THE CONSTRUCTION OF PLATE FINITE ELEMENTS USING WAVELET BASIS FUNCTIONS

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ABSTRACT

In the last years, applying wavelets analysis has called the attention in a wide variety of practical problems, in particular for the numerical solutions of partial differential equations using different methods, as finite differences, semi-discrete techniques or the finite element method. In the construction of wavelet-based elements, instead of traditional polynomial interpolation, scaling and wavelet functions have been adopted to form the shape function to construct elements. Due to their properties, wavelets are very useful when it is necessary to approximate efficiently the solution on non-regular zones. Furthermore, in some cases it is convenient to use the Daubechies wavelet, which has properties of orthogonality and minimum compact support, and provides guaranty of convergence and accuracy of the approximation in a wide variety of situations. The aim of this research is to explore the Galerkin method using wavelets to solve plate bending problems. Some numerical examples, with B-splines and Daubechies, are presented and show the feasibility of our proposal.

KEY WORDS: finite-element method, wavelet analysis, spline wavelets, Daubechies, plate element.

MSC: 90C59

RESUMEN

En los últimos años el análisis wavelet ha sido aplicado en una amplia variedad de problemas prácticos, en particular en distintos métodos para la solución numérica de ecuaciones diferenciales parciales, como diferencias finitas, técnicas semidiscretas o el método de elementos finitos. En la construcción de elementos con bases wavelets, en lugar de la interpolación polinómica clásica, se consideran wavelets y funciones de escala como funciones de forma. Debido a sus propiedades, las wavelets son muy útiles cuando es necesario aproximar eficientemente la solución en regiones no regulares. Más aún, en algunos casos es conveniente la utilización de las wavelets de Daubechies, que tienen propiedades de ortogonalidad y soporte compacto mínimo, asegurando convergencia y precisión en una amplia gama de situaciones. El objetivo de este trabajo es estudiar el método de Galerkin con bases wavelets para resolver problemas de flexión de placas. Los ejemplos numéricos presentados, utilizando B-splines y Daubechies, muestran la factibilidad de lo propuesto.

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1. Introduction

In structural analysis, classical and standard numerical methods as the finite element method (FEM), boundary element method (BEM), and Meshless methods have been applied during the last decades. In solving problems involving partial differential equations, the conventional finite element method use piecewise Lagrange interpolation functions for approximating displacements.
Since the mid 90ies and due to its desirable advantages, researchers have also payed attention to wavelet analysis. As it was discussed in [8], wavelets provide a robust and accurate alternative to traditional methods. The numerical evidence of different approaches [9, 10, 12, 5, 14], confirm this. Furthermore, the advantage of wavelet methods is really appreciated when they are applied to problems having solutions with localized singular behavior [8].
For beam and plate problems, finite wavelet based element are constructed for instance in [10, 15, 12, 5]. Han et al. [10] presented a spline wavelet finite element method, by using spline wavelet scaling functions as displacement interpolation functions in the finite element formulation, Xiang et al. [15] constructed a Mindlin-Reissner plate finite element using B-spline wavelet on the interval (BSWI). On the other hand, with Daubechies scaling functions, Ma et al. [12] constructed an Euler-Bernoulli beam element and its good approximation properties were shown in numerical examples with singularities. For thin plate bending problems, Chen et al. [5] developed a finite element, following the classical Kirchhoff theory. Furthermore, in Alvarez et al. [1, 13], a Mindlin-Reissner plate element is constructed using Daubechies scaling functions (DSCW) and the element formulation is not only applicable to the analysis of thin plates but to thick plates as well.
In this paper, we presented a wavelet based Galerkin method to solve boundary value problems for differential equations, in particular for plate bending problems. Multiresolution analysis of $L^2(\mathbb{R})$ were used as the approximating spaces in the Galerkin method and two different wavelet families, B-spline and Daubechies wavelets, were considered.
Many of current wavelet-based finite element formulations are constructed in *wavelet space*, in which the field variables like displacements are expressed by a product of wavelet functions and wavelet coefficients. However, the boundary conditions cannot be easily treated as in the case of conventional FEMs, and furthermore, for complex structural problems it becomes very difficult to deal with the interface between elements.
In this work we illustrate the use of wavelet based Galerkin method to solve plate bending problems in *physical space*, as it was done in [10, 15, 12]. The wavelet functions are used as the displacement interpolation functions and the shape functions are expressed by wavelets. By adopting local co-ordinates and co-ordinate transformation, plate wavelet finite element formulations are derived. The numerical examples show that the proposed wavelet FEM has high accuracy and fast convergent rate.
Two plate models were considered, which are the most common dimensionally reduced models of a thin linearly elastic plate: the classical Kirchhoff-Love and Reissner-Mindlin models. As it was demonstrated by Arnold [2], the second model approximation, is more accurate for moderately thin plates and when transverse shear plays a significant role.
The outline of this paper is the following: in Section 2. we introduce some basic concepts of wavelet analysis; in Section 3. how to construct plate finite element using wavelet basis and the Galerkin method; Kirchhoff-Love and Mindlin-Reissner plate finite element formulations are developed in Section 4. and Section 5., respectively. In Section 6. some applications are shown and in Section 7. conclusions are presented.

