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MATHEMATICS AND IDEOLOGY: THE POLITICS OF INFINITESIMALS

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Dedicated to the Memory of Mariano Hormigón

Pero en lo que se refiere al análisis de los asuntos cotidianos de orden sociopolítico lo que se filtra y se infiltra en el proceso de información y de opinión es, lisa y llanamente, ideología.
[HORMIGÓN, 1996a, p. 347]

RESUMEN

Los *Manuscritos Matemáticos* de Karl Marx fueron traducidos al chino en 1975, al final de la Revolución Cultural, por dos equipos de especialistas, uno de Beijing y otro de Shanghai. Aquí se estudia la respuesta a estas traducciones de matemáticos, filósofos e incluso del público en general, incluyendo el papel desempeñado por el análisis no estándar en la recepción de las ideas de Marx en China. La aproximación de Marx al cálculo, basada en el concepto de infinitesimal, obtuvo renovada difusión gracias a la teoría del análisis no estándar de Abraham Robinson. También se aborda cómo el análisis no estándar fue un tema de interés creciente entre matemáticos, filósofos y lógicos en China, incluyendo el primer simposio chino de análisis no estándar, celebrado en 1978.

ABSTRACT

The *Mathematical Manuscripts* of Karl Marx were translated into Chinese in 1975, at the end of the Cultural Revolution, by two teams of scholars working in China, one in Beijing, the other in Shanghai. The response to these translations by mathematicians, philosophers, and even the public at large, is discussed here, including the role nonstandard analysis played in the reception of Marx's ideas in China. Marx's understanding of the calculus, which he based on the concept of infinitesimal, was given new currency thanks to Abraham Robinson's theory of nonstandard analysis. How nonstandard analysis was a subject of growing interest among mathematicians, philosopher and logicians in China is also considered, including the first Chinese symposium for nonstandard analysis held in 1978.

Palabras clave: Matemáticas, Análisis no estándar, China, Siglo XX, Karl Marx, Mao Zedong, Revolución Cultural, Ciencia e ideología.

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The topic of this paper was originally suggested by Mariano Hormigón for the Third International Galdeano Symposium held at the University of Zaragoza in September of 1996, devoted to the subjet of «Ciencia e Ideología». Had it not been for Mariano's invitation to participate in that meeting, and to consider the subject addressed here as my contribution to the symposium, this paper would not have been written. It is to the memory of Mariano Hormigón that I wish to dedicate this paper, as a token of my gratitude for his many contributions to the history of mathematics, especially to the development of the subject internationally through such meetings as those he organized in Zaragoza in the name of Zoel García de Galdeano.

The *Mathematical Manuscripts* of Karl Marx were first published (in part) in Russian in 1933, along with an analysis by S.A. Yanovskaya. Friedrich Engels was the first to call attention to the existence of these manuscripts in the preface to his *Anti-Dühring* [1885]. A more definitive edition of the *Manuscripts* was eventually published, under the direction of Yanovskaya, in 1968, and subsequently numerous translations have also appeared. Marx was interested in mathematics primarily because of its relation to his ideas on political economy, but he also saw the idea of variable magnitude as directly related to dialectical processes in nature. He regarded questions about the foundations of the differential calculus as a *touchstone of the application of the method of materialist dialectics to mathematics.*

Nearly a century later, Chinese mathematicians explicitly linked Marxist ideology and the foundations of mathematics through a new program interpreting calculus in terms of nonstandard analysis. During the Cultural Revolution (1966-76), mathematics was suspect for being too abstract, aloof from the concerns of the common man and the struggle to meet the basic needs of daily life in a still largely agrarian society. But during the Cultural Revolution, when Chinese mathematicians discovered the mathematical manuscripts of Karl Marx, these seemed to offer fresh grounds for justifying abstract mathematics, especially concern for foundations and critical evaluation of the calculus. At least one study group in the Department of Mathematics at Zhejiang Teachers College issued its own account of *The Brilliant Victory of Dialectics — Notes on Studying Marx's 'Mathematical Manuscripts'* [ZHEJIANG, 1975]

Inspired by nonstandard analysis, introduced by Abraham Robinson only a few years previously, some Chinese mathematicians adapted the model Marx had laid down a century earlier in analyzing the calculus, and especially the nature of infinitesimals in mathematics, from a Marxist perspective. But they did so with new technical tools available thanks to Robinson but unknown to Marx when he began to study the calculus in the 1860s. As a result, considerable interest in nonstandard analysis has developed subsequently in China, and almost immediately after the

Cultural Revolution was officially over in 1976, the first all-China conference on nonstandard analysis was held in Xinxiang, in Henan Province, in 1978.

