ESTIMATION OF THE POPULATION MEDIAN OF SYMMETRIC AND ASYMMETRIC DISTRIBUTIONS USING DOUBLE ROBUST EXTREME RANKED SET SAMPLING

Amer Ibrahim Al-Omari
Al al-Bayt University, Faculty of Sciences, Department of Mathematics, Mafraq, Jordan
alomari_amer@yahoo.com

RESUMEN
Mediana de la población se sugieren. El rendimiento de DRERSS con respecto al muestreo aleatorio simple (SRS), se clasificó de muestreo (RSS) y la extrema se clasificó de muestreo (ERSS) a los regímenes se considera. Real conjunto de datos que consisten en alturas de 346 estudiantes se utilizan para evaluar el método de DRERSS. Se encontró que cuando la distribución subyacente es simétrica, los estimadores de DRERSS son imparciales de la mediana de la población. Cuando el tamaño de la muestra es impar, es eficiente que DRERSS SRS, ERSS, y RSS. Cuando el tamaño de la muestra es par, DRERSS es más eficiente que el RSS y ERSS si la distribución subyacente es simétrica, y para las distribuciones asimétricas, DRERSS es más eficiente que el RSS y ERSS, basándose en el mismo número de unidades de medida.

ABSTRACT
Double robust extreme ranked set sampling (DRERSS) and its properties for estimating the population median are suggested. The performance of DRERSS with respect to simple random sampling (SRS), ranked set sampling (RSS) and extreme ranked set sampling (ERSS) schemes is considered. Real data set that consist of heights of 346 students are used to evaluate the DRERSS method. It is found that when the underlying distribution is symmetric, the DRERSS estimators are unbiased of the population median. When the sample size is odd, DRERSS is efficient than SRS, ERSS, and RSS. When the sample size is even, DRERSS is more efficient than RSS and ERSS if the underlying distribution is symmetric, and for asymmetric distributions, DRERSS is more efficient than RSS and ERSS for $k > 0$ based on the same number of measured units.

KEYWORDS: Median; ranked set sampling; robust extreme ranked set sampling; efficiency.

MSC: 62D05

1. INTRODUCTION

The superiority of RSS over SRS in estimating the population mean is well known since (1952) when McIntyre apply the method for estimating mean pasture and forage yields. Takahasi and Wakimoto (1968) studied the mathematical properties of the RSS. Samawi et al. (1996) suggested using extreme ranked set samples (ERSS) for estimating a population mean. Al-Saleh and Al-Kadiri (2000) have extended the RSS to double ranked set sampling (DRSS) for estimating the population mean. Jemain and Al-Omari (2006a, 2006b) suggested double quartile and double percentile ranked set samples methods for estimating the population mean. Al-Nasser (2006) suggested L ranked set sampling (LRSS) as a generalization robust sampling method. Al-Nasser and Bani-Mustafa (2008) suggested robust extreme ranked set sampling (RERSS) for estimating the population mean. Al-Omari (2010) extended RERSS to double robust extreme ranked set sampling for estimating the population mean. Mahdizadeh and Arghami (2009) considered the entropy estimation and investigate entropy-based goodness-of-fit test for the inverse Gaussian distribution using RSS.

Assume that $X_1, X_2, ..., X_m$ is a random sample from probability density function $f(x)$ and cumulative distribution function $F(x)$, with mean μ and variance σ^2. Let X_{11h}, X_{12h}, ..., X_{1mh}; X_{21h}, X_{22h}, ...,