2. Wavelet analysis

Wavelets are functions generated by simple operations of dilation and translation, from one single function called mother wavelet. A mother wavelet ψ gives rise to a decomposition of the Hilbert space $\mathrm{L}^2(\mathbb{R})$, into a direct sum of closed subspaces W_j, $j \in \mathbb{Z}$.

Let $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$ and

$$W_j = \mathrm{clos}_{\mathrm{L}^2}[\psi_{j,k} : k \in \mathbb{Z}] \tag{2..1}$$

Then,

$$\mathrm{L}^2(\mathbb{R}) = \sum_j W_j = \cdots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \cdots \tag{2..2}$$

and using this decomposition of $\mathrm{L}^2(\mathbb{R})$, a nested sequence of closed subspaces V_j, $j \in \mathbb{Z}$ can be obtained, where

$$V_j = \sum_{l=-\infty}^{j-1} W_l = \cdots \oplus W_{j-2} \oplus W_{j-1} \, . \tag{2..3}$$

These closed subspaces $\{V_j, \ j \in \mathbb{Z}\}$ of $\mathrm{L}^2(\mathbb{R})$, form a "multiresolution analysis" [6] with the following properties:

1. $\cdots \subset V_{-1} \subset V_0 \subset V_1 \cdots$
2. $\mathrm{clos}_{\mathrm{L}^2}(\bigcup V_j) = \mathrm{L}^2(\mathbb{R})$
3. $\bigcap_j V_j = \{0\}$
4. $V_{j+1} = V_j \oplus W_j$
5. $f(x) \in V_0 \Leftrightarrow f(x-k) \in V_0 \, , k \in \mathbb{Z}$
6. $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1} \, , j \in \mathbb{Z}$
7. $\phi \in V_0$ exists that the set of

$$\{\phi(x-k) : k \in \mathbb{Z}\} \tag{2..4}$$

is a Riesz basis of V_0.

The function $\phi \in V_0$ is called "scaling function" and generates the multiresolution analysis $\{V_j\}_{j \in Z}$ of $\mathrm{L}^2(\mathbb{R})$ and by setting

$$\phi_{j,k}(x) := 2^{j/2}\phi(2^j x - k) \tag{2..5}$$

it follows that, for each $j \in \mathbb{Z}$, the family

$$\{\phi_{j,k} : k \in \mathbb{Z}\} \tag{2..6}$$

is also a Riesz basis of V_j.
Consequently, a unique sequence $\{p_k\} \in l^2(\mathbb{Z})$ exists, ($l^2(\mathbb{Z})$ denotes the integer space of all square-summable bi-infinite sequences), such that the scaling function $\phi(x)$ satisfies a refinement equation

$$\phi(x) = \sum_{k=-\infty}^{\infty} p_k \, \phi(2x - k), \qquad k \in Z \tag{2..7}$$

which is also called "two-scale relation".

