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Kai-Ju Yang[1]

1) Indiana University, USA.

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How Do Elementary Preservice Teachers Form Beliefs and Attitudes Toward Geometry Learning? Implications for Teacher Preparation Programs

Kai-Ju Yang
Indiana University

Abstract

In my personal interactions with elementary preservice teachers (EPSTs) at a large Midwestern university in United States, many EPSTs held negative beliefs and attitudes about geometry learning. Although finding ways to help EPSTs change their negative beliefs and attitudes is an important issue, it can be best addressed by first investigating how they are formed. This study sought to document how EPSTs' beliefs about and attitudes toward geometry were formed prior to and during a mathematics and pedagogy course at a large Midwestern university in United States. McLeod's (1989) theoretical framework of influencing one's beliefs and attitudes toward specific action events, and objects –Representation, Discrepancy, and Metacognition– was used to analyze data from two interviews with each of four EPSTs. The results of the analysis confirmed McLeod's framework but also identified a fourth factor, understanding, as playing an important role in affecting EPST's beliefs about and attitudes toward geometry.

Keywords: beliefs, attitudes, geometry, elementary preservice teachers

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In my personal teaching experiences and interactions with elementary preservice teachers (EPSTs) at a large Midwestern university in United States, I found that many EPSTs held negative beliefs and attitudes about geometry learning. This negativity concerned me; if these EPSTs continued to hold these beliefs and attitudes, they would not be well-prepared to teach geometry to their future students. Although finding ways to help EPSTs change their negative beliefs and attitudes is an important issue, it can best be addressed by first investigating how they are formed. The purpose of this study, therefore, explores the origins of EPSTs' beliefs and attitudes toward geometry learning.

Conceptual framework and reseach questions

Mathematics educators (Lester & Garofalo, 1982; Schoenfeld, 1983 and 1985; Charles & Lester, 1984; McLeod, 1994; Leder, Pehkonen & Törner, 2002; Maaß & Schlöglmann, 2009) have investigated student beliefs about and attitudes toward mathematics and how they influence students' mathematics performance. Schoenfeld (1985), having found that students were not able to make use of the necessary mathematical knowledge gained from their coursework to solve problems, attributed this failure not to misunderstanding or forgetting mathematical knowledge but rather to not believing that it would be useful to them. The beliefs and attitudes that the students held, then, limited their understanding of mathematics and their ability to solve mathematical problems. In a similar study, Törner (2001) analyzed students' ad hoc answers to mathematical questions and concluded that the mental net of "knowledge" is dominated by beliefs, raising the question: How did students form these negative beliefs about and attitudes?

The formations of one's beliefs and attitudes

McLeod (1989) proposed a theoretical framework for investigating beliefs about and attitudes toward specific actions, events, or objects as they are affected by three factors: representation, discrepancy, and metacognition (see table 1).

Table 1
McLeod's (1989) framework for forming beliefs and attitudes

Beliefs and attitudes		
Representation * The *format* of the objects or events determines one's beliefs and attitudes toward those particular objects or events. * The *order* of the objects or events determines one's beliefs and attitudes toward those particular objects or events.	Discrepancy * Error: One expects the action to be correct but in fact it produces unexpectedly negative results, causing negative beliefs and attitudes. * Success: One's particular action produces unexpectedly positive results, producing positive beliefs and attitudes.	Metacognition * One reflects on one's own cognitive processes. * So one should be aware of one's emotional reactions toward the things experienced. * Next, one uses this awareness to control one's cognitive processes. * Then, one rethinks and possibly changes one's beliefs and attitudes.

Representation. McLeod argued that representation plays a crucial role in problem solving, because it influences how students learn, which in turn affects how they view what they are learning. Representation that promotes mathematical understanding, therefore, might positively change their view of doing math problems (Fennell & Rowan, 2001). The format of the mathematical concepts and problems (e.g., written statements alone or written statements with pictures) and the order in which those concepts and problems are presented (e.g., moving from concrete to abstract concepts) may be assumed to affect students' beliefs about and attitudes toward mathematics. In sum, representation may give students useful tools for building understanding, communicating information, and demonstrating reasoning (Greeno and Hall, 1997; NCTM, 2000).

