REDIMAT
Journal of Research in Mathematics Education
Hipatia Press
www.hipatiapress.com

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Date of publication: February 24th, 2012

To cite this article: D'ambrosio, U. (2012). Mathematicians, Mathematics Educators and the State of the World. REDIMAT - Journal of Research in Mathematics Education, 1 (1), 5-28. doi: 10.4471/redimat.2012.01

To link this article: http://dx.doi.org/10.4471/redimat.2012.01

REDIMAT - Journal of Research in Mathematics Education Vol. 1 No. 1 February 2012 pp. 5-28.

Mathematicians, Mathematics Educators and the State of the World

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Abstract

This paper is, essentially, the transcription of my conference at the conference CIEAEM 63, promoted by the CIEAEM/Commission for the Study and Improvement of Mathematics Teaching, held in Barcelona, Spain, from 24 to 29 July 2011. The main objective of the CIEAEM, since its foundation, in the fifties, has been to analyze the actual conditions and the possibilities for the development of mathematics education in order to improve the quality of teaching mathematics. The annual conferences are the essential means for realizing this goal. The main theme of the annual conference of 2011 was "Facilitating access and participation: Mathematical practices inside and outside the classroom". I understand access and participation as Social Justice.

Keywords: Mathematics, Mathematics Educators, Social Justice.

ISSN 2014-3621
DOI: 10.4471/redimat.2012.01

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> Human well-being: A context- and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience. (ICSU, 2010)

For almost fifty years I have been discussing proposals of Mathematics for Social Justice, with different formulations, as the main focus of Mathematics Education.

I believe that Mathematics Educators are educators who regard Mathematics as an important instrument to prepare future generations to live in a World with Peace and Human Dignity for all. I see as a mistaken view to consider the main objective of Mathematics Educators to transmit Mathematics without reference to the ethics of its uses. Although Mathematics is taught with the declared intention that it will be useful for everyday life, Mathematics Educators cannot ignore the fact that the most successful students may be engineers who design lethal weapons or reinforce the practices of brutal capitalism. Without a clear understanding of how Mathematics can help in attaining Peace and Human Dignity for all, hence Social Justice, Mathematics Educators may miss an important ethical responsibility.

In 1976, in my controversial discussion paper on "Why do we teach Mathematics?" in the Third International Congress of Mathematics Education/ ICME 3, in Karlsruhe, Germany, I said:

> We see the educational process as the conjugation of global socio-economic aspects aiming at the betterment of the quality of life. In this conjugation intervene, the same as in the technological process, the philosophy to which society subscribes, as well as considerations about manpower and available material resources." (D'Ambrosio, 1976, p. 224)

Willy Servais was one of the members of the Panel discussing this paper. It is very present in my memory the ardent support he gave to the basic issues raised in that paper. He entirely supported the broader view on Mathematics Education, which I presented in the paper. This

encouraged my continued research looking for objectives of Mathematics Education that go beyond excellence in the transmission of established academic Mathematics.

I interpret quality as the appropriate response to the pulsions of survival and transcendence. Thus, quality becomes all pervading and aims at human well being as conceptualized in the opening caption. I believe Mathematics has been, in the evolution of the human species, an efficient instrument to attain this aim. Hence, I understand quality of teaching mathematics as conveying this essential feature of Mathematics.

Almost twenty years later, in 1993, the 15th conference of the PME-NA/North American Chapter of the Psychology of Mathematics Education, in Pacific Grove, CA, I went further in these ideas:

> Although the main concern of this meeting is Mathematics Education, I believe I will be allowed to subordinate my comments to a higher objective: the survival of civilization on Earth with dignity for all. This is not merely jargonizing. The world is threatened, not only by aggressions against nature and the environment. We are equally concerned with increasing violations of human dignity. We face more and more cases of life under fear, hatred and violation of the basic principles upon which civilization rests. (D'Ambrosio, 1996, p. 31)

In other writings, I ask for a new thinking in mathematics education. My objective, in this paper, is to stress the fact that our most urgent concern is to teach Mathematics for access and participation, understood in the broad sense of helping humans to attain well-being, which comprise the basic components of a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience.