2..1. B-spline wavelet functions

The m-order B-spline can be defined as follows [6]:

$$N_m(x) = (N_{m-1} * N_1) = \int_0^1 N_{m-1}(x-t)dt, \qquad m \geq 2 \tag{2..8}$$

where N_1 is the characteristic function within interval $[0,1]$. The following are some important properties of N_m:

- $supp\ N_m = [0, m]$
- $N_m(x) > 0$ for $0 < x < m$
- $\sum_k N_m(x-k) = 1$ for all x
- $N_m(x) = \frac{x}{m-1} N_{m-1}(x) - \frac{m-x}{m-1} N_{m-1}(x)$
- $N_m'(x) = N_{m-1}(x) - N_{m-1}(x-1)$

Let $\phi_m(x) = N_m(x)$ and V_j^m its corresponding subspace. Since $N_m(x) \in V_0^m$ and $V_0^m \subset V_1^m$, the following two-scale relation is obtained:

$$\phi_m(x) = \sum_{k=0}^{m-1} p_k\ \phi_m(2x-k) \tag{2..9}$$

There are advantages of the spline wavelets over the polynomial functions, and they can improve the efficiency of numerical calculation. One important aspect is that spline wavelets have explicit expressions, which facilitate not only theoretical formulation, but also numerical implementation with computer.

2..2. Daubechies-Wavelets

The family of orthogonal wavelets with compactly supported property was constructed by Daubechies [7].
The Daubechies scaling and wavelet functions, ϕ_N and ψ_N, have the following properties:

- $supp\ \phi_N = [0, N-1]$
- $supp\ \psi_N = [1 - N/2, N/2]$
- $\int_{-\infty}^{\infty} \phi(x)dx = 1$
- $\int_{-\infty}^{\infty} \phi(x-k)\phi(x-k')dx = \delta_{k,k'}$
- $\int_{-\infty}^{\infty} x^k \psi(x)dx = 0 \qquad \text{for} \quad 0 \leq k \leq N/2-1$

Thus, according to the last property, Daubechies scaling functions of order N can exactly represent any polynomial of order up to, but not greater than $N/2-1$.
In this case, the two-scale relation, Eq.(2..7), yields:

$$\phi_N(x) = \sum_{k=0}^{N-1} p_k\ \phi_N(2x-k) \tag{2..10}$$

It is important to point out that these wavelets have no explicit expression. The scaling functions and derivatives can be evaluated at integer values solving an eigenvalue problem. Technical details are treated in [12].
Using the two scaling relation values of $\phi_N^{(m)}(x)$ at dyadic points, $x = \frac{i}{2^n}$, with $n \in Z$, for $i = 1, 3, 5, \ldots, 2^n(N-1)-1$ can be determined. Therefore, the functions are first evaluated at the integer points $\{0, 1, \ldots, N-1\}$, then at half integers and so on, increasing the value of n from 0 to the desired resolution.

3. Plate wavelet finite element

In this section we describe the way we proposed to solve plate bending problems, using wavelet basis and Galerkin method.
Consider the following Elliptic Boundary Value Problem

$$\begin{cases} L(w(x,y)) &= f(x,y) \quad \text{on} \quad \Omega \\ S(w) &= 0 \quad \text{on} \quad \partial\Omega \end{cases} \tag{3..1}$$

where L denotes a differential operator, Ω the solving domain in $\mathbb{R}^2$, $\partial\Omega$ the boundary of the domain and $S(w)$ the boundary conditions.
Galerkin's method consists of seeking an approximate solution in a finite dimensional subspace of the space of admissible functions, while replacing Ω by a suitable discretization. It is important to note that the quality of the aproximation is completely determined by our choice of the basis functions. Due to wavelet good properties, we propose to expand and approximate the solution w in terms of the scaling functions presented before.