Yang (2008) investigated the effects of Cognitive Tutor, a math software program, on the problem solving behaviors of 12 tenth-grade students. The program presented linear algebra word problems with simultaneous verbal and visual representations that moved from familiar, concrete problem situations (e.g., a truck averages 12 miles per hour and has already traveled 70 miles. In two more hours, how many total miles will the truck have traveled?) to more abstract, symbolic forms (e.g., to write an expression, define a variable for the additional time traveled and use this variable to write a rule for the total distance the truck has traveled). The results showed that the combined verbal and visual representations helped students grasp the target math concept. Students also reported that Cognitive Tutor functioned in the same way as a human tutor, guiding them step-by-step from the concrete through the abstract problems to develop their mathematical thinking. These experiences helped students begin to regard learning mathematics as less difficult than they had previously thought, confirming that how concepts are presented and organized affects how students think about mathematics.

Discrepancy. Referring to Mendler's (1989) work, McLeod stated that any discrepancy between an expected outcome and the actual outcome in the course of problem solving in general, and in mathematical reasoning in particular, affects students' beliefs and attitudes. Discrepancies can be experienced as either errors or successes. An error occurs when students engage in actions that they believe to be correct but in fact are incorrect, resulting in a mismatch between an expected outcome ("I thought I did what would solve the problem") and the reality ("It didn't work"). An error can create a negative evaluation of the current situation, which may result in negative beliefs about and attitudes toward the subjects in general. A success occurs when actions produce unexpectedly positive results ("I just tried. I am not sure if the method I used is the correct way to solve the problem. But it works"). This success is linked to a positive evaluation, which may positively orient the learners' beliefs about and attitudes toward the subject.

Metacognition. Schoenfeld (1985) described metacognition as referring to a cognitive process in which one plans a strategy to solve a task, monitors the comprehension of task-related knowledge, evaluates the progress towards the completion of the task, and makes a decision about whether the strategy is appropriate to apply in performing the task or he needs to select a new strategy. Brown, Bransford, Ferrara, and Campione (1983) suggested that metacognition includes two phases: (a) awareness of one's own cognitive process, which is the knowledge of cognition, and (b) use of this awareness to make a decision, which is the regulation of cognition. In both Schoenfeld's and Brown et al.'s views, metacognition is a cognitive activity that leads to reflection on one's own thought process in problem solving.

In addition to playing a significant role in students' success in problem solving, McLeod argued that metacognition is closely tied to students' beliefs and attitudes. Learning to reflect on their own cognitive processes, therefore, can not only help students realize how much they have learned (derived from their successes) as well as how much they still need to learn (indicated by their errors), but also increase their awareness of their emotional reactions to learning endeavors. Being aware of their emotional reactions toward learning mathematics will give students greater control over their cognitive processes, thus affecting their beliefs about and attitudes toward mathematics.

For example, students who receive low scores on a math quiz initially feel sad, even angry, and then conclude they cannot learn mathematics, resulting in a negative attitude. But if they can be helped to understand the relevant mathematics knowledge, they may at least temporarily suspend their negative reactions and reflect on their errors in light of this knowledge. Such reflection gives students a sense of control over their learning and, at the same time, raises their awareness of their previous negative emotional reactions, so they may begin to think that mathematics is not as hard as they had believed and develop more positive attitudes.

Is McLeod's (1989) theoretical framework for determining beliefs about and attitudes toward specific actions, events, or objects applicable to elementary preservice teachers' (EPSTs) experiences with geometry? What other factors might influence their beliefs and attitudes? Those are the research questions that I would like to study.