I. Mathematics and Ideology

Euclidean geometry, and the mathematical methods it embodies, have been hallmarks of critical thinking and universally valued as one of the greatest of human intellectual achievements. This is reflected directly in the frontispiece contrived by the Oxford University engraver Michael Burghers for David Gregory's dual-language edition in Greek and Latin of Euclid's *Elements*, wherein the shipwrecked Socratic philosopher Aristippus, washed ashore with his fellow shipmates, points to geometric diagrams he sees in the sand on shore and exclaims: *«Hominum enim vestigia video»* — *I see a vestige of man!* [Figure 1].

This shipwreck is reminiscent of another less fortunate incident at sea, the death of the fabled Pythagorean Hippasus. One of the basic tenets of early Pythagorean philosophy was the concept of harmony and commensurability —all things could be measured by numbers, i.e. integers, or by ratios of integers. But discovery of the incommensurability of the side of a square with its diagonal, a geometric equivalent to the algebraic assertion that $\sqrt{2}$ is irrational, was a patent contradiction of the basic Pythagorean idea. As the story goes, the Pythagoreans were so dismayed by this discovery that they swore all who knew the existence of incommensurable magnitudes to secrecy. Hippasus, an initiate who is said to have divulged the secret, died at sea:

> It is well known that the man who first made public the theory of irrationals perished in a shipwreck in order that the inexpressible and unimaginable should ever remain veiled [...] and so the guilty man, who fortuitously touched on and revealed this aspect of living things, was taken to the place where he began and there is forever beaten by the waves[1].

Although discovery of the incommensurability of the diagonal of the square may have been a crisis for Pythagorean ideology, if not for Pythagorean mathematics, it clearly raised a troubling ideological problem that linked the certainty of mathematics with the certainty of the claims of Pythagorean philosophy —and with it the image of the Pythagorean sect itself. If one of its most basic principles were known to be false, then surely the credibility of Pythagoreanism in general would be open to question. Thus discovery of incommensurability revealed an aspect of infinity with a dangerously political edge at least for the Pythagoreans[2].

The edge was different for Plato, who took mathematics to play an important role in educating citizens in the best of all forms of government, as he explains in the *Republic*. But more specifically, he took the proof that the diagonal of the square is incommensurable with its side to be a direct reflection of the triumph of the human mind, and this in turn provided an important focus for Plato's ideological polemic in his dialogue, the *Theatetus*[3].

In the 17th century the infinite returned as an ideological as well as a political issue in debates over the infinitesimal calculus. The debate was ideological in the objections to infinitesimals and their ambiguous zero/non-zero character, which led to disagreements articulated by Berkeley in England and Nieuwentijdt on the continent. But the debate was also political, especially in the acrimonious disagreement between Newton and Leibniz over the question of priority —who discovered the calculus first? The resulting battle, *Philosophers at War* (as A. Rupert Hall has so colorfully described the opposition of Newtonians and Leibnizians), had an explicitly political subtext in so far as it was a battle between English mathematicians concerned about the Hannoverian succession and Leibniz's prominent role as advisor, historian and leading scientific figure in the German court [HALL, 1980].

Arnold Thackray has also described these *politically troubled times* in England: *With the very succession to the throne insecure, the fact that Leibniz had the ear of the Electress Sophia — the destined English monarch was— in light of Newton's neurotic, suspicious and jealous character —little short of a disaster* [THACKRAY, 1970, p. 52; see also THACKRAY, 1968]. In fact, the Newton-Leibniz controversy may be taken as a prime example of what Mariano Hormigón has called the *intervención de la ideología* in cognitive progress [HORMIGÓN, 1996b]. Political tensions between the English supporters of Newton and the leading scientific figure among the Hanoverians, namely Leibniz, were reflected not only in the priority dispute over the calculus, but in other differences as well —over gravity, the Newtonian theory of matter, optics, space, time— in fact, on virtually any issue the one side felt it could use against the other[4].

