mental computation. The non-availability of technological gadgets for computational use and the need for carrying out quick transactions necessitated and nurtured participants' mental computational acts. A purposeful analysis of participants' descriptions of computational strategies illustrated that they conceived and used cognitive functions that were situation-appropriate and those that made the most sense to them. On several occasions, they broke a main problem into sub-problem (s), devised extemporaneous mental schema that encompassed a variety of strategies. Although the participants did not name specific properties, based on their explanations we used labels such as decomposition, counting-up, repeated grouping, reformulations, compensation, translation, to describe their strategies. Logic mathematical properties, which denote the relationships that underlie the computational strategies, were evident in their solution processes. The logic mathematical properties also represent the "logic implicit in the problem solving behaviour of subjects like for instance, principles of identity, commutability, associativity, distributivity, and proportionality" (Jurdak & Shahin, 1999, p. 167). On the theoretical level, research evidence supported a model of mental computation as a vehicle for promoting thinking, conjecturing, and generalizing based on situational and conceptual understanding (Trafton, 1986), rather than the traditional view of such computation as a set of skills to be mastered (Naresh, 2008).

In the Aftermath of the Findings

We deliberately chose a cognitive perspective to address our research goals. The use of this perspective is justified because (at the time) our main goal was to uncover and describe the computational strategies implicit in participants' work. However, now we realize that, in using such a lens we had paid little attention to the contextual parameters that significantly influenced our participants' mathematical actions. New questions emerged: What if we could investigate our participants' mental computational processes using an alternate perspective – one that takes into consideration the influence of context and context related parameters? What theoretical perspective (s) would support this investigation? What would this analysis reveal? In light of these questions, we now feel compelled to revisit our ECS and related data sources to investigate the role of context and related parameters on our participants' mental computational acts. Through this investigation, we hope to explore questions such as: *What contextual parameters are unique to the context of bus conducting and street vending? How did these parameters impact participants' work-related mathematical activities? How did the participants make sense of a mathematical problem inherent in a seemingly non- mathematical scenario?*

So What?

What Personal, Experiential and Theoretical Knowledge Base Can We Draw From?

We sifted through observation notes, field reports, interview transcriptions, and journal entries from our ECS to carry out a purposeful analysis centred on our new research goals. We hoped to discern commonalities in the descriptions of the workplace characteristics and or the descriptions of mathematical activities of the two groups. We acknowledge that each workplace is unique, the demands at these workplaces fluctuate, and the complexities associated are many. Nevertheless, we envision the activities of street vending and bus conducting as analogous practices in which the participants (bus conductors and street vendors) engaged in goal-oriented actions directed toward the attainment of specific outcomes. We focus on such outcomes and claim that certain unique contextual parameters guided and impacted participants' goal-oriented mathematical actions. We attempt to parse these unique context-related parameters and describe if and how these parameters framed and influenced participants' mathematical activities at work. To this end, we turned to our theoretical base in search for an appropriate theoretical lens.

What New Knowledge Can We Generate?

Our main assumption is that participants' mathematical activities in everyday situations –in particular, at work – are influenced by their working conditions and context-related parameters. Thus, we needed a theoretical framework that would take into consideration the complex relationships between participants' mathematical activities and the settings in which they were performed. Saxe's four parameter model (1991) lent itself well to this purpose. This model offers a venue to explore goals and related goal-oriented actions in the context of work-related mathematical activities. *Emergent goals* referred are not fixed or static operations, but take form and shift as children and adults participate in everyday activities (Saxe, 1991). Depending upon what a situation demands, an individual constantly has to shape and reshape his/her goals and the ways in which those goals can be accomplished. Goals are interwoven with cognitive and sociocultural aspects of participants' functioning and are further explored using the following parameters: (a) activity structures, (b) conventions and artefacts, (c) social interactions, and (d) an individual's prior understandings.