X_{2mh}; …; $X_{m1h}, X_{m2h}, ..., X_{mmh}$ be m independent simple random samples each of size m in the hth cycle $h = 1, 2, ..., n$. Let $X_{i(1:m)h}, X_{i(2:m)h}, ..., X_{i(m:m)h}$ be the order statistics of the sample $X_{i1h}, X_{i2h}, ..., X_{imh}$ for $i = 1, 2, ..., m$. The cumulative distribution function (cdf) of the ith order statistics $X_{(i:m)}$ is given by

$$F_{(i:m)}(x) = C_{i,m-i+1} \int_0^{F(x)} v^{i-1}(1-v)^{m-i} dv,$$

where $C_{i,m-i+1} = \dfrac{m!}{(i-1)!\,(m-i)!}$, and the probability density function (pdf) $f_{(i:m)}(x)$ is given by

$$f_{(i:m)}(x) = \frac{m!}{(i-1)!(m-i)!} \left[F(x)\right]^{i-1} \left[1-F(x)\right]^{m-i} f(x).$$

The mean and the variance of $X_{(i:m)}$ are given by

$\mu_{(i:m)} = \int_{-\infty}^{\infty} x f_{(i:m)}(x)dx$ and $\sigma^2_{(i:m)} = \int_{-\infty}^{\infty} \left(x - \mu_{(i:m)}\right)^2 f_{(i:m)}(x)dx$, respectively, see David and Nagaraja (2003).

In the case of estimating the population mean μ, Takahasi and Wakimoto (1968) showed that the efficiency of RSS relative to SRS is

$$1 \le eff\left(\bar{X}_{RSS}, \bar{X}_{SRS}\right) = \frac{\text{Var}\left(\bar{X}_{SRS}\right)}{\text{Var}\left(\bar{X}_{RSS}\right)} \le \frac{m+1}{2},$$

where $\bar{X}_{SRS}$ and $\bar{X}_{RSS}$ are the sample means using SRS and RSS, respectively. Also, they showed that

$$f(x) = \frac{1}{m}\sum_{i=1}^{m} f_{(i:m)}(x), \quad \mu = \frac{1}{m}\sum_{i=1}^{m}\mu_{(i:m)}, \text{ and } \sigma^2 = \frac{1}{m}\sum_{i=1}^{m}\sigma^2_{(i:m)} + \frac{1}{m}\sum_{i=1}^{m}\left(\mu_{(i:m)} - \mu\right)^2.$$

For more about RSS see Al-Omari and Jaber (2008), Arnold, et al. (2009), Bouza (2009), Islam, et al. (2009), Bouza (2009), and Sengupta and Mukhuti (2009).

In this paper, we used the DRERSS suggested by Al-Omari (2010) for median estimation. The advantage of the robust statistic is to discards the unit(s) in the extremes of a data set or by replaces these unit(s) with next extreme data value.

The remaining part of this paper is organized as follows: in Section 2, RSS, ERSS, and DRERSS are defined. Estimation of the population median using SRS, RSS, ERSS, and DRERSS is given in Section 3. In Section 4, a simulation study is conducted for different distribution functions. In Section 5, example of real data is given. Finally, conclusions are given in Section 6.

2. SAMPLING METHODS

The ranked set sampling (RSS) can be described as follows:

Step 1: Randomly select m^2 units from the target population.

Step 2: Allocate the m^2 selected units as randomly as possible into m sets, each of size m.

Step 3: Without yet knowing any values for the variable of interest, rank the units within each set with respect to variable of interest. This may be based on personal professional judgment or done with concomitant variable correlated with the variable of interest.

Step 4: Choose a sample for actual quantification by including the smallest ranked unit in the first set, the second smallest ranked unit in the second set, the process is continues in this way until the largest ranked unit is selected from the last set.
Step 5: Repeat Steps 1 through 4 for n cycles to obtain a sample of size mn for actual quantification.

The extreme ranked set sampling (ERSS) method can be summarized as follows:

Step 1: Select *m* random samples each of size *m* units from the target population,
Step 2: Rank the units within each sample with respect to a variable of interest by visual inspection or any other cost free method.
Step 3: For actual measurement, if the sample size *m* is even, from the first $\frac{m}{2}$ sets select the lowest ranked unit and from the other $\frac{m}{2}$ sets select the largest ranked unit. If the sample size is odd, from the first $\frac{m-1}{2}$ sets select the lowest ranked unit, from the other $\frac{m-1}{2}$ sets select the largest ranked unit, and from the remaining set, the median is selected.
Step 4: The procedure can be repeated *n* times if needed to increase the sample size to *nm* units.