3..1. 2D-basis functions

The domain Ω can be divided into subdomains which are the image of the master element, $\hat{\Omega} = \{(\xi, \eta); \xi, \eta \in [0,1]\}$ under a coordinate map. A direct and simple approach to construct two-dimensional basis functions is to use the tensor product of the wavelet expansions at each coordinate.
Supposing that one-dimensional scaling functions $\phi^1(\xi)$ and $\phi^2(\eta)$ generate multiresolution analyses $\{V_j^1\}$ and $\{V_j^2\}$ respectively, the tensor product space of V_j^1 and V_j^2, $j \in Z$, is

$$V_j = V_j^1 \otimes V_j^2 \tag{3..2}$$

To construct plate elements $\hat{\Omega}$ is divided into $n \times n$ meshes, where $n = m-1$ or $n = N-2$, and m and N are spline and Daubechies orders, respectively.
If we call

$$\begin{aligned} \varphi^1 &= \{\phi^1(\xi), \phi^1(\xi+1), \ldots, \phi^1(\xi+n)\} \\ \varphi^2 &= \{\phi^2(\eta), \phi^2(\eta+1), \ldots, \phi^2(\eta+n)\} \end{aligned} \tag{3..3}$$

then, the scaling functions of $\{V_j\}$ can be expressed using the tensor product of the wavelets expansions at each coordinate, i.e.:

$$\varphi = \varphi^1 \otimes \varphi^2 \tag{3..4}$$

The unknown field function $w(\xi, \eta)$ can be expressed as follows

$$w(\xi, \eta) = \varphi \, \alpha \tag{3..5}$$

where α is the vector of the $(n+1)^2$ wavelet coefficients, corresponding to the elemental nodes. These elemental degrees of freedom should be transformed from wavelet space into physical space [1], and this transformation can be expressed as,

$$T = T^1 \otimes T^2 \tag{3..6}$$

where T^1 and T^2 are the transformation matrices corresponding to one-dimensional problem.

3..2. Computation of Connection Coefficients

When wavelet Galerkin method is applied to solve boundary problems of the form (3..1), stiffness matrices and load vectors, can be calculated using tensorial product of different types of connection coefficients, such as:

$$\Gamma_{i,j}^{d_1 d_2} = \int_0^1 \phi^{(d_1)}(\xi - i)\, \phi^{(d_2)}(\xi - j)\, d\xi \tag{3..7}$$

$$R_i^{(s)} = \int_0^1 \xi^s \, \phi(\xi - i)\, d\xi \tag{3..8}$$

where $i, j \in \mathbb{Z}$, ϕ denotes the basis function and the superscripts d_1 and d_2 refer to differentiation orders.
The problem that arises using Daubechies wavelets is how to calculate these connection coefficients when ϕ is a Daubechies scaling function. In first place, the difficulty is due to the lack of an explicit Daubechies scaling function expression, as it was mentioned in the previous section. Moreover, the highly oscillatory nature of the Daubechies basis functions makes standard numerical quadrature impractical for computing connection coefficients. However, efficient algorithms to calculate these coefficients have been developed in the last years, as the ones proposed by Latto et al. [11] and Beylkin et al. [4].
In the following sections we describe the construction of the stiffness matrices and load vectors, corresponding to the two plate models we considered.