Framework for Inquiry: Saxe's Four-Parameter Model

Every day, our participants reported to work expecting to complete all of their work-related duties. Mathematical activities are intertwined with many of their work-related duties. Associated with each of these mathematical activities are little goals that emerged and disappeared as the activities were initiated and completed. As Magajna and Monaghan (2003)

note, these emergent goals should be considered "as little I need to do… things" (p. 105) that are necessary to complete the task at hand. Emergent mathematical goals are guided by overall economic goals of the practice (Saxe, 1991). In the current scenario, the economic goals refer to the profit that the participants generate in their daily practices. Bus conductors' notion of profit refers to meeting or exceeding the target ticket sales. In the context of street vending, the goals of achieving high sales are guided by purposively employing a supply and demand model of price determination.
Examples of emergent goals are provided in Table 3. Similar goals surfaced and faded with the initiation and completion of additional transactions as well as other work-related mathematical activities. Emergent goals are not static phenomena –they are fluid, constantly changing and develop at the confluence of the four parameters exemplified in Saxe's model.

Activity cycle. Activity Structures refer to a series of cyclic tasks that must be accomplished to fulfil the emergent goals. Exemplars of emergent goals and related activity cycles specific to product transactions are provided in Table 3.

Table 3
Descriptions of Activity Cycles

Emergent goals	Activity cycle	
	Participants' description (contextual)	Researchers' description (computational)
Bus Conducting: Issue 5 tickets to a passenger going from A to B.	Initiate transaction; Gather trip detail; Validate trip details; Issue tickets.	Determine the actual ticket fare; Compare with passenger-paid fare; Issue balance amount back, if any.
Street Vending: Sell 4 kilos of produce to a client.	Initiate transaction; Receive a purchase order; Weigh the produce; Get paid.	Negotiate price to increase sales; Engage in dialogue to promote other produce; Exchange money;

Emergent goals	Activity cycle	
	Participants' description (contextual)	Researchers' description (computational)
		Barter to exchange produce instead of money.

In order to fulfil the emergent goals, participants completed several tasks set in activity cycle.

Participants' descriptions of activity cycles were strictly contextual. We introduced a mathematical dimension to the descriptions to highlight components of mental mathematics. Participants completed numerous tasks (set in similar activity cycles) to successfully complete other goals that emerged in situ. They had a thorough understanding of the "why" and "how" of each task embedded in an activity cycle and viewed each task "in the context of the whole activity cycle." (Magajna & Monaghan, 2003, p. 114) This holistic understanding, we claim, enabled them to synchronize their mental computational acts with the contextual activities and consequently fulfil related emergent goals.

Conventions and artefacts. Individuals who participate in a practice develop a set of conventions that may be unique to their particular situation, which significantly influence the ways they work and reason. We refer to artefacts as the tools employed by the participants in their practice. Our participants used tools specific to their workplace to complete the tasks set in activity cycles. Serial numbers on ticket bundles (of different denominations) helped bus conductors keep track of the number of tickets sold between each fare stages and during each trip. Similarly, non-standard weight measures such as rock and iron weights helped vendors monitor the amount of produce sold hereby adjusting the price to increase sales.

Often times, our participants interpreted the computational tasks in the context of money. This enabled them to view and interpret mental mathematical problems that involved whole numbers, decimals, and fractions into simpler problems by breaking them into bills (rupee, lira) notes and coins (paisa 250 and 500 liras) The availability and the use of monetary units highly influenced our participants' choice of mental strategies and also helped (them) complete incognito mathematical tasks mentally quickly and correctly.

Prior understandings. These refer to the experiential, contextual, and mathematical knowledge that individuals bring into practice. Conductors' prior understandings included knowledge about different bus depots and bus routes, fare stages along different routes, and knowledge about ticket fares. Street vendors' prior understandings comprised a keen awareness of the dynamics thriving in wholesale markets and the fluctuating prices of produce that is linked to the economic stability in the country. Participants' prior understandings also helped them manage "breakdown situations" (Pozzi, Noss & Hoyles, 1998) – situations that deviated from the normal routines. During these situations participants drew upon their prior encounters with similar situations to make on the spot decisions to decide the next course of action (see Table 4).