Description of four proposed approaches

This study took place in a mathematics and pedagogy course, focusing on geometry, at a large Midwestern university, in order to investigate how EPSTs' beliefs and attitudes toward geometry were formed. Following is an overview of how the instructor introduced the geometry concept of an Inscribed Circle Within a Triangle (ICWT) through the four learning approaches: a paper-folding activity, making a construction with a compass and a straightedge, determining proof, and operating Geometer's Sketchpad (GSP), a dynamic geometry software.

Hands-on activity: folding paper

Each EPST was given a white sheet and asked to draw a triangle ABC. Next, s/he folded the paper by finding the angle bisector of each angle of triangle ABC. After folding the paper, s/he located the point at which the three angle bisectors (the three folds) intersected and then used a compass to draw a circle that just touched each side of triangle ABC (see figure 1). After completing the activity, the geometry instructor led a whole-class discussion by asking such questions as "What is the mathematical name of point P, where the three bisectors intersect? What relationships have you discovered concerning the distance from point P to each side of triangle ABC?"

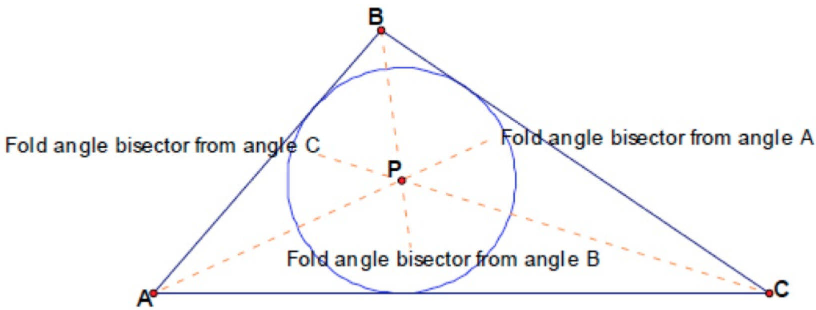


Figure 1. **Folding angle bisectors from each angle of a triangle to investigate the inscribed circle within a triangle**

Making constructions with a compass and a straightedge

Each EPST was given a blue sheet and asked to draw a triangle ABC. Next, a compass was used to construct an angle bisector for each angle of this triangle to produce an intersecting point P where the three angle bisectors intersected. Then this intersecting point P was used as a center to make a circle that touched each side of the triangle ABC (see figure 2). After the activity, EPSTs were asked, "Do you see any relationship between the distances from the incenter to each side of the angle?" By now, many EPSTs were able to understand that the distance from the incenter to each side of the triangle is the same since it is actually the radius of the inscribed circle. Next, the EPSTs were asked, "Can you prove it?"

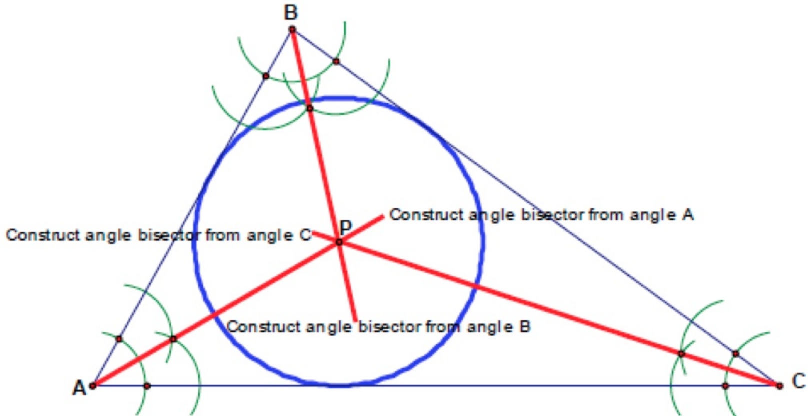
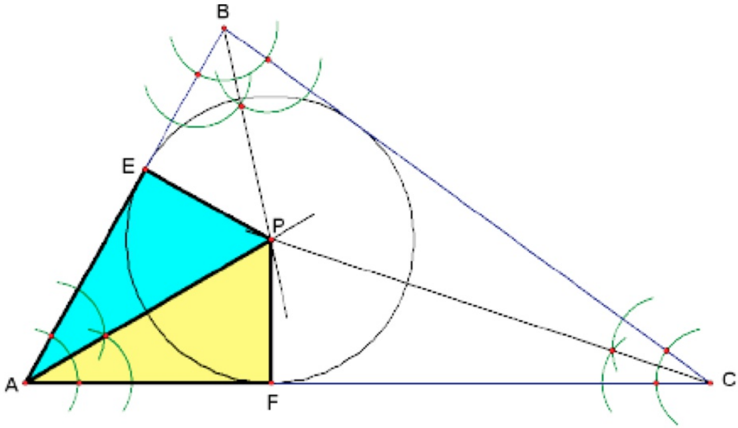