Mathematics is present in all the major achievements of civilization. Advances in mathematics are associated with progress. But, paradoxically, mathematics has been the main instrument in weaponry and in economics. I have often referred to mathematics as the imprint of modern society, for good and evil. As historian Mary Lefkowitz says, "the evolution of general mathematical theories from those basics

[mathematics of Egyptians, Sumerians and others] is the real basis of Western thought (italics mine)."[1]

As a consequence of wars, of greedy capitalism and of uncontrollable consumerism, people are killed in a broad sense, either physically or morally, as the termination of life and also as the loss of dignity.

I understand violation of Social Justice in this broad conception. As political scientist Glenn D. Paige says,

> "There are no social relationships that require actual or threatened killing to sustain or change them. No relationships of dominance or exclusion—boundaries, forms of government, property, gender, race, ethnicity, class, or systems of spiritual or secular belief—require killing to support or challenge them. This does not assume that such a society is unbounded, undifferentiated, or conflict-free, but only that its structure and processes do not derive from or depend upon killing. There are no vocations, legitimate or illegitimate, whose purpose is to kill. (Paige, 2002, p. 30)

I go even further in this broad vision of violation of Social Justice. A form of killing is to deprive human beings of their means of subsistence, such as food, air and water. Hence, the sustainability of the environment is to be considered a major issue when our goal is to avoid the great threat of global famine.

The Charter For a World Without Violence, endorsed by Mikhail Gromov Nobel laureates ends with the appeal:

> To address all forms of violence we encourage scientific research in the fields of human interaction and dialogue, and we invite participation from the academic, scientific and religious communities to aid us in the transition to non-violent, and non-killing societies. (Raussen, & Skau, 2010, p. 395)

Mathematicians and mathematics educators are among the addressees. How do we respond to this appeal?

The State of the World and Mathematics

Mathematician Mikhail L. Gromov, laureate with the Abel Prize 2009, says:

> Earth will run out of the basic resources, and we cannot predict what will happen after that. We will run out of water, air, soil, and rare metals, not to mention oil. Everything will essentially come to an end within fifty years. What will happen after that? I am scared. It may be okay if we find solutions, but if we don't then everything may come to an end very quickly. Mathematics may help to solve the problem, but if we are not successful, there will not be any mathematics left, I am afraid! (Raussen, & Skau, 2010, p. 403)

I am also afraid. What kind of world are we leaving to the future generations? The future may not be. All our proposals for better educating the future generations may be voided. The tensions within our contemporary societies, both intranational and international, add to the feeling of scare and fear. As mathematicians and mathematics educators we have a responsibility with the future. We have to find ways to both recognize and respond to this responsibility.

Christiane Rousseau, the Vice-President of Executive Committee of IMU/International Mathematics Union, a pure mathematician, speaks about her growing interest in the science of sustainability and the attention IMU is devoting to this, as response to an appeal of the ICSU/International Council of Scientific Unions. She says:

> While it is not new that scientists are involved in the study of climate change and sustainability issues, a new feeling of emergency has developed. The warning signs are becoming more numerous that urgent action is needed if we want to save the planet from a disastrous future, since we may not be far from a point of no return: climate change with more extreme weather events, rising of the sea level with the melting of glaciers, shortage of ood and water in the near future

> because of the increase of the world population and the climate change, loss of biodiversity, new epidemics or invasive species, etc.[2]

A new 10-year research initiative, the Earth System Research for Global Sustainability, sponsored by ICSU, has the following goals:

> (1) Deliver at global and regional scales the knowledge that societies need to effectively respond to global change while meeting economic and social goals;
> (2) Coordinate and focus international scientific research to address the Grand Challenges and Belmont Challenge;
> (3)Engage a new generation of researchers in the social, economic, natural, health, and engineering sciences in global sustainability research.

There are references to two important proposals of ICSU. The Belmont Challenge and the Grand Challenges. The Grand Challenges are:

> Challenge #1: Improve the usefulness of forecasts of future environmental conditions and their consequences for people.
> Challenge #2: Develop the observation systems needed to manage global and regional environmental change.
> Challenge #3: Determine how to anticipate, avoid and cope with dangerous global environmental change.
> Challenge #4: Determine what institutional and behavioral changes can best ensure global sustainability.
> Challenge #5: Develop and evaluate innovative technological and social responses to achieve global sustainability.