4. Classical Kirchhoff Plate Theory

The basic assumptions for the classical Kirchhoff plate bending theory are very similar to those for the Euler-Bernoulli beam theory. One of the most important assumptions is that a straight line normal to the midplane of the plate before deformation remains normal even after deformation. In other words, the transverse deformation is neglected. With this supposition, in plane displacement of a point of coordinates x, y and z can be expressed as

$$u = -z\frac{\partial w}{\partial x} \qquad\qquad v = -z\,\frac{\partial w}{\partial y} \tag{4..1}$$

where x and y are the inplain axis located at the midplane of the plate, and z is along the plate thickness direction. In addition u and v are the displacements in the x and y axis respectively, while w is the transverse displacement (or called deflection) along the $z-$axis.

According to this theory, the elemental generalized function of potential energy in linear static analysis for a thin plate is,

$$\pi = \frac{1}{2}\int_{\Omega_e} \kappa^T C_b \kappa \ dxdy \quad - \int_{\Omega_e} q\, w \ dxdy \tag{4..2}$$

where κ is the generalized strain,

$$\kappa \;\; = \;\; \left\{ -\frac{\partial^2 w}{\partial x^2}, -\frac{\partial^2 w}{\partial y^2}, -2\frac{\partial^2 w}{\partial x \partial y} \right\}^T \tag{4..3}$$

$$C_b \;\; = \;\; \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{4..4}$$

Ω_e is the elemental solving domain, q is the distributed load, t is the thickness of the plate (assumed constant), E is Young modulus, and ν is Poisson's ratio.

4..1. Kirchhoff plate finite element

In Eq.(4..2), the transversal displacement function can be replaced using the following expression

$$w = \varphi\, T^{-1}\, \widehat{w} \tag{4..5}$$

where $\widehat{w}$ has the physical DOFs of elemental nodes.
Then, substituting Eq.(4..3) into Eq.(4..2) and according to the stationarity condition of π ($\delta\pi = 0$), we obtain the elemental stiffness matrix K^1, and the elemental FEM solving equations are:

$$K^1 \widehat{w} = R \tag{4..6}$$

In the above equation, the stiffness matrix K^1 and load vector R are made up connection coefficients, and have the following expression

$$R \;\; = \;\; ((T)^{-1})^T \int_0^1 \int_0^1 q(\xi,\eta)\varphi^T d\xi d\eta \tag{4..7}$$

$$\begin{aligned} K^1 \;\; = \;\; & D_0\{A_1^{00} \otimes A_2^{22} + \nu(A_1^{20} \otimes A_2^{02} + A_1^{02} \otimes A_2^{20}) \\ & +2(1-\nu)\, A_1^{11} \otimes A_2^{11} + \; A_1^{22} \otimes A_2^{00}\} \end{aligned} \tag{4..8}$$

where $D_0 = \frac{Et^3}{12(1-\nu^2)}$ and

$$A_s^{d_1 d_2} = l_{e,s}^{1-(d_1+d_2)} \;\; (T_s^{-1})^T \; \Gamma_s^{d_1 d_2} \; T_s^{-1}, \qquad\qquad s = 1, 2 \tag{4..9}$$

$l_{e,s}$ is the finite element side length, $\Gamma_s^{d_1 d_2}$ is the connection coefficients matrix defined in Section 3..2., Eq.(3..7), and the subscript s denotes the scaling function φ^s considered in Eq.(3..3).

The elemental stiffness matrix if of size $m \times m$ in using splines of order m, and $(N-1) \times (N-1)$, in case of Daubechies. It is important to point out that, as we have values only at dyadic points, N must be of the form $2^k + 2$. After all elements are assembled together to form the global stiffness matrix, global node displacement vector and global node load vector, the wavelet finite element formulation of plate structures can be obtained by the conventional finite element procedure. The structural analysis can then be performed by finding the solutions of the global finite element equations.