Table 4
Prior Understandings' in Play

Bus Conducting	Street Vending
Situation: A passenger boards the bus and asks for tickets to a destination that does not lie along that bus route.	*Situation*: A customer approaches the vendors to purchase a particular amount of produce.
Emergent goal: Negotiate with passenger and propose alternate courses of action.	*Emergent goal*: Negotiate with customer and propose higher sale by decreasing the price.
Draw upon prior understandings	*Draw upon prior understandings*
Suggest alternate route(s) that will get passenger closer to if not exactly to the destination.	Suggest alternate transaction in which the customer gets more produce for a decreased price.
Final outcome: Complete a ticket transaction to the passenger's satisfaction.	*Final outcome*: Complete a transaction and establish a connection with the customer to ensure future sales.

Social relationships. Social interactions refer to context specific conversations that help individuals construct, transform, and fulfil emergent goals. Participants' social relationships with their clients played a significant role in their mathematical enactments. Although the participants were the main actors in these scenarios, clients' role as *commuters* and *buyers* was equally important. Conductor participants continuously interacted with their clients to gather travel information, suggest alternate exit points (when needed), indicate points of exit, collect ticket fare, and issue the balance amount. Street vendors are continuously building and using social ties as a means of engaging in exchange activities and promoting their business. Participant-client interactions during breakdown episodes are the most interesting. On such occasions, conductors engaged in activities that required them to offer "spontaneous explanations and considerably more articulated (and therefore explicit) reasoning" to placate their customers (Hoyles & Noss, 2002).

Fresh perspectives

The aforementioned four parameters helped participants fulfil the emergent goals and related tasks quickly and efficiently. It is important to notice the interrelationships between the four parameters and its impact on our participants' successful completion of work-related duties. In order to complete the tasks set in the activity cycle, it was necessary to interact with customers. A thorough understanding of the dynamics associated with product transactions was required to conceive and complete activity cycles. During their interactions with the clients, participants summoned and used tools of the trade to execute tasks relevant to the activity cycle. Thus, the interrelationships between the four parameters are evidently exemplified in our participants' actions related to the emergent goals. The use of Saxe's model helped us understand participants' cognitive processes by taking into the consideration the sociocultural context inherent in their practice. We now envision the activities of street vending and bus conducting as on-going practices in which the participants engage in goal-oriented actions directed toward the attainment of specific outcomes.

Now What?

Personal Practical Theories

We now transition from the knowledge-building phase to an action-oriented, reflexive phase. Our participatory research experiences and personal learning experiences helped us address questions such as: *What is the role of mathematics at work? What counts as workplace mathematics? What is the insiders' perspective? Whose perspective matters?* Informed and inspired by our work with the participants, we construed personal practical theories related to the characteristics of the workplace settings, our participants' perceptions of work and workplace mathematics and our notion of workplace mathematics.

Structure of the work. In many formal work settings, a highly efficient workflow structure in place. For instance, consider the case of CAD/CAM technicians (Magajna & Monaghan, 2003). As part of their routine activities, these professionals designed and produced moulds for glass factories. Specific aspects of their work involved using prewritten computer programs for producing moulds and for determining the volume of the moulds. Participants in structured work settings know in advance where they work, whom they report to and what they will earn. In contrast, bus conductors and street vendors work in quasi-structured settings. It is possible to conceive their overall work structure as follows:

> Bus conductors: Report to duty – complete bus trips – negotiates trade transactions – End duty
> Street Vendors: Purchase produce – display and price produce – negotiate sales – complete transactions and sell out produce.

However, little do these participants know in advance the specifics of their daily work-related activities. All trade transactions involve two parties – workers and the clients. Workers do not know in advance the client's specific needs and or preferences. Based on what they demand, participants draw upon their prior understandings, summon and use one or more contextual parameters to fulfil trade transactions. Furthermore, in contrast to workers in formal settings, our participants' daily income is determined by the revenue they generate through ticket sales or profit generated through selling produce.