Figure 2. **Constructing angle bisectors for each angle of a triangle with a compass and a straightedge to explore the inscribed circle within a**

Constructing proof

In the proof activity EPSTs (in pairs or groups) were first given some time to discuss how to prove that the distance from the incenter (point P) to each side of the triangle is the same. After 10-15 minutes, geometry instructors led a whole-class discussion to facilitate EPSTs' thinking by asking several questions, such as, which two triangles could be used to solve this proof problem? What are the prerequisite (given corresponding congruent parts) you could find from the two triangles you have chosen? Based on what condition (SSS, SAS, or ASA) of triangle congruence, you could say these two triangles were congruent (see figure 3)? With proof, EPSTs were able to gain a deeper understanding of what the inscribed circle within a triangle is. More specifically, they would learn the triangle property that any point at angle bisector to the sides of the triangle will be the same. Too often, these two math ideas (inscribed circle within a triangle & property of angle bisectors) are considered separate concepts. With proof, EPSTs were able to make the fundamental connections.



Step 1: Angle EAP = Angle FAP (segment AP bisects angle A)

Step 2: Segment EP = segment FP?

Step 3: In triangle EAP and triangle FAP

Angle EAP = angle FAP (given info)

Segment AP = segment AP (common side)

Angle FPA = angle EPA (In a triangle, two angles are the same then the third angle is the same

So triangle EAP is congruent to triangle FAP.

Therefore, segment EP = segment FP

Figure 3. **Proving that the distance from the incenter to each side of the triangle is the same**

Geometer's Sketchpad (GSP)

EPSTs also work modeling real life situations such as the shipwreck survivor problem using GSP, dynamic geometry software (see figure 4), to understand property of angle bisectors. In this problem one needs to find the place where a survivor could set camp in an island that closely approximates the shape of a triangle. Specifically, EPSTs (two EPSTs worked as a pair) were asked to construct an inscribed circle within a triangle through angle bisectors with GSP. After 10- 15 minutes, geometry instructors led a whole-class discussion to talk about how to make the construction via GSP. In order to accurately make the construction, EPSTs need to have good understanding about the inscribed circle within a triangle related to angle bisector property from previous activities and apply what they have learned. After the construction with GSP, EPSTs were able to "see" that no matter where and how they drag the vertex of the triangle, the inscribed circle always touches each side of the triangle and that the distance from the incenter to each side of the triangle remains the same (see figure 4). This GSP activity reconfirmed their understanding about the property of angle bisector as well as inscribed circle within a triangle. It also allows EPSTs see the fundamental connections among those representations.