In the 19th century, debate over the infinite continued on many fronts, among others notably in the context again of the calculus in the hands of Augustin-Louis Cauchy and his approach to rigor[5]. Although discussion continued concerning infinitesimals and whether to include or exclude them from rigorous mathematics, the most dramatic use of the infinite was due to the controversial ideas of Georg Cantor, creator of transfinite set theory. Cantor's set theory, above all his theory of transfinite numbers, precipitated disagreeable differences between Leopold Kronecker on the one and, and Georg Cantor and Karl Weierstrass on the other[6]. The debate with Kronecker may have been ideological, but the problem of the infinite, as well as the calculus, also assumed political significance at about this same time in the hands of Karl Marx.

II. Marx and Mathematics

Marx wrote his dissertation at Jena on the Greek atomists, interpreted from an Hegelian perspective. Of the two major atomists, he preferred Epicurus to Democritus *because Epicurus gave a social content to his atomism* [STRUIK, 1992, p. 741]. In 1858, in preparing the first draft of *Das Kapital*, Marx began a serious self-study of mathematics in earnest. Meanwhile, his colleague Friedrich Engels took up physics and other natural sciences. Both were convinced that *without knowledge of natural science* their analysis of society would be severely handicapped. *Natural science,* wrote Marx in a draft of 1863 for his *Kapital, is the foundation of all knowledge* [STRUIK, 1992, p. 742].

As for the interest Marx showed in mathematics, Dirk Struik explains that *The dialectical interplay of the finite and infinite intrigued both Marx and Engels* [STRUIK, 1992, p. 747][7]. Unfortunately, Marx undertook his study of the calculus and subsequent critical evaluation of its foundations apparently in ignorance of the works of Cauchy and before the increasingly technical demands for rigor advanced by Weierstrass, Dedekind and Cantor had become hallmarks of modern, rigorous mathematics. Marx makes no direct mention of Cauchy, and seems to have limited his reading to basic textbooks of the 18th and early 19th centuries, primarily books that he was able to read in the British Library in London[8].

Marx was interested in mathematics primarily because of its relation to his ideas on political economy. Engels was also interested in the calculus but doubted its foundations, which he felt were dubious, especially the introduction of variable magnitudes and the extension of their variability over infinitely small and large quantities. As he said in his famous polemic against Eugen Dühring, *most people differentiate and integrate not because they understand what they are doing but from pure faith, because up to now it has always come out right*[9].

Engels wrote, perhaps too poetically, that the introduction of variables (and the extension of their variability as far as infinitesimals and the infinitely large) committed the *original sin —whereby the otherwise so pure mathematics had eaten of the apple of knowledge, which opened the way to greatest success, but also the way to errors. The original condition of the absolute validity, the irrefutable provability of all things mathematical were long past, and the reign of controversies had broken out* [ENGELS, 1894, p. 81].

Marx looked deeper, and saw the idea of variable magnitudes as directly related to dialectical processes in nature. In 1881 he wrote up treatments of both derivatives and integrals which he sent to Engels [KENNEDY, 1977, p. 307]. In doing so, much of Marx's thinking was clearly influenced by Hegel. Marx approached his analysis of the derivative, for example, in terms of a dialectical process, the *negation of the negation.*

Engels, who was less theoretical and much more practical when it came to mathematics, preferred to advance a materialistic view of the calculus, and looked for analogs of infinitesimals in the real world: *Nature offers prototypes of all these imaginary magnitudes* [ENGELS, 1962b, pp. 529-534, esp. p. 530][10].

Despite the limited quality of his reading, Marx, as the Italian mathematician Lombardo Radice has observed, was interested in the calculus for philosophical reasons related to the foundations of his own philosophy:

Marx gave so much attention and so much effort of thought in the last years of his life to the foundations of differential calculus because he found in it a decisive argument against a metaphysical interpretation of the dialectical law of the negation of the negation[11].