This leads to Priority Research Questions for all areas of Science. Details can be seen in the site of ICSU[3]. Among the many expected outcomes, I distinguish:

> Human well-being: A context- and situation-dependent state, comprising basic material for a good life, freedom and choice, health and bodily well-being, good social relations, security, peace of mind, and spiritual experience.

I see in the expected outcome of Human well being the quintessence of the objective of Social Justice, as stated in the beginning of this paper. The Belmont Challenge complements the Grand Challenges and aims at delivering knowledge to support human action and adaptation to regional environmental change[4].

A Conference on Sustainable Development (Rio+20), organized by the United Nations in June 4-6, 2012, in Rio de Janeiro, will have the participation of all the disciplines. Mathematics plays a transdisciplinarian role in these discussions. Indeed, Mathematics in deeply involved in the interdisciplinary research that needs to take place in preparation for the conference. Considering the importance role of mathematics in the themes of the conference, the IMU has endorsed the broad project Mathematics of Planet Earth 2013. This endorsement states:

> Earth is a planet with dynamic processes in the mantle, oceans and atmosphere creating climate, causing natural disasters, and influencing fundamental aspects of life and life-supporting systems. In addition to these natural processes, humans have developed systems of great complexity, including economic and financial systems; the World Wide Web; frameworks for resource management, transportation, and energy production and utilization; health care delivery; and social organizations. Human activity has increased to the point where it influences the global climate, impacts the ability of the planet to feed itself and threatens the stability of these systems. Issues such as climate change, sustainability, man-made disasters, control of diseases and epidemics, management of resources, and global integration have come to the fore. Mathematics plays a key role in these and many other processes affecting Planet Earth, both as a fundamental discipline and as an essential component of multidisciplinary and interdisciplinary research[5].

As educators, we must respond to this. A number of mathematics educators, including Nicoletta Lanciano, of the Department of Mathematics of the Università di Roma "La Sapienza", have

developed a project, independently of the IMU project, to involve schools with the major objective of showing the students local specificities in the Planet Earth. The Project Parallel Globe http://www.globolocal.net/ tries to recreate, for students, important stages in the development of mathematics, which are related to the observation of the skies.

The Parallel Globe helps students to visualize their position on the Earth surface in relation to the position occupied by other countries, recurring to observing the way the Sun illuminates different regions of the Earth in real time. This helps to understand time zones and the alternation of the seasons on the planet. This tool of didactic practice is rich in mathematics contents, allying observation, concrete experimentation with reflection, and data collection. In the development of the Project, it is fundamental to share the results from countries of different longitude and latitude, thus helping to clarify the semantic and symbolic differences of the distinctions North-South, top-bottom, over-under, up-down in different languages and cultures. Intercultural awareness has obvious political implications for a democratic perception of globalization, helping the respect for difference and the recognition that all nations are part of the same global system which is threatened. To convey to the students the message that civilization is threatened and that all nations share different but common environmental conditions and that mathematics is an important instrument to monitor the conditions is the main objective of the project.

Challenge to Mathematics Educators and the preparation of teachers

A remarkable conference on "Visions in Mathematics – Towards 2000" was held in Tel Aviv from August 25th to September 3rd, 1999, considered one the most important reunions of leading mathematicians worldwide to discuss past and future mathematics in the eve of the 21st century, its importance and methods. In this conference Gromov gave an address where he points to new directions for the development of mathematics, resulting from the socio-cultural context instead of the conceptual necessities and details intrinsic to established mathematics theories. We need another Mathematics. He calls these new

mathematical structures "soft", since they consist of greatly flexible hypotheses. These remarkable ideas, although very difficult, clearly indicate that the new generation of scientists, engineers and, obviously, mathematicians, will need broader attitudes towards mathematics.

The challenging problems require, besides new mathematical techniques, the training of a new generation of researchers in the mathematical sciences. Again, quoting Gromov:

> We shall need for this the creation of a new breed of mathematical professionals able to mediate between pure mathematics and applied science. The cross-fertilization of ideas is crucial for the health of the science and mathematics. (Gromov, 1998, p. 847)

All the considerations above are mainly addressed to research mathematicians, but it is undeniable that it poses an even great challenge to mathematics educators.