The double robust extreme ranked set sampling (DRERSS) can be described as follows:

Step 1: Randomly select m^2 samples each of size *m* from the target population.
Step 2: Select the coefficient $k=[\beta m]$ where $0 \le \beta < 1$ and $[x]$ is the largest integer value less than or equal to *x*.
Step 3: If *m* is even, from the first $\frac{m^2}{2}$ samples select the $k+1$ th smallest ranked unit and from the second $\frac{m^2}{2}$ samples the $m-k$ th smallest ranked unit. If *m* is odd, select from the first $\frac{m\ m-1}{2}$ samples the $k+1$ th smallest ranked unit, and from the next *m* samples the $\left(\frac{m+1}{2}\right)$th smallest ranked unit and from the last $\frac{m\ m-1}{2}$ samples the $m-k$ th smallest ranked unit. This step yields *m* samples each of size *m*.
Step 4: For the *m* samples obtained in Step 3, if *m* is even, select for actual measurement from the first $\frac{m}{2}$ samples the $k+1$ th smallest ranked unit and from the second $\frac{m}{2}$ samples the $m-k$ th smallest ranked unit. If *m* is odd, select from the first $\frac{m-1}{2}$ samples the $k+1$ th smallest ranked unit, the median from the next sample and from the last $\frac{m-1}{2}$ samples the $m-k$ th smallest ranked unit. This step yields one sample of size *m* units from the DRERSS data.
Step 5: The cycle can be repeated *n* times if needed to obtain a sample of size *nm* out of nm^2 units.

Note that, in RSS and ERSS m^2 units are selected, while in DRERSS m^3 units are chosen, but at the end of the process we get *m* units and compare between the three methods based on the *m* units.

3. ESTIMATION OF THE POPULATION MEDIAN

The SRS estimator of the population median ϖ is defined as:

$$\hat{\varpi}_{SRS} = \begin{cases} X_{\left(\frac{m+1}{2}\right)h} & ,\text{if } m \text{ is odd} \\ \frac{1}{2}\left(X_{\left(\frac{m}{2}\right)h} + X_{\left(\frac{m+2}{2}\right)h} \right) & ,\text{if } m \text{ is even,} \end{cases} \quad h = 1, 2, ..., n. \tag{1}$$

The probability density functions of the $\frac{m}{2}, \frac{m+2}{2}, \frac{m+1}{2}$ ranked units respectively are

$$f_{\left(\frac{m}{2}:m\right)}(x) = \left[B\left(\frac{m}{2}, \frac{m+2}{2} \right) \right]^{-1} \left[F(x)\right]^{\frac{m-2}{2}} \left[1 - F(x)\right]^{\frac{m}{2}} f(x),$$

$$f_{\left(\frac{m+2}{2}:m\right)}(x) = \left[B\left(\frac{m+2}{2}, \frac{m}{2} \right) \right]^{-1} \left[F(x)\right]^{\frac{m}{2}} \left[1 - F(x)\right]^{\frac{m-2}{2}} f(x),$$

and

$$f_{\left(\frac{m+1}{2}:m\right)}(x) = \left[B\left(\frac{m+1}{2}, \frac{m+1}{2} \right) \right]^{-1} \left[F(x)\right]^{\frac{m-1}{2}} \left[1 - F(x)\right]^{\frac{m-1}{2}} f(x),$$

where $B(\alpha, \beta)$ is the beta function with parameters α and β.