A similar opinion was expressed by the Soviet historian and philosopher of mathematics, Konstantin Alexeyevich Rybnikov, who points out that *the difficult task of the foundation of differential calculus became for Marx the touchstone of the application of the method of materialistic dialectics to mathematics* [KENNEDY, 1977, p. 316]. A sense of what Marx had in mind may be gleaned from the following passage taken from Friedrich Engels' *Anti-Dühring*:

Therefore, dy/dx, the ratio between the differentials of x and y, is equal to 0/0, but 0/0 taken as the expression of x/y. I only mention in passing that this ratio between two quantities which have disappeared, caught at the moment of their disappearance, is a contradiction; however, it cannot disturb us any more than it has disturbed the whole of mathematics for almost two hundred years. And now, what have I done but negate x and y, though not in such a way that I need not bother about them any more, not in the way that metaphysics negates, but in the way that corresponds with the facts of the case? In place of x and y, therefore, I have their negation, dx and dy, in the formulas or equations before me. I continue then to operate with these formulas, treating dx and dy as quantities which are real, though subject to certain exceptional laws, and at a certain point I negate the negation, i.e., I integrate the differential formula, and in place of dx and dy again get the real quantities x and y, and am then not where I was at the beginning, but by using this method I have solved the problem on which ordinary geometry and algebra might perhaps have broken their jaws in vain [ENGELS, 1894, pp. 164-165].

III. Marx and the Mathematical Manuscripts in the Soviet Union

Soon after the Bolshevik Revolution of 1917, Soviet authorities became interested in Marx's manuscripts, including those devoted to mathematics. Following a series of negotiations throughout the 1920s, photocopies of the papers were obtained for the Marx-Lenin Institute in Moscow[12]. The director of the Institute, a well-known historian, Professor David Borisovic,ˇ Ryazanov, created a special group to study and publish the manuscripts, headed by Professor Sofia Alexandrovna Yanovskaya —a well-known historian of mathematics and an ardent partisan of the Soviet mathematical community. Study of the *Mathematical Manuscripts* became a special preoccupation for Soviet mathematicians, historians and philosophers of mathematics.

The first serious report describing the mathematical manuscripts of Karl Marx was presented by Ernst Kol'man at the famous 1931 *crossroads meeting* in London [KOL'MAN, 1931b]. The following year he also made a brief report about the manuscripts at the International Congress of Mathematicians in Zürich[13]. The earliest results of Yanovskaya's research group, however, were published in an article of 1933, *On Marx's Mathematical Manuscripts* [YANOVSKAYA, 1933]. After World War II, the best-known of her group was Rybnikov, who succeeded Yanovskaya in the chair for history and philosophy of science at Moscow University [VOGT, 1983, p. 54]. They finished their work on the manuscripts only in the 1960s, and published a substantial volume in 1968[14]. The first of Marx's mathematical manuscripts to be published, however, actually appeared in 1933 (devoted to his thoughts about the nature of differentiation). Marx had written this piece for Friedrich Engels, and it appeared appropriately enough on the 50th anniversary of Marx's death in 1933 [YANOVSKAYA, 1933]. But it was not for another twenty-five years, until 1958, that another translation of a manuscript by Marx was published —this time on the concept of function. Another ten years elapsed before a complete translation of all the mathematical manuscripts by Marx appeared in the dual Russian-German edition under the editorship of Yanovskaya in 1968 [VOGT, 1983, p. 54, note 24].

IV. Mao Ze-Dong / Mao Tse-tung (毛澤東 / 毛泽东)[15]

Mao Ze-Dong (1893-1975) was born in Shao Shan, a village in Hunan Province in Southern China, on December 26, 1893. When he was eighteen years old, having passed the necessary entrance examination, he went to Ch'ang-sha, the provincial capital, where he entered middle-school in 1911. Before the year was out, in October he joined the revolutionary army that soon overthrew the Qing dynasty. Within a year Mao had left the army, after which he spent six months reading every day back in the local library. He soon felt prepared to further his education at Ch'ang-sha Teacher's College, from which he graduated at the age of 24. Mao then moved to the capital where he not only worked in the Beijing University library, but he joined a Marxist study group as well.

In 1919 Mao again returned to Ch'ang-sha. There he assumed a position as principal of a primary school, and at the same time also served as secretary to the local Communist group. When the Chinese Communist Party (CCP) was officially organized in 1921, Mao was on hand in Shanghai. Two years later he was elected a member of the party's Central Committee, and the following year he joined the Central Executive Committee of the Kuomintang (KMT, the republican party of followers of Sun Yat-sen).

Illness forced Mao to return to his peasant village, and it was during this extended period of isolation and self-reflection that he came to appreciate the revolutionary potential of the peasantry in China. Faith in its power to support revolution would eventually set Mao apart from both the CCP and the KMT, both of which were much more interested in winning over the urban proletariat than the rural peasantry.