5. Mindlin-Reissner Plate Theory

The plate element formulation we are presenting in this section is based on the theory of plates with the effect of transverse shear deformations included (like Timoshenko beam theory). This theory, due to E.Reissner and R.D.Mindlin, needs only C^0 continuity and uses the assumption that particles of the plate originally on a straight line that is normal to the undeformed middle surface remain on a straight line during deformation, but this line is not necessarily normal to the deformed middle surface.With this assumption, in small displacement bending theory, the displacement components of a point of coordinates x, y and z are

$$u = -z\theta_x(x, y) \qquad v = -z\theta_y(x, y) \qquad w = w(x, y) \tag{5..1}$$

where u and v are inplain displacements, w is the transverse displacement (or called deflection) and θ_x and θ_y are the rotations of the midplane about y and x axes, respectively.
According to Mindlin-Reissner theory, the elemental generalized function of potential energy for Mindlin-Reissner plate bending problem in linear static analysis is,

$$\pi = \frac{1}{2}\int_{\Omega_e} \kappa^T C_b \kappa \ dxdy \quad + \frac{1}{2}\int_{\Omega_e} \gamma^T C_s \gamma \ dxdy \quad - \int_{\Omega_e} q\, w \ dxdy \tag{5..2}$$

where

$$\kappa = \left\{ \frac{\partial\theta_x}{\partial x}, -\frac{\partial\theta_y}{\partial y}, \frac{\partial\theta_x}{\partial y} - \frac{\partial\theta_y}{\partial x} \right\}^T \qquad \gamma = \left\{ \frac{\partial w}{\partial x} + \theta_x, \frac{\partial w}{\partial y} - \theta_y \right\}^T \tag{5..3}$$

$$C_s = \frac{Etk}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

C_b, Ω_e, q, t, E and ν are the same as in Section 4. and k is the shear correction factor equal to $\frac{5}{6}$.
One thing to be noted here is that the first term in (5..2) corresponds to bending energy, while the other is the transverse shear energy and this last term becomes dominant compared to the bending energy as the plate thickness becomes very small compared to its side length.

5..1. Mindlin-Reissner plate finite element

For the plate problem, Eq.(5..2), independent interpolation is considered and the same shape functions are used for the displacements and slope interpolations. In this way, the elemental displacement functions, Eq.(5..1), can be replaced by

$$\theta_x = \varphi\, T^{-1}\, \widehat{\theta}_x, \qquad \theta_y = \varphi\, T^{-1}\, \widehat{\theta}_y, \qquad w = \varphi\, T^{-1}\, \widehat{w} \tag{5..4}$$

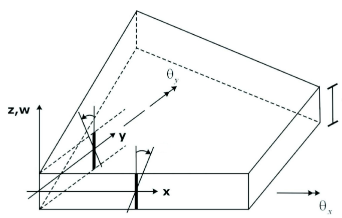

Figure 1: Mindlin-Reissner Plate element

where $\widehat{\theta}_x$, $\widehat{\theta}_y$ and $\widehat{w}$, are the physical DOFs of elemental nodes, see Fig.(1).
Then, substituting Eq.(5..4) into Eq.(5..2) and according to the stationarity condition of π ($\delta\pi = 0$), we obtain the elemental stiffness matrix.
Finally, the elemental FEM solving equations can be expressed by:

$$\begin{bmatrix} K^1 & K^2 & K^3 \\ K^4 & K^5 & K^6 \\ K^7 & K^8 & K^9 \end{bmatrix} \begin{bmatrix} \widehat{\theta}_x \\ \widehat{\theta}_y \\ \widehat{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ R \end{bmatrix}, \tag{5..5}$$

$w_c/(qL^4/100D_0)$			
Mesh	Spline	DSCW10	MITC4
1x1	–	0.4019	
2x2	0.2866	–	0.3189
4x4	0.3699	–	0.3969
8x8	0.3967	–	0.4042
Thin plate sol.	0.40625		

Table 1: Central displacements for simply supported square plate subjected to uniform load $t/L = 0.01$. Kirchhoff-Love Theory.