The ranked set sample units are $X^*_{1(1:m)h}$, $X^*_{2(2:m)h}$, …, $X^*_{m(m:m)h}$ $(h = 1, 2, ..., n)$. The RSS estimator of the population median ϖ, from a sample of size m is given by:

$$\hat{\varpi}^*_{RSS} = \text{median}\left(X^*_{1(1:m)h}, X^*_{2(2:m)h}, ..., X^*_{m(m:m)h} \right), h = 1, 2, ..., n. \tag{2}$$

The measured units using extreme ranked set sample if m is even (ERSSE) are $X^*_{1(1:m)h}$, $X^*_{2(1:m)h}$, …, $X^*_{\frac{m}{2}(1:m)h}$, $X^*_{\frac{m+2}{2}(m:m)h}$, $X^*_{\frac{m+4}{2}(m:m)h}$, …, $X^*_{m(m:m)h}$ $(h = 1, 2, ..., n)$, and if m is odd, the ERSSO are $X^*_{1(1:m)h}$, $X^*_{2(1:m)h}$, …, $X^*_{\frac{m-1}{2}(1:m)h}$, $X^*_{\frac{m+1}{2}\left(\frac{m+1}{2}:m\right)h}$, $X^*_{\frac{m+3}{2}(m:m)h}$, …, $X^*_{m(m:m)h}$. The ERSSE and ERSSO estimators of the population median ϖ, respectively are defined as

$$\hat{\varpi}^*_{ERSSE} = \text{median}\left\{ X^*_{1(1:m)h}, X^*_{2(1:m)h}, ..., X^*_{\frac{m}{2}(1:m)h}, X^*_{\frac{m+2}{2}(m:m)h}, X^*_{\frac{m+4}{2}(m:m)h}, ..., X^*_{m(m:m)h} \right\}, \tag{3}$$

and

$$\hat{\varpi}^*_{ERSSO} = \text{median}\left\{ X^*_{1(1:m)h}, X^*_{2(1:m)h}, ..., X^*_{\frac{m-1}{2}(1:m)h}, X^*_{\frac{m+1}{2}\left(\frac{m+1}{2}:m\right)h}, X^*_{\frac{m+3}{2}(m:m)h}, ..., X^*_{m(m:m)h} \right\}. \tag{4}$$

Based on DRERSS, if the sample size m is even, in the hth cycle $h = 1, 2, ..., n$, let $X^{**}_{i(k+1:m)h}$ be the $k+1$ th smallest ranked unit of the ith sample $\left(i = 1, 2, ..., \frac{m}{2}\right)$, and $X^{**}_{i(m-k:m)h}$ be the $m-k$ th smallest ranked unit of the ith sample $\left(i = \frac{m+2}{2}, \frac{m+4}{2}, ..., m\right)$. In this case, the measured units $X^{**}_{1\ k+1:m\ h}$, $X^{**}_{2\ k+1:m\ h}$, ..., $X^{**}_{\frac{m-1}{2}\ k+1:m\ h}$ are iid and $X^{**}_{\frac{m+3}{2}\ m-k:m\ h}$, $X^{**}_{\frac{m+5}{2}\ m-k:m\ h}$, ..., $X^{**}_{m\ m-k:m\ h}$ are iid. However, all units are mutually independent but not identically distributed and will be denoted by DRERSSE. The DRERSSE estimator of the population median is given by

$$\hat{\varpi}^{**}_{DRERSSE} = \text{median}\left\{ \begin{array}{l} X^{**}_{1(k+1:m)h}, X^{**}_{2(k+1:m)h}, ..., X^{**}_{\frac{m}{2}(k+1:m)h}, \\ X^{**}_{\frac{m+2}{2}(m-k:m)h}, X^{**}_{\frac{m+4}{2}(m-k:m)h}, ..., X^{**}_{m(m-k:m)h} \end{array} \right\}. \quad (5)$$