Nature of social interactions. In structured work settings, participants work in teams and work collaboratively to fulfil the job obligations. In many such settings, social interactions are limited to members within the practice. For example, in the case of nurses' work, they consulted fellow nurses when they had trouble interpreting information in the fluid balance chart (Pozzi, Noss, & Hoyles, 1998). In contrast, in informal settings (e.g. candy selling, newspaper vending) workers interacted with non-members of their practice. In the present case, social exchanges with clients (non-members) were central to our participants' workplace activities and played a crucial role in defining and shaping their mathematical actions. It must be noted here that the tone of participant-client interactions was not always pleasant. Our participants put up with such rude behaviour as they rely on the clients for their business. Another aspect of bus conductors' work that sets this group apart from many other groups concerns the nature of social interactions between the members of the practice. An atmosphere of mutual respect helped CAD/CAM technicians and their supervisors transcend the hierarchical structures and smoothly complete tasks and activity cycles (Magajna & Monaghan, 2003). With our participants, a sense of camaraderie permeated and prevailed among the members of the community. With our participants' work, we have a unique situation. Seldom do bus conductors closely work with each other or discuss the specifics of their work. Two conductors, who are very good friends, posted along same bus routes on different bus services become professional rivals instantly. They have to compete with each other to woo the same group of commuters in order to gain more earnings on their bus service. While off duty, street vendors freely mingled with their peers in a friendly manner. In the face of a precarious status and lack of support from the authorities, street vendors' main source of security is their family. Because of physical proximity and the length of time that they spend in the market, they talk to their fellow vendors; however, building trust among street vendors is not easy as they always find themselves competing to earn a living.

Participants' perceptions of work. Every day, our participants reported to work hoping to fulfil their duties to the best of their abilities. Their work-related activities were primarily guided by the desire to meet or exceed target economic goals. They were also genuinely concerned about completing the duties to their clients' satisfaction. Despite this commitment, most of our participants held lowly perceptions of the conducting and vending occupations. Mostly, they expressed frustrations about their fluctuating income, uncertainty associated with their daily activities, and the rude attitude of some of their clients. They lamented about the stigma associated with the 'conductor' and 'vendor' labels and about outsiders' perceptions of their jobs.

Perceptions of mathematics used at (in) work. Our participants were unaware that they were using mathematics in their everyday work practice. While we as researchers were alert to pinpoint and unfold the mathematical concepts that underlie goal-driven actions, these concepts were unnoticed or less acknowledged by the participants themselves. This was evidenced by the inferences we drew when the participants were asked about the nature of the actions and whether they have employed any mathematics while carrying out their daily work, the characters responded that they do not use any of these concepts in their work-related actions. When we shared a description of their solutions processes to situational computational tasks, they responded by pointing out that they just did 'simple mathematics' and not 'complex mathematics' used by other professionals such as 'accountants' or 'engineers'.

How Do We Use Our Theories to Negotiate Broader Understandings?

Immersed in the field, our participants have acquired work-related skills that contributed to their expertise in carrying out their work-related actions. This expertise emerged and developed as the characters engaged in daily actions over an extended period of time. The participants in our studies had a thorough understanding of the specifics of their work and related activities. For example, when conductors completed ticket transactions, they drew upon their understanding of work-related notions such as determination of fare stages, fare stage numbers, and ticket prices associated with fare stages. This *expertise* helped conductors determine the ticket fares and the balance amount due to the passengers and thus execute ticket transactions smoothly. For street vendors, knowledge of consumers and the street market had equipped them with strategies that will achieve quality in their investments and sales. We contend that our participants have contributed to a repertoire of knowledge which we term as 'conductors' mathematics' or 'mathematics of street vending' whose characteristics are shaped by the dynamics, context, and the tools specific to these workplaces. As researchers, we are guided by our research goals and argue that uncovering the mathematical ideas underlying the participants' acts have provided insights into their goal-directed activities in the workplaces. While such mathematical descriptions are warranted by our research purposes, it must be mentioned that the participants held independent, unrelated perceptions of the notion of workplace mathematics. Most of our workers are sole breadwinners of their families. Their primary motivation for work was their need and desire to provide for their family. Amidst tough working conditions, they continue to do their jobs constantly looking out for ways to maximise their daily income. Their acts at work are geared towards meeting this economic goal and seldom did they view themselves as doing mathematics at work. To the participants of the practices described here, the notion of mathematics in the workplace was irrelevant. According to them, 'they need to work and work well' so they can 'bring food to the table at the end of the day'. If (according to our interpretation) what they do at work is mathematics of their workplace then 'so be it' (our participants' reaction)! One of the conductor participant's comments vividly encapsulates this notion.