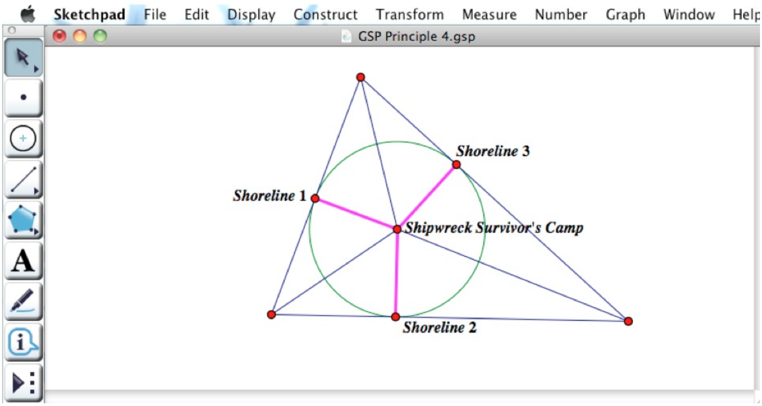


Figure 4. **Finding the incenter to solve the shipwreck survivor problem with GSP**

Methodology

Data for this study were collected during the Fall 2007 semester in a mathematics and pedagogy course, focusing on geometry, at a large Midwestern university. After the Study Information Sheet approved by the university's research office had been distributed, four EPSTs volunteered to participate: John, Karen, Becky, and Carrie (pseudonyms). The geometry instructor, Mr. Grow (pseudonym), gave lectures every Monday and Wednesday from 9:30 a.m. to 10:45 a.m. Each EPST participated in two interviews, each lasting sixty to ninety minutes.

The first interview questions focused on the EPSTs' geometry learning experiences prior to their current geometry-related mathematics and pedagogy course. Examples of the first interview questions included: When did you learn geometry? What kind of geometry knowledge had you learned before? How did you learn geometry? Do you think the ways you learned geometry were effective for you and why? What did you do when you were learning geometry inside or outside the classroom? Did you like geometry during or after learning geometry back then and why?

The second interview questions focused on the EPSTs' current geometry learning experiences in their geometry-related mathematics and pedagogy course. Examples of the second interview questions included: What kind of geometry knowledge have you learned from Mr. Grow's class? How do you learn geometry from Mr. Grow's class? Were the ways of learning geometry from Mr. Grow's class different from the ones you had experienced before? Do you think the ways you learned geometry from Mr. Grow's class were effective for you and why?

Questions in the first and second interviews were similar but used different verb tenses in order to investigate how the EPSTs' previous and current geometry learning experiences affected their beliefs and attitudes toward geometry. The prepared interview questions were used as guiding questions and then, based on individual responses, the interviewees were asked sub-questions related to their initial answers. Such now-and-then investigation offered the researcher the opportunity to confirm how the four EPSTs' beliefs and attitudes toward geometry were formed.

Data analysis and results

Three sets of questions and sub-questions –ways of learning geometry, geometry performance, and reflection on geometry learning– emerged as most useful for understanding how the four EPSTs' beliefs about and attitudes toward geometry were formed. Table 2 shows those three interview question sets.

Table 2

Three interview question sets for investigating the origins of the four EPSTs' beliefs and attitudes toward geometry learning

Ways of learning geometry	* How did/do you learn geometry? What did/do you learn about geometry? * Did/do you think the ways you learned geometry were effective for you and why? * Did/do you like geometry after taking the course?

	* Did/do you think your beliefs and attitudes toward geometry were influenced positively or negatively by the ways of learning geometry?
Geometry performance	* Did/do you have any geometry homework? * How was/is your homework performance? How about the midterm or final exam? * Did/do those performance influence what you think about geometry?
Reflection on geometry learning	* During the class, did/does the instructor help you reflect on what you have learned and what you have missed? * After the instructor has returned homework or an exam, did/does the instructor help you reflect on what you have learned and what you have missed?