For odd sample size, let $X^{**}_{i(k+1:m)h}$ be the $k+1$ th smallest ranked unit of the ith sample $\left(i = 1, 2, ..., \frac{m-1}{2}\right)$, and $X^{**}_{i\left(\frac{m+1}{2}:m\right)h}$ be the median of the ith sample of the rank $i = \frac{m+1}{2}$, and $X^{**}_{i(m-k:m)h}$ be the $m-k$ th smallest ranked unit of the ith sample $\left(i = \frac{m+3}{2}, \frac{m+5}{2}, ..., m\right)$. The measured units $X^{**}_{1\ k+1:m\ h}$, $X^{**}_{2\ k+1:m\ h}$, ..., $X^{**}_{\frac{m-1}{2}\ k+1:m\ h}$ are iid and $X^{**}_{\frac{m+3}{2}\ m-k:m\ h}$, $X^{**}_{\frac{m+5}{2}\ m-k:m\ h}$, ..., $X^{**}_{m\ m-k:m\ h}$ are iid, while all units are mutually independent but not identically distributed, and will be denoted by DRERSSO. The DRERSSO estimator of the population median is defined as

$$\hat{\varpi}^{**}_{DRERSSO} = \text{median}\left\{ \begin{array}{l} X^{**}_{1\ k+1:m\ h}, X^{**}_{2\ k+1:m\ h}, ..., X^{**}_{\frac{m-1}{2}\ k+1:m\ h}, X^{**}_{\frac{m+1}{2}\left(\frac{m+1}{2}:m\right)h}, \\ X^{**}_{\frac{m+3}{2}\ m-k:m\ h}, X^{**}_{\frac{m+5}{2}\ m-k:m\ h}, ..., X^{**}_{m\ m-k:m\ h} \end{array} \right\}.. \quad (6)$$

With references to Tables 1-4, the properties of the suggested estimators are:

- When the underlying distribution is symmetric about μ, $\hat{\varpi}^{**}_{DRERSSa}$ $a = E, O$ are unbiased estimators of the population median. For asymmetric distributions, the estimators have a small bias, close to zero in most cases.
- If the sample size is odd, DRERSSO is more efficient than RSS and ERSS for all values of k and m considered in this study with symmetric and asymmetric distributions.
- If the sample size is even, DRERSSE is more efficient than RSS and ERSS for all values of k and m considered in this study when the distribution is symmetric. For asymmetric distributions, DRERSSE is more efficient than RSS and ERSS if $k > 0$, and when $k = 0$ for some distributions.

In the following section we will give some particular properties with some illustrated examples.

4. SIMULATION STUDY

In this section, we compared the performance of the proposed estimators of the population median. Several symmetric and asymmetric distributions are considered. The simulation study considers set sizes $4 \leq m \leq 7$.

If the underlying distribution is symmetric, the efficiency of $\varpi^*_{RSS}, \varpi^*_{ERSS}$ and $\hat{\varpi}^{**}_{DRERSSa}$ $a = E, O$ with respect to $\hat{\varpi}_{SRS}$ is defined as:

$$eff\left(J, \hat{\varpi}_{SRS}\right) = \frac{\text{Var}\left(\hat{\varpi}_{SRS}\right)}{\text{Var}\left(J\right)}, J = \hat{\varpi}^*_{RSS}, \hat{\varpi}^*_{ERSS}, \hat{\varpi}^{**}_{DRERSSa}, \quad (7)$$

and for asymmetric distributions the efficiency is given by

$$eff\left(J, \hat{\varpi}_{SRS}\right) = \frac{MSE\left(\hat{\varpi}_{SRS}\right)}{MSE\left(J\right)}, J = \hat{\varpi}^*_{RSS}, \hat{\varpi}^*_{ERSS}, \hat{\varpi}^{**}_{DRERSS}. \quad (8)$$

Simulation results in terms of the efficiency and bias values are presented in Tables 1-4.

Table 1: The efficiency of RSS, ERSS and DRERSSE with respect to SRS for estimating the population median with $m = 4$.