It is questionable if we should insist in keeping contents which are consuming school time and energy instead of moving more rapidly into the new concepts of mathematics, as suggested by Gromov and others. The same question is applicable to the new Physics, the new Biology and other scientific fields. It is undeniable that this new face of Mathematics is more attractive to students. The digital natives feel that the traditional mathematics that still dominates the curricula are obsolete, boring and useless. I am convinced that this is the main cause of the bad results in tests.

The new Mathematics depends, of course, on basic mathematics. But to what extent shall we insist on the basics? Thanks to the amazing technology available, it is possible to accelerate the acquisition of the basic mathematics, which is necessary – a small part of what is in the usual programs -- and to step, rapidly, into the new mathematics. The basic includes mainly concepts, not techniques. Curricular development should focus on accelerating the teaching of what is effectively basic in traditional mathematics, which are concepts. Instead, much of the time and energy of teachers still goes into insisting on skills.

Mathematics, as a science, has specificities. According to Steve

Kennedy:

> Math is different from the other sciences. In a very real sense the problems, motivations and verification of mathematics come from inside the discipline itself, whereas the other sciences look to the world of phenomena for problems and affirmation. The chemist whose experiment yields a result within six decimal places of his theoretical prediction has good reason to feel pretty pleased with his theorizing. A mathematician rarely finds herself in such an empirically happy place vis-à-vis her theories. Usually a mathematician has only the cold reassurance of logic for comfort; the universe does not deign to validate our work except indirectly, when the work proves useful as a model in another science. (Kennedy, 2003, p. 180)

The difficulty is to bridge the gap between internal advances of mathematics and their utilization. Language is a key obstacle. As Felix Browder says:

> The richer the repertoire of modern mathematical research, the broader the arsenal of concepts and tools available for the use of the mathematicized sciences. The difficulty lies in the problem of communication, of the scientific practitioners being able to penetrate through the difficulties of translation between the languages of different disciplines, of knowing what is relevant in the concepts and techniques that are available." (Aspray, & Kitcher, 1988, p. 29)

To approximate mathematics to the sciences is to show, in Mathematics Education, that Mathematics in fully integrated with the scientific method, which is an essential component of multidisciplinary and interdisciplinary research.

This is intrinsic to the proposal of laboratory practices in Mathematics Education by Eliakim H. Moore in the beginning of the 20th century. For example, when he says:

> The boy will be learning to make practical use in his scientific investigations - to be sure, in a naive and elementary way - of the finest mathematical tools which the centuries have forged; that under skilful guidance he will learn to be interested not merely in the achievements of the tools, but in the theory of the tools themselves, and that thus he will ultimately have a feeling towards his mathematics extremely different from that which is now met with only too frequently - a feeling that mathematics is indeed itself a fundamental reality of the domain of thought, and not merely a matter of symbols and arbitrary rules and conventions. (Moore, 1903, p. 408)

More than proposing a short cut, Moore proposes restoring Mathematics Education to the original roots of mathematics development in Modernity. The advances proposed since the 16th century recognizes Mathematics as the main support of scientific inquiry.

Examples of short cuts to present advanced mathematics in a simple and contextual way are the proposals exposed in the books Calculus Made Easy, published in 1910, by Silvanus P. Thompson, F.R.S., generally repudiated by mathematicians (Thompson, 1914), and Lectures on Physics, of 1962, by Richard P Feynman (Feynman, Leightn, & Sands, 1988).

In both books, authored by distinguished scientists, non-mathematicians which are users of advanced mathematics, contents are rapidly presented, with adequate rigor for its purpose. To find the equilibrium between accessible presentation and acceptable rigor is a major challenge to Mathematics Educators.

The greatest challenge to mathematics educators is to perceive these changes, to understand the new and to develop methods for transmitting this to teachers.

Children must be prepared for a future that we cannot envisage. To prepare children to be proficient in obsolete mathematics is to prepare them to the anguish of being marginal in the future, because they possess outdate knowledge. To avoid this anguish is, to me, an important feature of Social Justice.

Social Justice should be understood as a response to satisfying the

basic needs for a good life, aiming at freedom and choice, at health and bodily well-being, and establishing at good social relations, anchored on security, peace of mind and respect for spiritual experience. We must avoid giving students the illusion that passing the current tests, obtaining good grades, they are prepared for the future. This is fallacious and the denial of Social Justice.