Ways of learning geometry

During the first interview, all EPSTs mentioned that they first experienced geometry when they were elementary school students. Although geometry was not a specific subject, they learned basic geometric shapes and their characteristics, such as the four equal sides of a square. In the ninth or tenth grade, they took courses on geometry concepts, relationships, and operations such as the properties of parallel lines, their relationship to angles, and calculation of the areas of geometric shapes, making basic constructions with a compass and determining proofs. Most instruction was teacher-and textbook-centered with students listening to lectures and observing projected or written demonstrations, followed by worksheets and textbook assignments, but with little group interaction, whole-class discussion, or opportunity to explore the concepts being taught. The EPSTs regarded this approach as an ineffective way for them to learn geometry, which they came to believe it was too abstract and difficult to understand, leading to negative attitudes toward the subject. Becky said:

> …When he [the geometry instructor] taught the criteria for congruent triangles, he used an overhead projector to give us lectures about triangle congruence. He selected a question from the textbook to demonstrate how to prove it. After he taught the lesson, he gave us a worksheet to practice it until he thought that we had grasped the concept…. No, I don't think it [the way of learning geometry] was effective because I did not connect the concept and questions very well. I mean I understand the concept but I don't know how to apply these concepts to questions…. I did not get much understanding from that class. I hate geometry.

During the second interview, the EPSTs explained that the geometry concepts they learned from Mr. Grow in current mathematics and pedagogy course were similar to those they had learned in high school, so they were relearning but in ways that helped them understand the concepts better. They felt that learning geometry concepts through the four approaches in a sequence moving from the easiest one (folding paper) to the more complicated ones (determining proof or using GSP), helped them develop a deeper understanding of geometry a step at a time. These positive experiences encouraged them to rethink their ideas about geometry. Again, Becky said:

> …In Mr. Grow's class, we used different ways to learn a geometry concept and I especially like hands-on activity because I am experiencing the concept. I am actually learning by doing it.... Yes, I think it [the way of learning geometry] was effective because I am able to practice the concept by myself, not watching or listening to [an explanation of] the concept from the instructor. Mr. Grow introduced the concept by engaging us in an activity that we can "see" or "touch" the concept and later he helped us generalize the concept in a mathematical term…. I think I engaged in Mr. Grow's class more, compared to my previous geometry learning experiences. Mr. Grow made me feel geometry learning is easier.

This question set, *ways of learning geometry*, was related to the role of representations in affecting EPSTs' beliefs and attitudes toward geometry. The results of both interviews showed that the way of presenting geometry concepts strongly affected how EPSTs formed their beliefs and attitudes. Specifically, EPSTs' beliefs and attitudes about geometry were influenced by the *format* of how the geometry concepts were presented, for example, in written or diagram form only (EPSTs' previous geometry learning) or in a combination of visual presentation and interactive experiences (EPSTs' current geometry learning). EPSTs' beliefs and attitudes were also influenced by the *order* in which approaches were presented. In Mr. Grow's class, EPSTs first experienced geometry concepts by performing operations and then later Mr. Grow helped them make generalizations about the concepts. In this way, EPSTs' learning progressed from concrete examples to abstract ideas. This result confirmed McLeod's (1989) position that representation strongly affects the formation of one's beliefs and attitudes toward specific actions, events, or objects.

Ways of learning geometry

During the first interview, the EPSTs mentioned that in high school, their geometry instructors assigned them homework consisting of approximately 20 questions selected from the textbook, mostly short, discrete questions such as definitions or area calculations and a few more complex questions such as proof or construction problems. Because they did not have a good understanding of what was taught in class, they sometimes had to ask a tutor or their parents for help. Although they felt they had tried to prepare themselves to the best of their ability, their performance on the geometry tests often did not meet their expectations, which made them dislike geometry. Carrie said:

> …Doing homework was not an easy job for me. I had to hire a tutor to re-teach me in order to finish the assignment because I did not understand geometry at all in the class…. When the midterm was approaching, I needed to meet with my tutor several times to go over the geometry concepts that were needed for doing the midterm…. Even though I spent so much time at preparing for the midterm, the midterm results I got were very disa-

> ppointing. I was very frustrated. I felt that I made a lot of effort to prepare for the exam but the results were not very good…. I did not like geometry before I took the midterm. But after the midterm result came back, I just disliked geometry more.