Distribution	RSS	ERSS	DRERSSE	
			$k = 0$	$k = 1$
Uniform (0,1)	2.007	2.435	17.850	5.678
Normal (0,1)	2.173	1.989	3.640	7.245
Logistic (0,1)	2.267	1.881	2.311	7.410
Laplace (0,1)	2.486	1.553	1.313	9.440
Exponential (1)	2.357	1.326	0.387	7.815
	(0.094)	(0.218)	(0.704)	(0.051)
Gamma (2,1)	2.226	1.594	0.687	7.472
	(0.100)	(0.225)	(0.739)	(0.053)
Weibull (1,3)	2.254	1.327	0.388	7.713
	(0.287)	(0.660)	(2.109)	(0.147)
Beta (7,4)	2.155	2.039	3.620	6.745
	(0.003)	(0.006)	(0.020)	(0.002)

Table 2: The efficiency of RSS, ERSS and DRERSSO with respect to SRS for estimating the population median with $m = 5$.

Distribution	RSS	ERSS	DRERSSO		
			$k = 0$	$k = 1$	$k = 2$
Uniform (0,1)	1.875	1.399	2.959	3.521	9.700
Normal (0,1)	2.113	1.493	3.497	4.271	12.122
Logistic (0,1)	2.178	1.562	3.695	4.479	12.901
Laplace (0,1)	2.553	1.703	4.681	5.893	18.817
Exponential (1)	2.331	1.588	4.074	4.924	14.357
	(0.043)	(0.061)	(0.025)	(0.021)	(0.007)
Gamma (2,1)	2.162	1.544	3.677	4.532	13.469
	(0.042)	(0.060)	(0.028)	(0.024)	(0.008)
Weibull (1,3)	2.310	1.628	4.027	4.846	14.851
	(0.126)	(0.176)	(0.073)	(0.067)	(0.020)
Beta (7,4)	2.030	1.456	3.443	4.108	11.719
	(0.001)	(0.002)	(0.001)	(0.000)	(0.000)

From Tables 1-4, we can conclude that:

- A gain in efficiency is obtained by using DRERSS in estimating the population median when the underlying distribution is symmetric. For example, for $m = 4$ and $k = 1$, the efficiency of DERSSE is 9.440 for estimating the population median of Laplace distribution with parameters 0 and 1.

Table 3: The efficiency of RSS, ERSS and DRERSSE with respect to SRS for estimating the population median with $m=6$.

Distribution	RSS	ERSS	DRERSSE		
			$k=0$	$k=1$	$k=2$
Uniform (0,1)	2.453	3.158	61.185	14.687	10.981
Normal (0,1)	2.715	2.266	4.785	10.595	13.598
Logistic (0,1)	2.832	2.009	2.563	10.161	14.558
Laplace (0,1)	3.218	1.472	1.216	7.532	20.098
Exponential (1)	2.930	1.127	0.172	3.237	16.147
	(0.046)	(0.227)	(0.973)	(0.183)	(0.012)
Gamma (2,1)	2.750	1.543	0.335	4.987	14.778
	(0.051)	(0.238)	(0.986)	(0.192)	(0.0146)
Weibull (1,3)	2.866	1.127	0.171	3.175	15.748
	(0.146)	(0.679)	(2.810)	(0.550)	(0.036)
Beta (7,4)	2.627	2.411	3.356	10.320	13.021
	(0.002)	(0.006)	(0.026)	(0.005)	(0.000)

Table 4: The efficiency of RSS, ERSS and DRERSSO with respect to SRS for estimating the population median with $m=7$.