> You see my work and you see mathematics... In my job, I cannot afford to make mistakes It all depends on how you see it – like a stone or a sculpture… At the end of the day the job must be done and very well done. I have little time to think about anything else.

We conclude that to the participants in our studies, the mathematical activities are invisible in the press of their practice; in our perception it is an example of their ethnomathematics. In a similar vein, we also state that our participants' toils at the workplace are invisible (to us) in the press of our practice; in our participants' perceptions, we exemplify the persona of an outsider.

Our personal practical theories are framed by our practical research experiences and our personal learning experiences. The ideas portray our beliefs and highlight their influences on the decisions we make and the theories we propose. Our engagement in a critical analysis of our personal and practical experiences has revealed to us the true nature of our work as reflective researchers. We faced emotional and intellectual struggles as we went through the process of questioning our own research studies, its methods and the findings. Nevertheless, we have shared our deliberations with the larger research community because we firmly believe that

> This messiness, soul searching and sharing with the research community is certainly useful, in part, because it reveals that research is a journey that, while guided by the formal theory of research canons, is by nature fluid and troublesome and filtered through our personal practical theories of research (Cornett, 1995, p. 123).

Where Do We Go from Here?

Our journey as researchers was inspired and fuelled by the principles and practitioners of ethnomathematics. We grew up in our respective countries amidst the social practices of street vendors and bus conductors. However, in all those years, it never occurred to us to acknowledge the significance of their work – let alone engage in an inquiry of their workplace mathematics. Exposure to research on ethnomathematics (and its sub sets) has broadened our research interests and outlooks. Thus we see a value in investigating and learning from the mathematical practices of social groups engaged in non-academic work settings and view this form of research on par with traditional mathematics education research held in academic settings. However, that there is much more to be learned.

Our participants' perceptions about the role of mathematics at work have already raised a red flag and we propose the following question: *If and how do workers in some professional settings do complex mathematics compared to certain other groups*? We believe that for a more subtle and profound answer to this question, we need to look beyond the reported versions of mathematics of the workplaces and focus on questions such as: *What educational background is required of the workers in different settings? If and how does participants' background mathematical knowledge impact their workplace mathematical actions? What contextual factors are at play? How do these factors impact participants' professional goals?* Furthermore, we need to develop a greater awareness regarding the different perceptions of mathematics that is in existence and understand how such views shape and influence societal perceptions of workplace mathematics and those who practice it. Our journey as reflective researchers will continue; these new research purposes will guide our future reflections and subsequent research endeavours.

References

Brenner, M. E. (1998). Development of mathematical communication in problem solving groups by language minority students. *Bilingual Research Journal*, *22*(2), 149–174. doi: 10.1080/15235882.1998.10162720

Brown, A.J. & Dowling, P.C. (1998). *Doing Research/Reading Research: a mode of interrogation for education*. London: RoutledgeFalmer.

Carraher, D.W. (1991). Mathematics in and out of school: A selective review of studies from Brazil. In M. Harris (Ed.), *Schools, mathematics and work* (pp. 169-202). Bristol, PA: The Falmer Press.

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and oral mathematics. *Journal for Research in Mathematics Education, 18*, 83-97.

Cornett, J.W. (1995). The Importance of Systematic Reflection: Implications of a Naturalistic Model of Research. *Anthropology & Education Quarterly*, *26*(1), pp. 123-129. doi: 10.1525/aeq.1995.26.1.04x0790j

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44-48.

D'Ambrosio, U. (2007). Peace, Social Justice and Ethnomathematics. In B. Sriraman (Ed), *International Perspectives on Social Justice in Mathematics Education. The Montana Mathematics Enthusiast,* *Monograph 1*, (pp. 25-34). Retrived February 10,2012 from http://www.math.umt.edu/tmme/monograph1/d%27ambrosio_final_pp25_34.pdf.

Gay, J., & Cole, M. (1967). *The new mathematics and an old culture: A study of learning among the Kpelle of Liberia.* New York: Holt, Rinehart, and Winston.