After the Shanghai massacre in 1927, when Chang Kai-shek turned his KMT forces against the CCP, Mao literally took to the hills where he set up his revolutionary base camp in the Ching-kang mountains. Eventually the KMT succeeded in driving the Communists to the North-West of China, and the famous *Long March* of 1934 brought Mao and his followers to Yenan, where what was left of the CCP took refuge and established a new base of operations. Slowly regrouping his forces, and following the War of Resistance against Japan, Mao and the Communists were able to win the Civil War in China against Chang Kai-Shek and the KMT, at least controlling most of the Mainland. On October 1, 1949, the People's Republic of China was founded in Beijing, whereupon Mao Ze-dong was elected Chairman of the Republic.

V. Marxism, Scientism and Chinese Thought in General

Well before Mao Ze-dong, many prominent Chinese intellectuals had already advocated strong, necessary links between science and the hoped-for transformation of Chinese society from a feudal to a modern socialist or communist state. As Chen Du-xiu / Ch'en Tu-hsiu (陳獨秀) wrote in 1920:

> [Political] evolution goes from feudalism to republicanism and from republicanism to communism. I have said that the [Chinese] republic has failed and that feudalism has been reborn, but I hope that soon the feudal forces will be wiped out again by democracy and the latter by socialism [...] for I am convinced that the creation of a proletarian state is the most urgent revolution in China[16].

In his role as editor of the influential journal *New Youth* (新青年), Chen Du-xiu linked the future of China directly with science and modernity. Chen maintained that *Republican government in politics and science in the domain of ideas, these seem to me the two treasures of modern civilization* [17] . Furthermore, in the famous debate over the appropriateness and efficacy of modern science versus traditional philosophy in modern China, reflected in two volumes of essays devoted to the myriad opinions that were voiced on the subject and entitled *Science and the Philosophy of Life* (科學與人生觀), Chen Du-xiu insisted that *The old ethics is no longer adapted to the modern world, which is ruled by economy*[18].

However, at the end of the Qing dynasty, the old ethics was still dominated by basically three religions or philosophies, traditional Confucianism, Buddhism, and Taoism. None of these was compatible with the progress of modern science or the new China as it was envisioned by most intellectuals. As Chen Du-xiu explained in an essay on *The Confucian Way and Modern Life,* again in *New Youth* (新青年):

> Confucius was born and lived during feudal times, and the ethics he advocated were feudal. What he taught and handed down as the religion of rites was but an attitude toward life, and this feudal religion of rites was nothing but a feudal attitude of life. His politics were feudal politics. The feudal ethics, religion of rites, attitude toward life, and politics had as their central reference the power and reputation of a small aristocracy; it had no connection whatsoever with the life and happiness of the large majority of common citizens[19].

Wu Zhi-hui / Wu Chih-hui (吳稚暉) raised similar objections to Buddhism as inimical to the scientific way of approaching nature, and in an essay called *A New Belief* (一個新信仰) went so far as to characterize Buddhism as a *tragic form of terrorism* inflicted on China. Above all, it was held responsible for the lifeless quality of Chinese society in the wake of its influence:

> Buddhism was a religion which teaches man to forsake this world and prepare for life in the other world. But, when Chu Hsi and his co-workers unconsciously adopted this religion of the other world and superimposed its ideas upon the moral and political codes for life in this world, then the new codes became terrors and made Chinese society a tragedy. How lifeless has Chinese society become since the twelfth and thirteenth centuries![20]

Taoism, on the other hand, was little better. It was described by the French Jesuit and sinologist O. Brière as the *enemy of all progress* which he equated, in the end, with the same deleterious effects as Buddhism:

> Taoism, by its worship of pure «nature» and its contempt of civilization, and by its theory of *laissez-faire* (which not long since inspired Tolstoy), set itself up as the enemy of all progress. Buddhism, by preaching the «emptiness» of all things and the escape from this world, had eventually the same result. The philosophies of these two systems, eminently anti-social and unsuitable to the governing of a State, provoked a reaction in favour of Confucianism, an essentially positive system of politics and morals [BRIÈRE, 1956, p. 13].

Marxist scholars, even those interested in Chinese antiquity and the classics, were not impressed by the Tao, Buddha or Confucius, but instead emphasized the necessity of studying Marx and Engels, praising the emphasis they placed on a timely, realistic point of view. The call for change, which Mao clearly heard, was evident in the words of Guo Mo-ruo / Kuo Mo-jo (郭沫若):

When one wishes to speak of antiquity, it does not suffice to study the written documents and the old historians. It is necessary first of all to know Marx and Engels, for they give us the key to all interpretation to come, that is to say, the materialistic point of view. It is necessary then to get rid of the ordinary, classical, self-styled scientific method of the historians, which is in fact influenced by their prejudices[21].