The inadequacy of tests is not new. Évariste Galois very clearly denounced this, over two hundred years ago:

> Are you quite happy to do well in the test? Do you believe you will be finally appointed as one of the two hundred geometers that will be admitted? You believe you are prepared: you are mistaken, this is what I will show you in a next letter. (Galois, 1831)

He died before writing the next letter. Education, in this era of science and technology, challenges the established approaches "validated" by results in standardized tests. The goals of education go much beyond merely preparing for professional success. Education has a responsibility in building up saner attitudes towards the self, towards society, towards nature.

We are primarily faced with the preparation of teachers to assume a different attitude in their teachings, responding to the greatest challenge proposed above. Educators must be creative.

I believe the key problems in the preparation of teachers of mathematics are related to inadequate visions of the purposes of education and of the role of mathematics teachers as educators. Prospective and in-service teachers of mathematics should be always reflecting about the changes in education, as a consequence of profound changes in society, particularly in the demographic scenario, in production, in information, in communication and in the environment.

I will elaborate on the purposes of education as a preliminary to discussing the role of the mathematics teacher as educators. I identify a double purpose why societies establish educational systems:

> (1) to promote citizenship (which prepares the individual to be integrated and productive in society), which is achieved by transmitting values and showing rights and responsibilities in society; and
> (2) to promote creativity (which leads to progress), which is achieved by helping people to fulfill their potentials and rise to the highest of their capability.

The practice of education is in the present. The major challenge to educators is to manage, in this process, the encounter of the past and of the future, that is:

> (1) The transmission of values rooted in the past, which leads to citizenship,
> (2) The promotion of the new, for an uncertain future, which means creativity.

But in this process, we must be careful. We do not want:

> (1) To transmit docile citizenship – we do not want our students to accept rules and codes, which violate human dignity, to be permanently frightened, we want them to assume a critical attitude towards obedience.
> (2) Nor to promote irresponsible creativity – we do not want our students to become bright scientists creating new instruments to increase inequity, arrogance and bigotry; we want them to be conscious of their acts and of the consequences of their creation.

Hence, the goals that I hold important in Education, hence in Mathematics Education, are:

> (1) The transmission of values rooted in the past, which leads to citizenship, but not docile citizenship;
> (2) The promotion of the new, for an uncertain future, which means creativity, but not irresponsible creativity.

The transmission of values is intrinsic to cultural encounters. Cultural

encounters have a very complex dynamics. This encounter occurs between peoples, as occurred in the conquest and colonization, between groups. It also occurs in the encounter between the young man or woman, who have their own culture, and the culture of the school, with which the teacher identifies. The so-called civilizing process, carried on by colonizers, is essentially the management of this dynamics. I claim the same occurs in the educational process. Didactics and pedagogy are strategies to manage cultural encounters of students and teachers.

An important component of Mathematics Education is to reaffirm and, in many cases, to restore cultural dignity of children. Much of the contents of current programs are supported by a tradition alien to the children. On the other hand, children are living in a civilization dominated by mathematically based technology and by unprecedented means of information and communication, but schools present an obsolete worldview.

It is equally important to recognize that improving the opportunities for employment is a real expectation that students and parents have of school. But preparation for the job market is indeed preparation for the capability of dealing with new challenges.

There are many careers, which require different kind of knowledge and experience that remain unfilled because of lack of able candidates. There is a need for change. But what to change and how to change? Ideally, the advances of research in Mathematics Education produce better-qualified teachers, capable of promoting innovative education. But, regrettably, the focus on passing tests dominates school systems. Many teachers are attracted by rewards, such as salary increase, if their students are successful in the tests. Schools support this practice, because they are rewarded with grants and other government subsidies. This is a subtle form of corruption, which paves the way to explicit corruption, a flagrant violation of Social Justice. Responsible governance should look carefully at the disequilibrium among preparation of graduates and the needs of the job market. This was extensively discussed, some years ago, by Robert Reich[6].

Education for all, which is frequently given as a strategy for Social Justice, has many problems and the fact that more and more people are becoming educated, with emphases in science, technology and

engineering, sounds like a good thing. It is, indeed, a progress. But it is an illusion that this is the key to economic growth and prosperity and good jobs. We have to analyze the context in which this progress takes place and the fitness and quality of it. There is no point in preparing children for jobs that will probably be extinct when they reach adulthood[7].