During the second interview, the EPSTs explained that answers for the approximately 10 homework questions Mr. Grow gave them every week could not be found in the textbooks or on worksheets but required students to comprehend what was taught and consult their class notes. None needed to hire a tutor because Mr. Grow's four learning approaches, especially the folding activity, helped them grasp the geometry concepts. To prepare for their midterm, they studied the textbook and reviewed their homework, hand-outs, and in-class practice with problems or other activities. They even searched for websites with information about the geometry concepts being taught in order to better understand them. Overall, they felt well-prepared for the midterm and thought they had performed well on it, at least, according to their own criteria. Again, Carrie said:

> …Now there is no need for me to hire a tutor for my geometry class. Mr. Grow helped me gain a deeper understanding about geometry by actually "doing" the concept…. I used the notes, textbook, homework, or worksheets to prepare the midterm…. When I received my midterm I was happy with that…. That makes me realize that "Wow! Actually I can do well in geometry".

This question set, *geometry performance*, was associated with the role of discrepancy in affecting EPSTs' beliefs and attitudes. The results of both interviews revealed the effects of discrepancy between expected actual and performance on exams. In high school, because they had put a lot of effort into preparing for geometry exams, they believed they would perform well, but, in fact, they didn't, resulting in negative beliefs and attitudes. This finding resonates with McLeod's (1989) *error* discrepancy, a mismatch between an expected outcome and the actual outcome resulting in negative beliefs and attitudes. Because of their previous geometry learning experiences, the EPSTs thought they might not be able to perform well on Mr. Grow's geometry midterm although they continued to practice the geometry concepts. In reality, the results showed that they were able to do well in geometry. This discrepancy led to more positive beliefs and attitudes, which, resonates with McLeod's (1989) *success* discrepancy, when particular actions or thoughts produce unexpectedly positive results. In short, both geometry learning experiences confirm McLeod's (1989) beliefs and attitudes framework.

Reflection on geometry learning

The EPSTs recalled that in high school they spent little time reflecting on their geometry test results to assess what they had learned and what they needed to learn. The instructors just checked the correct answers with them to make sure that the grade was accurate, leaving them disappointed and with negative feelings about geometry learning.

Compared to their previous geometry learning experiences, the participants in this study commented that Mr. Grow spent more time going over geometry concepts, especially when they received back homework assignments, quizzes, major exams. This process helped them realize which parts of the concepts they had mastered and which parts they still needed to work on. Although when they first saw low grades they might feel frustrated and averse to geometry, Mr. Grow's guidance helped them understand the nature of their mistakes and reconsider their initial emotional reactions toward geometry, which they could change by paying closer attention to problems and, with their instructor's help, improving their comprehension of target concepts. This greater sense of control, in turn, motivated them to study geometry harder and rethink their initial perceptions of geometry. Karen said:

> …Sometimes it [geometry performance results] did bother me and made me so frustrated when I saw the grade… I even thought to quit learning geometry because I thought I understand the concepts and I should get a good grade, better than the one I received…. When Mr. Grow explained the questions, I realized that I did not think though the concepts completely. I did not grasp the concepts totally…. I realized there was no need for me to feel frustrated. Instead, I should study geo-

> metry harder so I will not miss the points next time when I see the similar questions.

This question set, *reflection on geometry learning*, was linked to the role of metacognition in affecting EPSTs' beliefs about and attitudes toward geometry. The results of both interviews indicated that metacognition –a cognitive process which helps EPSTs think about their own thinking and be aware of their emotional reactions to a subject– strongly affects how EPSTs form their beliefs and attitudes. Specifically, Mr. Grow helped these EPSTs reflect on what they knew and what they still needed to master, overcome negative emotions, and acquire a sense of control, which positively affected how they viewed themselves as learners of mathematics. This reflection process confirms McLeod's (1989) theoretical claim that metacognition plays a significant role in formation of one's beliefs and attitudes.