VI. Mao Ze-dong and the Early Influences on His Thought

Marxism was expected to reinvigorate what many regarded as the *static and dormant* character of traditional Chinese thought, and to reverse the literary spirit of Chinese thinking in favor of a scientific outlook. As Ren Hong-jun / Jen Hung-chün (任鴻雋) put it in an essay published in 1919 on *The Future of Academic Thinking in Our Country* (我國學術思想之將來):

A general view of four thousand years of the history of Chinese thought reveals that it has been literary rather than scientific. The acceptance of an idea and the establishment of schools of thought have been formed on the basis of intuition, not material facts [...]. [Chinese thought] has followed subjective observation and not objective analysis; it has exhausted the vicissitudes of human affairs and not studied the diversity of matter. The material [for thought] has been simple, and therefore the application of thought has not been widespread. Is it any wonder that [thought] has been static and dormant, not showing a single thread of light?[22]

Objective analysis for Mao Ze-dong meant criticism using dialectics leading to correct ideas. Mao believed that correct ideas came only from social practice, and correct social practice depended on a Marxist epistemology:

Where do correct ideas come from? Do they drop from the skies? No. Are they innate in the mind? No. They come from social practice, and from it alone [...]. It is man's social being that determines his thinking. Once the correct ideas characteristic of the advanced class are grasped by the masses, these ideas turn into a material force which changes society and changes the world [...]. Furthermore, the one and only purpose of the proletariat in knowing the world is to change it. Often, correct knowledge can be arrived at only after many repetitions of the process leading from matter to consciousness and then back to matter, that is, leading from practice to knowledge and then back to practice. Such is the Marxist theory of knowledge, the dialectical materialist theory of knowledge [MAO ZE-DONG, 1967, pp. 405-406].

Much of what the Chinese learned about Marxism and revolution came initially through contacts from the Soviet Union and drew on the lessons of the great Revolution in Russia:

> Bortman, even before my arrival, had fairly wide ties with progressive Chinese students from the higher educational institutions and colleges in Tientsin and Peking, and personally with Professor Li Ta-chao [Li Da-zhao 李大钊(1889-1927)], whom Bortman spoke of as an excellent Marxist [...]. When I met Bortman in September 1919, the ties with the students were maintained as before, and one group after another visited our apartment nearly every evening.
>
> We acquainted the Chinese students with Lenin's work on Imperialism, the Highest Stage of Capitalism, often touching on problems relating to China, analyzed the works of Sun Yat-sen, and told them about the leading role of the working class in the October Socialist Revolution in Russia [MULLER, 1957].

Although the Chinese had been exposed to two of the leading philosophers from the West when John Dewey and Bertrand Russell visited China —and many were impressed by the Pragmatism of the American, Dewey, as well as by the mathematical logic of Russell— it was Hegelian dialectics and especially its expression in the form of Marx's dialectical materialism that were to become extremely influential:

> If the experimental logic of Dewey and the mathematical logic of Russell have had their day, now they have lost their authority. They have been supplanted by a new philosophical method, the Dialectic[23].

But, as Mariano Hormigón has described Hegelian dialectics —*las piruetas discursivas de raiz hegeliana que enmascaran la claridad de los conceptos a costa del rigor metodológico*— applying Marxist principles to science in general, and to mathematics in particular, required just as many inspired intellectual leaps and turns, but with little gain in clarity and rigor [HORMIGÓN, 1996a, p. 337]. In addition to Hegel, Chinese writers stressed the importance of knowing Marx and Engels as they hoped China would march headlong into the 20th century embracing the virtues of modern science.

VII. Scientism in China

One of the influential new journals in China after World War I was the *Twentieth Century* (二十世纪). The advertising prospectus for the journal makes clear the importance of science, especially the critical nature of the scientific method as a means of criticizing thought in general:

> The present review treats of theoretical science, [and] is devoted to criticism of thought and to ideological construction. It seeks to judge history and theory by relying upon scientific

truth. Its aims are to unify science and philosophy, to create a theoretical system of natural sciences, to blend the social sciences and the natural sciences, and to construct the science of thought[24].