Hoyles, C., & Noss, R. (2002). Problematising statistical meanings: A sociocultural perspective. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching of Statistics* (pp. 1-6). Retrieved February 10, 2011 from http://www.stat.auckland.ac.nz/~iase/publications/1/2e3_hoyl.pdf.

Jurdak, M., & Shahin, I. (1999). An Ethnologic Study of the Computational Strategies of a Group of Young Street Vendors in Beirut. *Educational Studies in Mathematics*, *40*(2), 155-172. doi: 10.1023/A:1003894908704

Lave, J. (1988). *Cognition in practice: mind, mathematics and culture in everyday life*. New York: Cambridge University Press

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Magajna, Z., & Monaghan, J. (2003). Advanced mathematical thinking in a technological workplace. *Educational Studies in Mathematics, 52*(2). 101-122. doi: 10.1023/A:1024089520064

Millroy, W.L. (1992). *An Ethnographic Study of the Mathematics of a Group of Carpenters* (Monograph 5). Reston, VA: NCTM.

Naresh, N. (2008). *Workplace Mathematics of Bus Conductors in Chennai, India*. PhD Dissertation. Illinois State University, Normal, IL, United States of America.

Noss, R. Hoyles, C., & Pozzi, S. (2000). Working knowledge: Mathematics in use. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 17-36). Dordrecht, The Netherlands: Kluwer Academic Publishers

Pea, R. D. (1991). Book Review. *Educational Studies in Mathematics, 22*(5), 481-490. doi: 10.1007/BF00367910

Pozzi, S., Noss, R., & Hoyles, C. (1998). Tool use in practice, Mathematics in use. *Educational Studies in Mathematics, 36*(2), 105-122. doi: 10.1023/A:1003216218471

Reys, B. J., & Barger, R. H. (1994). Mental computation: Issues from the United States perspective. In R. E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century* (pp. 31-47). Reston, VA: NCTM.

Reys, R. E., Reys, B. J., Rybolt, J. F., & Wyatt, J. W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education*, *13*, 183-201.

Rolfe, G., Freshwater, D., Jasper, M. (2001) (Eds.) *Critical Reflection for Nursing and the Helping Professions*. Basingstoke, U.K: Palgrave.

Santos, M., & Matos, J. F. (2002). Thinking about mathematical learning with Cabo Verde ardinas. In G. de Abreu, A. J. Bishop & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 81-122). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Saxe, G. (1991). *Culture and cognitive development: Studies in mathematical understanding.* Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Scribner, S. (1986). Thinking in action: Some characteristics of practical thought. In R. J. Sternberg & R. K. Wagner (Eds.), *Practical intelligence: Nature and origins of competence in the everyday world* (pp. 13-60). Cambridge, MA: Harvard University Press.

Shimizu, K., & Ishida, J. (1994). The cognitive processes and use of strategies of good Japanese estimators. In In R. E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century* (pp. 161-178). Reston, VA: NCTM.

Silver, E. A. (1994). Treating estimation and mental computation as situated mathematical processes. In R. E. Reys & N. Nohda (Eds.), *Computational alternatives for the twenty-first century* (pp. 147-161), Reston, VA: NCTM.

Trafton, P. R. (1986). Teaching computational estimation: Establishing an estimation mind-set. In H. L. Schoen & M. J. Zweng (Eds.), *Estimation and mental computation* (pp. 16-30). Reston, VA: National Council of Teachers of Mathematics.

Vergnaud, G. (1988). Theoretical frameworks and empirical facts in the psychology of mathematics education. In A. Hirst and K (eds.), *Proceedings of the Sixth International Congress on Mathematical Education*, ICMI, Budapest.

Vergnaud, G. (2000). Introduction. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. xvii-xxiv). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Zevenbergen, R. (2000). Research methods for mathematics at work. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 181-188). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Nirmala Naresh is assistant professor at the Department of Mathematics, in Miami University, USA.

Iman Chahine is assistant professor at the College of Education, in Georgia State University, USA.

Contact Address: Direct correspondence concerning this article should be addressed to the author at: 123 Bachelor Hall, Department of Mathematics, Miami University; Oxford, Ohio 45056, United States of America. E-mail: nareshn2@miamioh.edu.