Another important insight derived from analysis of the interview data is that another factor, *understanding*, is important in affecting the formation of the EPSTs' beliefs and attitudes toward geometry, as reflected in the following statements:

> …I know that Mr. Grow used different ways to help us understand geometry concepts…. I think the way Mr. Grow taught geometry is more influential for me in learning geometry. He made geometry simple and I understand more. (John)
>
> … Mr. Grow made me think about geometry from different ways such as hands-on activity, constructions, or proof. Those ways helped me master geometry concepts. Now I understand geometry more compared to previous geometry learning…. I enjoyed learning geometry. (Karen)
>
> … Basically, I used geometry performance to tell how much I have learned. If my geometry performance is good, that means I understand more, then my beliefs and attitudes about geometry will be more positive. But if I get low geometry performance, that means I understand less, then my beliefs and attitudes about geometry will be negative. (Carrie)

> … The four geometry learning approaches helped me understand geometry concepts, hands-on activity particularly… Although I still don't like geometry, at least I don't dislike it that much. (Becky)

From these statements, it suggested that Mr. Grow's use of four learning approaches, including a paper-folding activity, making a construction with a compass and a straightedge, determining proof, and operating Geometer's Sketchpad (GSP), a dynamic geometry software, provided more learning opportunities for EPSTs in exploring the geometry concepts being taught, and the better understanding that resulted was an important link in the chain leading to the participants' development of more positive beliefs about mathematical learning and their attitude toward geometry learning as a subject to be both learned and taught.

Conclusions and discussion

In this study, I explored how EPSTs both formed their beliefs about and attitudes toward geometry by analyzing data from two interviews as different points in the learning period. This analysis revealed the strong influences of (1) the format and the order of presenting geometry concepts, (2) mismatches between expected and actual performance on the geometry midterm, and (3) reflection leading to self assessment of learning and awareness of emotional reactions to geometry, confirming McLeod's (1989) beliefs and attitudes framework explicating the roles of representation, discrepancy, and metacognition. In addition, the analysis identified understanding as another important factor affecting beliefs and attitudes (see table 3).

If in fact we can understand how EPSTs beliefs about and attitudes toward learning geometry are formed, we will be able to find out ways to help EPSTs change negative orientations, which can have positive implications for designing mathematics and pedagogy courses for EPSTs that will better prepare them to be effective mathematics teachers.

Table 3

Results confirming McLeod's (1989) framework for forming beliefs and attitudes through representation, discrepancy, and metacognition and the emergence of a fourth factor: understanding

Beliefs and attitudes			
Representation	Discrepancy	Metacognition	Understand
* The *format* of the objects or events determines one's beliefs and attitudes toward those particular objects or events. * The *order* of the objects or events determines one's beliefs and attitudes toward those particular objects or events.	* Error: One expects the action to be correct but in fact it produces unexpectedly negative results, causing negative beliefs and attitudes. * Success: One's particular action produces unexpectedly positive results, producing positive beliefs and attitudes.	* One reflects on one's own cognitive processes. * So one should be aware of one's emotional reactions toward the things experienced. * Next, one uses this awareness to control one's cognitive processes. * Then, one rethinks and possibly changes one's beliefs and attitudes.	* One learns about the objects, events, or persons from multiple perspectives. * One masters the ideas about the objects, events, or persons. * One, then, increases the level of understanding of the objects, events, or persons. * Thus, one rethinks previous beliefs and attitudes or forms new ones.

Toward this end, further inquiry might investigate how each of the different instructional approaches affected EPSTs' beliefs and attitudes as well as the cumulative effects of the four approaches in the sequence in which they were taught. Similarly, further research could pursue deeper understanding of the roles that representation, discrepancy, metacognition, and understanding play in changing EPSTs' beliefs about and attitudes toward geometry learning.

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Kai-Ju Yang is doctoral candidate in Curriculum Studies (Mathematics focus), at Indiana University, USA.

Contact address: Direct correspondence concerning this article should be addressed to the author at: 1179, Cooper River Drive, San Jose, CA, 95126, USA. E-mail address: kayang@indiana.edu.