Another journal devoted to progress in China, understood in terms of science and technology, was *New China* (新中華). In an article devoted to *Sincere Views on Saving the Country with Motors* (摩托救國之肊見), Wu Zhi-hui / Wu Chih-hui (吳稚暉) used traditional Chinese seal characters to praise technology, especially motors and the steam engine [Figure 2]:

The world changed since the invention of the steam engine. The world changed again when the petroleum-driven engines were invented. The nineteenth century is an age governed by the steam engine; the twentieth century is an age ruled by the petroleum-driven engine. How miraculous, the motor! How sacred, the motor![25]

In addition to an understandable fascination with technology, the reformers were also convinced that science could help dispel errors and inexactness in general. Again and again one reads in both the Marxist and the Chinese literature that one goal of critical, dialectic philosophy, is to *destroy all mystery:*

Each man has his own ideas which come from his heredity, from the milieu, and from various circumstances —ideas which are not always sound and exact. It is important to rectify them. This is the mission of progressive philosophy. It helps us to penetrate problems more quickly and more exactly, to destroy all mystery, to discover the origin of all things, to combat all oppression[26].

Similar attitudes were often reflected in negative references to *subjective prejudice* and general faith in the value of scientific method, as stressed by Ding Wen-jiang (丁文江), for example, in his contribution to the debate with Zhang Jun-mai / Chang Chün-mai (張君勱) (Carsun Chang) over *science* versus *metaphysics* in 1923:

The aim of science is to eliminate from the philosophy of life preconceived and subjective ideas, the greatest enemy of the philosophy of life, [and] to search for the kind of truth that can be recognized by all [...]. Science is all-sufficient not so much in its subject matter as in its method and procedure[27].

Not everyone, however, was convinced that science and its methods were the answer. Zhang Jun-mai / Chang Chün-mai (張君勱) was among those who still valued the roots, historically, of Chinese philosophy in Confuciansim:

Science cannot solve the problems of life. The great philosophers of history are those who have tried to find a solution to the problems of life. Among us there has been a series of philosophers, from Confucius and Mencius to the Sung and Ming neo-Confucian literati, who have produced the great spiritual civilization of China[28].

Unfortunately, China's traditional philosophies were at odds with the struggle to bring China into the 20th century, to modernize it sufficiently to make it competitive with the rest of the industrialized world. As mentioned briefly above, Ren Hong-jun recognized this in ascribing much of the static, uninspired quality of traditional Chinese thought to the passivity and *other-worldly* character of the classics.

VIII. Mao Ze-dong and the Importance of Modern Science

As for Mao Ze-dong, he firmly believed that a proper understanding of science, especially the laws of nature, was crucial. He put it this way in a well-known lecture of 1937, *On Practice* (实践论):

> If a man wants to succeed in his work, in other words, to achieve the anticipated results, he must train his thought to correspond with the laws governing the objective external world; if they do not correspond, he will fail in practice[29].

Mao insisted that the methods of dialectical materialism would provide the key, the *spiritual weapon* as he called it, for bringing about the revolution that would lead to a new and proper world view. In Chapter Two of his *Lecture Notes on Dialectical Materialism,* Mao explained that *Dialectical Materialism Is the Revolutionary Weapon of the Proletariat:*

> Dialectical materialism is the world view of the proletariat. The proletariat, which has been given the task by history of eliminating classes, utilizes dialectical materialism as a spiritual weapon in its struggle and as the philosophical basis for its various viewpoints [...]. The Chinese proletariat, which is at present shouldering the historical task of the bourgeoisie-democratic revolution in order to arrive at socialism and communism, must adopt dialectical materialism as its spiritual weapon[30].

This in turn was linked in Mao's view to applying the understanding of nature, i.e. science, to actively *change* the world. This is how he put it in his essay, *On Practice,* in 1937:

> The epistemology of dialectical materialism is that rational knowledge depends upon perceptual knowledge and that perceptual knowledge remains to be developed into rational knowledge. In philosophy, neither «rationalism» nor «empiricism» grasps the historical or dialectical nature of knowledge, and although each of these contains some truth (here I am referring to materialist, not to idealist, rationalism and empiricism), both are wrong about epistemology as a whole [...]. But the process of knowing does not end here [...]. Marxist

philosophy holds that the most important problem does not lie in merely understanding the laws of the objective world in order to explain it, but in applying the knowledge of these laws to actively change the world[31].