Distribution	RSS	ERSS	DRERSSO			
			$k=0$	$k=1$	$k=2$	$k=3$
Uniform (0,1)	2.213	1.246	4.038	4.111	5.702	18.942
Normal (0,1)	2.541	1.320	4.849	4.788	6.703	23.190
Logistic (0,1)	2.590	1.317	4.891	4.962	7.046	23.923
Laplace (0,1)	3.050	1.406	6.328	6.430	9.288	35.458
Exponential (1)	2.742	1.362	5.233	5.196	7.507	26.414
	(0.026)	(0.049)	(0.014)	(0.014)	(0.010)	(0.003)
Gamma (2,1)	2.607	1.304	5.034	5.117	7.125	24.081
	(0.267)	(0.053)	(0.015)	(0.016)	(0.011)	(0.003)
Weibull (1,3)	2.718	1.360	5.365	5.102	7.557	26.631
	(0.072)	(0.152)	(0.043)	(0.045)	(0.029)	(0.006)
Beta (7,4)	2.407	1.299	4.560	4.660	6.541	21.918
	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)

- For $k>0$, DRERSS is more efficient than RSS and ERSS methods based on the same sample size. For example, for $m=6$ and $k=2$, the efficiency of DRERSSE is 13.598 for estimating the median of normal distribution with parameters 0 and 1 while the efficiency values of RSS and ERSS are 2.715 and 2.266, respectively.
- For asymmetric distributions considered in this study, DRERSS estimators are biased. The bias is small, depending on the value of *k*, *m*, and the underlying distribution. As an example, for $m=5$ and $k=2$ the efficiency is 14.851 with bias 0.020, for estimating the population median of Weibull distribution with parameters 1 and 3. For $m=7$, the bias is zero for all *k* when the underlying distribution is $B(7,4)$.

5. AN APPLICATION

In this section, we applied the SRS, ERSS, RSS, and DRERSS estimators to data concerning the height of 346 students at Primary School in UAE. Student's heights are ranked judgmental by nursing staff. Summery statistics of the data are

$$\mu = \frac{\sum_{i=1}^{346} X_i}{346} = 152.139\ ,\ \varpi = Median\ \ X_i ; i = 1, 2, ..., 346\ = 152,$$

and $Skewness = 0.0135948$.

Since these data are asymmetrically distributed, the efficiency values are obtained using Equation (8). Results are summarized in Table 5.

Table 5: The bias, MSE, and efficiency values of estimating the population median of the height of 346 students using RSS, ERSS, and DRERSS with respect to SRS for $4 \le m \le 7$.

	SRS	**RSS**	**ERSS**	**DRERSS**			
				$k=0$	$k=1$	$k=2$	$k=3$
			$m=4$				
Bias	0.1111	0.1663	0.1451	0.0697	0.1671	-----------	-----------
MSE	41.4815	19.9478	19.5372	6.0123	6.5086	-----------	-----------
Efficiency		2.0795	2.1232	6.8994	6.3733	-----------	-----------
			$m=5$				
Bias	0.1089	0.0000	0.1307	0.1027	0.0889	0.0308	-----------
MSE	41.6445	20.8615	28.6848	12.6034	10.4802	3.6104	-----------
Efficiency		1.9962	1.4518	3.3042	3.9736	11.5346	-----------
			$m=6$				
Bias	0.1241	0.1301	0.1501	0.0975	0.1929	0.0383	-----------
MSE	31.3073	12.0186	12.3545	2.0137	2.7200	2.4221	-----------
Efficiency		2.6049	2.5341	15.5472	11.5100	12.9257	-----------
			$m=7$				
Bias	0.1628	0.1195	0.1508	0.0495	0.0534	0.0519	0.0354
MSE	31.0571	13.1775	24.4569	6.7039	6.7465	4.8626	1.5619
Efficiency		2.3568	1.2699	4.6327	4.6034	6.3869	19.8842

Table 5, revealed that the DRERSS estimators are more efficient than the other estimators, SRS, ERSS, and ERSS for this data set.