where

$$\begin{aligned}
R &= ((T)^{-1})^T \int_0^1 \int_0^1 q(\xi,\eta)\varphi^T d\xi d\eta \\
K^1 &= D_0\{A_1^{11} \otimes A_2^{00} + (1-\nu)/2\ A_1^{00} \otimes A_2^{11}\} + C_0\ A_1^{00} \otimes A_2^{00} \\
K^2 &= D_0\{\nu A_1^{10} \otimes A_2^{01} + (1-\nu)/2\ A_1^{01} \otimes A_2^{10}\} \\
K^3 &= -C_0\ A_1^{01} \otimes A_2^{00} \\
K^4 &= (K^2)^T \\
K^5 &= D_0\{A_1^{00} \otimes A_2^{11} + (1-\nu)/2\ A_1^{11} \otimes A_2^{00}\} + C_0\ A_1^{00} \otimes A_2^{00} \\
K^6 &= -C_0\ A_1^{00} \otimes A_2^{01} \\
K^7 &= (K^3)^T \\
K^8 &= (K^6)^T \\
K^9 &= C_0\ A_1^{11} \otimes A_2^{00} + A_1^{00} \otimes A_2^{11}
\end{aligned}$$

and $D_0 = \frac{Et^3}{12(1-\nu^2)}$ and $C_0 = \frac{Etk}{2(1+\nu)}$.
In this model, each node has three DOFs, consequently one finite element has $3 \times n^2$ DOFs (as before, $n = m$ for splines and $n = N - 1$ for Daubechies).
The global stiffness matrix is obtained assembling elemental matrices, as it was done in the previous model.
In the following section the finite element implementation is validated and numerical solutions are presented for both models.

6. Applications

The formulations of two-dimensional tensor product spline and Daubechies plate elements developed in Section 4. and Section 5., are applied to a typical numerical example: a square isoparametric plate subjected to uniform load q, simply supported on all four edges. Let Poisson's rate μ be fixed as 0.3 and $t/L = 0.01$, where t and L denote plate thickness and side length, respectively.

$w_c/(qL^4/100D_0)$			
Mesh	Spline	DSCW6	MITC4
1x1		0.3125	
2x2	0.3636	0.345	0.3189
4x4	0.3941	0.3961	0.3969
8x8	0.4033	0.4042	0.4042
Thin plate sol.	0.40625		

Table 2: Central displacements for simply supported square plate subjected to uniform load $t/L = 0.01$. Mindlin-Reissner Theory.

In Table 1 results of central displacements for the Kirchhoff-Love plate model are presented, considering the thickness $t/L = 0.01$. Numerical solutions obtained with splines of order 4 and with Daubechies of order 10, are in good agreement with those obtained using MITC4 plate element, [3], and with the exact thin plate solution. It can be observed that with only one single DSCW10 element, 99 $\%$ of the exact value is obtained. It is important to point out that in this case, we had to use DSCW10 because as it was derived in Subsection 4..1., in this model second derivatives appear in the stiffness matrix, which have high oscillations and then, $k = 3$ was considered in the expression $N = 2^k + 2$, for ensuring stability. Further details about how to choose the order of the scaling functions, can be found in [1].

In Table 2 the results of central displacements with the hypothesis of Mindlin-Reissner theory are presented. As in the first model, good aproximations were obtained with both wavelet families, in particular, a 2×2 mesh yield excellent results. For the other meshes the values are similar to those obtained with the MITC4 element. In this case, we use DSCW6, a lower order of Daubechies than in the classical model because, as it was derived in Subsection 5..1., only first derivatives appear in the stiffness matrix [1].

Computational effort required using wavelet basis was also investigated. It was observed that with standard finite elements, the required CPU time is about three times larger than with wavelet-based elements to achieve similar accuracy.

7. Conclusions

The feasibility of the wavelet-Galerkin method in the numerical computations of structural mechanics is investigated. The details of the discretization process for the classical Kirchhoff and Mindlin-Reissner plate models are described and the corresponding approach is developed. Numerical results on a practical engineering problem show that the present method is competitive with FEM.

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