Education for all results in an extraordinary amount of people going to school with the hope of finding good jobs. But there are reasons for caution. The expansion can dilute the quality of graduates, giving space to less able individuals into the system. Bright students are poorly employed and they may be soon disillusioned by the ruthless and often fruitless fight for a permanent job. There is a need of research with the objective of finding out how the labor market will accommodate those who emerge from the school systems. There are results on this, but as yet many programs remain firmly attached to the traditional curricula, disregarding the disequilibrium among preparation of graduates and the needs of the job market.

In a seminar in the UNESCO Institute for Information Technology in Education, in 2001, Seymour Papert denounced the enormous amount of resources that are wasted in obsolete education:

> Using computers connected to the Internet students can obtain better and quicker access to sources of historical as well as scientific knowledge; they can explore economics as well as physics by making models and simulations; the rigor of mathematics can be extended to areas that were previously inaccessible. But in the midst of these explosions of change the institution of School has remained as remarkably constant over time as it is across countries. So why am I wasting time drawing attention to familiar facts and problems that are already being addressed? The answer is saddening: Although the problem is widely recognized, its depth is seldom appreciated. Most of those billions of dollars are being wasted (Papert, 2001)[8]

Indeed, this means that much of the traditional contents which exhaust current programs should be drastically changed. It may be a big

mistake to insist on mathematics curricula simply because they satisfy criteria of rigor. Some defend that the satisfaction of such criteria are enough to justify contents. Curriculum proposals are frequently disguised as new methods to teach the same contents, mostly inappropriate and obsolete. Much cost and energy is devoted to show how to do better what is disinteresting, obsolete and useless, as denounced by Seymour Papert in the quote above.

These remarks may be interpreted by many as suggesting a reduction of the importance of mathematical contents. This is a gross mistaken interpretation. We need more and better mathematical contents, but not the same contents. What I say is that methodological innovation should be directed to making advanced mathematics attractive and teachable. Compromising rigor, in benefit of generating interest and motivation, cannot be interpreted as conceptual errors neither as relaxing the importance of serious mathematics in schools.

Self-esteem is essential to venture and to propose the new. Self-esteem goes together with cultural dignity. Both, to acquire cultural dignity and to be prepared for full participation in society, requires more than what is offered in traditional curricula. This is true both for students and for teachers. Particularly serious is the situation of Mathematics, which is largely obsolete as present in the programs, both in schools and in teacher preparation.

Classroom mathematics has practically nothing to do with the world children are experiencing outside of school. Children consider mathematics as something above their reach and out of their world. It is important to listen again to Paulo Freire:

> In my generation of Brazilians in the Northeast, when speaking of mathematics, we were speaking something about gods. (Freire, D'Ambrosio, & Domite, 1997, p. 8)

Indeed, a divine character is attributed to Mathematics and this goes back to Ancient texts, in several cultural traditions (D'ambrosio, 2010). It important to recognize, as said above in this paper, that Social Justice can not ignore spiritual experience, which was the subject of extensive research by Klaus Witz (2007).

Paulo Freire championed in his writings the idea that Social Justice cannot be achieved without literacy. As he recognized, to be literate means, nowadays, much more than reading and writing. Other codes are essential in daily life, such as the manipulation of numbers and the basic operations. Mathematics is more than these skills, as I will discuss later.

Mathematics and mathematics education in a changing civilization

All the spectacular achievements of science and technology have their bases in mathematics. And the institutions of modern civilization, mainly economics, politics, management and social order, are rooted in mathematics. Consequently, both Mathematics, as a research field, and Mathematics Education, are changing. A new thinking is needed for education in the civilization that goes through so many changes (D'ambrosio, 2011).

Mathematics is a fascinating cultural endeavor. It is seen as the imprint of rationality and, indeed, it is the dorsal spine of modern civilization. No surprise that accomplished scholars are devoted to mathematics. A good number of successful citizens, who did not accomplish well in mathematics in their school years, sometimes even failed, are fiduciary of mathematics in the educational systems.