6. CONCLUSIONS

In this paper, DRERSS method is considered for estimating the population median. It is turns out that the suggested estimators are unbiased when the distribution is symmetric about μ. Also, the DRERSS is more efficient than SRS, RSS and ERSS when the underlying distribution is symmetric or asymmetric. For odd sample size, DRERSSO is more efficient than SRS, ERSS, and RSS for all cases considered in this study. When the sample size is even and the distribution is symmetric, DRERSSE is more efficient than SRS, ERSS, and RSS, and it is more efficient for asymmetric distribution when $k > 0$. DRERSS is more efficient with odd sample size. The recommendation is to use DRERSS if the underlying distribution is symmetric and if the distribution is asymmetric with small bias.

REFERENCES

[1] AL-NASSER, D.A. (2007). L ranked set sampling: a generalization procedure for robust visual sampling. **Communication in Statistics-Simulation and Computation**, 36: 33-43.

[2] AL-NASSER, D.A. and BANI-MUSTAFA, A. (2008). Robust extreme ranked set sampling. **Journal of Statistical Computation and Simulation**, 79: 859-867.

[3] AL-OMARI, A.I. (2010). Estimation of mean based on modified robust extreme ranked set sampling. **Journal of Statistical Computation and Simulation**, **to appear.**

[4] AL-OMARI, A.I. and JABER, K. (2008). Percentile double ranked set sampling. **Journal of Mathematics and Statistics**, 4: 60-64.

[5] AL-SALEH, M.F. and AL-KADIRI, M. (2000). Double ranked set sampling. **Statistics and Probability Letters**, 48: 205 - 212.

[6] ARNOLD, B.C., CASTILLO, E., and SARABIA, J.M. (2009). On multivariate order statistics. Application to ranked set sampling. **Computational Statistics and Data Analysis**, 53: 4555-4569.

[7] BOUZA, C.N. (2002). Estimation of the mean in ranked set sampling with non responses. **Metrika**, 56: 171–179.

[8] BOUZA, C.N. (2009). Ranked set sampling and randomized response procedures for estimating the mean of a sensitive quantitative character. **Metrika**, 70:267–277.

[9] DAVID, H.A. and NAGARAJA, H.N. (2003). **Order Statistics**, Third Edition, John Wiley & sons, Inc., Hoboken, New Jersey.

[10] ISLAM, T., SHAIBUR, M.R., and HOSSAIN, S.S. (2009). Effectivity of modified maximum likelihood estimators using selected ranked set sampling Data. **Austrian Journal of Statistics**, 38: 109-120.

[11] JEMAIN, A.A. and AL-OMARI, A.I. (2006a). Double percentile ranked set samples for estimating the population mean. **Advances and Applications in Statistics**, 6: 261-276.

[12] JEMAIN, A.A. and AL-OMARI, A.I. (2006b). Double quartile ranked set samples. **Pakistan Journal of Statistics**, 22: 217-228.

[13] MCINTYRE, G.A. (1952). A method for unbiased selective sampling using ranked sets. **Australian Journal Agricultural Research**, 3: 385-390.

[14] MAHDIZADEH, M. and ARGHAMI, N.R. (2009). Efficiency of ranked set sampling in entropy estimation and goodness-of-fit testing for the inverse Gaussian law. **Journal of Statistical Computation and Simulation**, 1-14.

[15] SAMAWI, H., ABU-DAYYEH, W. and AHMED, S. (1996). Extreme ranked set sampling. **The Biometrical Journal**, 30: 577-586.

[16] SAMAWI. H. M. and TAWALBEH, E. M. (2002). Double median ranked set sample: Comparison to other double ranked samples for mean and ratio estimators. **Journal of modern Applied Statistical Methods**, 1: 428-442.

[17] SENGUPTA, S. and MUKHUTI, S. (2009). Unbiased estimation of $P(X > Y)$ using ranked set sample data. **Statistics**, 42: 223–230.

[18] TAKAHASI, K. and WAKIMOTO, K. (1968). On the unbiased estimates of the population mean based on the sample stratified by means of ordering. **Annals of the Institute of Statistical Mathematics**, 20: 1-31.