Administrators, teachers, parents, students, and the population in general, see mathematics as the principal subject in schools. Society regards those who do well in mathematics as geniuses, and those who fail are stigmatized. There is a lack of recognition that there are different interests, different creativity, and different talents among different individuals, particularly among different children. Some do well in mathematics and do poorly in the manual arts, while other do poorly in mathematics and do well in the manual arts. It is an absurd to value those that do well in mathematics and do poorly in the manual arts and humanities over those that do poorly in mathematics and well in the manual arts humanities. Those in the first group are identified with the intelligent, the intellectually capable, while the others are regarded as performers. Mathematics acts a selector on intellectual elites. These elites will pursue the same pattern of society, impregnated with arrogance, inequity and bigotry, which is a clear violation of Social Justice.

When looking at mathematics education, we may identify two positions:

> (1) To use education as a strategy for teaching mathematics, defended by those described in the two paragraphs above;
> (2) To teach mathematics as a strategy for good education.

I like to use a metaphor. I recognize that the great energy we have in the planet, both physical and intellectual, creative, comes from children. Metaphorically, I see children as our Sun. Position 1 sees Mathematics presented as a discipline cold and austere, in the words of Bertrand Russell[9]. Position 1 implies children, which are full of energy, like the Sun, revolving around the cold and austere focus of Mathematics, metaphorically cold and austere as the Earth. Thus, I call Position 1 the Ptolemaic version of Mathematics Education.

I am fully identified with Position 2. The focus of our mission as educators reside in children, young adults, elderly adults, in general those who are the reason and the source of energy for educational action. In this Copernican view, the disciplines, which revolve around those being educated, are merely instruments in this action action. Disciplines are, thus, in permanent reformulation, reflecting social and cultural context and the queries, wishes and needs of those being educated. Is this a good strategy for a good education? I believe so.

We have to look into history and epistemology with a broader view. The denial and exclusion of the cultures of the periphery, so common in the colonial process, still prevails in modern society. The denial of knowledge that affects populations is of the same nature as the denial of knowledge to individuals, particularly children. To propose directions to counteract ingrained practices is the major challenge of educators, particularly mathematics educators. Large sectors of the population do not have access to full citizenship. Some do not have access to the basic needs for survival. This is the situation in most of the world and occurs even in the most developed and richest nations. Further discussion about these matters is the objective of the Program Ethnomathematics, which will not be discussed in this paper (D'Ambrosio, 2006).

A new world order is urgently needed. Our hopes for the future depend on learning -- critically -- the lessons of the past. When we look at the history of mathematics since the early mathematical manifestations of man, we recognize the development of techniques to compare, to classify and to organize, to measure and to count, to infer and to conclude, much before mathematics is formalized. We also recognize mathematical ideas in the confluence of various modes of understanding, such as the religions, the arts, the techniques, the sciences, that is, we must assume a transdisciplinarian posture, and we also need to look at all this in different cultural environments, in different traditions, that is, we must assume a transcultural posture. This may restore to Mathematics its characteristics of being the most universal mode of thought and to face the most universal problem facing humanity, which is survival with dignity.

With respect to cognition, it is largely accepted that the emergence of modern science is closely associated with the recognition of an exclusive rational dimension of thinking. Recently, there has been acknowledgement of other dimensions in the capacity of reasoning and understanding. Multiple intelligences, emotional intelligence, spiritual intelligence, and numerous approaches to rationality have important consequences for education. Also, mental tasks performed by individual human beings are better understood thanks to the advances of artificial intelligence. For Mathematics Education, these advances strongly challenge the concepts of skill and drilling.

The enormous changes in society, particularly due to demographic dynamics, raise the exclusion of large sectors of the population, both in developed and undeveloped nations, to unbearable level. The exclusion of countries of the benefits of progress and advancement is unsustainable. An explanation for the current perverse concept of civilization asks for a deep reflection on colonialism. This reflection should not aim at blaming one group or another and not an attempt to redo the past. Rather, it is the moment to understand the past as a first step to move into the future.

Since mathematics has everything to do with the State of the World, its autonomy in the curriculum, and its central role as the dominating discipline and as an educational sphere in itself, should be reconsidered. Paraphrasing Mikhael Gromov (1998), we shall need for

Curriculum is the strategy for the educational action. Educational action should provide the three instruments that, together, provide what is essential for citizenship in a world moving swiftly toward a planetary civilization. These instruments are the communicative instruments, the analytic/symbolic instruments and the technological instruments. They constitute the modern trivium, which I called respectively literacy, matheracy, and technoracy (D'Ambrosio, 1999).

This is a proposal for a curriculum based on developing a broad perception of the complexity of the world and of society and providing the instruments to deal with such complexity. Literacy is the critical capability of processing information, such as the use of written and spoken language, of signs and gestures, of codes and numbers. Nowadays, reading must include also the competency of numeracy, of interpretation of graphs and tables, and of the other several means of informing the individual. Reading even includes understanding the condensed language of codes. These competencies have much more to do with screens and keys than with pencil and paper. Matheracy is the critical capability of inferring, proposing hypotheses, and drawing conclusions from data. It is a first step toward an intellectual posture, which is almost completely absent in our school systems. Matheracy is closer to the way mathematics was present, both in classical Greece and in indigenous cultures. The concern goes much beyond counting and measuring. Matheracy, proposes a deep reflection about man and society and aims at explaining and understanding reality. It is, indeed, symbolic analysis. This is the central idea behind the origins of mathematics. This competency should not be restricted to an elite, as it has been in the past. It is not the result of appropriation of skills, but it is acquired through competency to analyze. Technoracy is the critical familiarity with technology. Of course, the operative aspects of it are, in most cases, inaccessible to the lay individual. But the basic ideas behind technological devices, their possibilities and dangers, the morality supporting the use of technology, are essential issues to be raised among children at a very early age. As a historian, my resource is the critical perception of past and of future as a guide for action in the present, and history show us that ethics and values are intimately related to technological progress.

Proficiency in Mathematics means much more than counting,

measuring, sorting, comparing and solving problems aimed at drilling. Regrettably, even conceding that problem solving, modeling, and projects are practiced in some mathematics classrooms; the main importance is usually given to developing skills, particularly in the manipulation of numbers and operations. But problems and situations present in daily life are new and unexpected. Students should be prepared to tackle the new. The three instruments together, which obviously include reading, writing and basic mathematics, constitute what is essential for citizenship in a world moving swiftly toward a planetary civilization.

Concluding Remarks

Civilization, as well as life of all the animal species, is threatened. There will not be a privileged group of humans, as we are told in the Epic of Gilgamesh or in the biblical episode of Noah that will survive. I understand the threat to the species as the broadest violation of Social Justice. I tried to avoid commenting or reinforcing the proposals of colleagues who are presenting their views on a better Mathematics Education. All are proposed with extreme competence, presenting improvements of Mathematics Education aimed at full citizenship. My objective in this talk was to bring to the attention of mathematics educators the need to give their thoughtful and serious consideration to a broader concept of Social Justice, focusing the State of the World and the real threat to civilization.

Paraphrasing Bertrand Russell and Albert Einstein in the Pugwash Manifesto (1955)[10], a New Thinking is needed to achieve Social Justice, meaning equilibrium and safety, in a world menaced by exhaustion of resources, which lead to war and fear. Mathematicians and Mathematics Educators have powerful means of developing new concepts and techniques to cope with the major threats to the survival of civilization.

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Notes

[1] Interview given to Ken Ringle, The Washington Post, June 11 1996.

[2] Editorial. Email Newsletter from the International Mathematical Union, IMU-Net 47: May 2011.

[3]http://www.icsu-visioning.org/wp-content/uploads/GrandChallenges_Pre-publication.pdf

[4]http://www.icsu.org/what-we-do/projects-activities/archived-projects-and-activities/belmont-challenge .

[5]http://www.mpe2013.org

[6] For a discussion of labor in the future, see Robert B. Reich: The Work of Nations: Preparing Ourselves for 21st Century Capitalism, Vintage Books, New York, 1992.

[7] Harsh views of the future of employment, revealing the inadequacy of current educational systems, can be read in Viviane Forrester: The Economic Horror, Routledge Pub., New York, 1999.

[8] http://www.iite.unesco.org/pics/publications/en/files/3214592.pdf.

[9] According to Bertrand Russell "Mathematics possesses not only truth, but supreme beauty – a beauty cold and austere, like that of scultpture".

[10]http://www.pugwash.org/about/manifesto.htm.

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