From a Marxist perspective, Mao took the *fundamental law* of nature to be a Hegelian *unity of opposites* which was not only universal, but had direct social implications. What was to be of greatest significance for his views on revolution, as well as the lessons mathematicians in China would later draw from Mao's views, were the roles that contradiction, struggle, motion, and constant change should play in bringing about changes on all levels of life and society. In an essay *On the Correct Handling of Contradictions Among the People,* Mao expressed his indebtedness to Lenin for making this *fundamental law* especially clear:

Marxist philosophy holds that the law of the unity of opposites (对立统一) is the fundamental law (根本规律) of the universe. This law operates universally, whether in the natural world, in human society, or in man's thinking. Between the opposites in a contradiction there is at once unity and struggle, and it is this that impels things to move and change. Contradictions exist everywhere, but their nature differs in accordance with the different nature of different things. In any given thing, the unity of opposites is conditional, temporary and transitory, and hence relative, whereas the struggle of opposites is absolute. Lenin gave a very clear exposition of this law[32].

Indeed, Mao quoted Lenin to make a point he wished to emphasize about Newtonian views of space and time. In physics, the Maoist critique took the abstract world of Newtonian physics to task:

Of particular note is Newtonian mechanics, which treats space and time as unrelated and static insubstantial entities, and which situates matter within this insubstantial context. Dialectical materialism, in opposition to this theory of mechanics, points out that our conception of time and space is a developmental one. «There is nothing in the world but matter in motion, and matter in motion cannot move otherwise than in space and time. Human conceptions of space and time are relative, but these relative conceptions go to compound absolute truth. These relative conceptions, in their development, move towards absolute truth and approach nearer and nearer to it», (Lenin)[33].

Imbedded in their views of social change and revolution, *change* was directly related to a proper understanding of motion, and motion in turn, including mechanical motion, was to be understood in terms of dialectical principles reflecting the contradiction of opposites and the unity of their synthesis. Thus a moving point at any given instant both *occupied* a given position, and in so far as it was moving continuously, according to Mao, was *not occupying it.* This analysis was applicable to any kind of change, and Mao specifically considered examples from physics, chemistry, biology and *social phenomena.* Here the sciences were critical in correctly understanding motion and the transformation of matter from one state to another, as Mao explained in a section of his *Lecture Notes on Dialectical Materialism* entitled *On Motion (On Development):*

> It must be understood that all forms of motion are dialectical, although there are enormous differences between them in the depth and diversity of their dialectical content. Mechanical motion is still dialectical motion. And as for the view that an object «occupies» a point in space at a certain moment, in actuality, it both «occupies» that point while simultaneously not occupying it. The so-called «occupation» of a point and «immobility» are only particular conditions of motion; the object is still fundamentally in motion [...]. Heat, chemical reaction, light and electricity, right through to organic and social phenomena, are all qualitatively particular forms of the motion of matter. The great and epoch-making contribution rendered by natural science at the turn of the nineteenth and twentieth centuries resides in its discovery of the principle of the transformation of motion, in pointing out that the motion of matter is always via the transformation of one form into another, and that the new form produced by this transformation is in essence different from the old form[34].

Dialectical materialism also prompted Mao to consider mathematics specifically, because it embodied, on his view, the dialectical unity of opposites. Put simply:

> In mathematics, any number contains internal contradictoriness, and can become a positive or negative number, a whole number, or a fraction. Positive and negative, whole number, and fraction, constitute the movement of contradictions within mathematics[35].

Moreover, dialectical materialism went further, and prompted Marx to conclude that the world was actually infinite, not in an abstract, mathematical way, but in an objective, concrete way. He also held that the study of motion which dialectics takes as fundamental must ultimately be applicable to the *highest form of motion,* namely the motion and transformation of society itself. Mao expressed all of this very clearly in his *Lecture Notes on Dialectical Materialism:*

> Dialectical materialism suggests that the world is infinite (limitless); not only is it so in its totality, but also in its parts. Are not electrons, atoms, and molecules manifestations of a complex and infinite world?
>
> The fundamental form of the motion of matter also determines the various subjects of the basic natural and social sciences. Dialectical materialism observes and studies the development of the world as a progressive motion which passes through the inorganic world to the organic world to arrive at the highest form of the motion of matter (society); the subordinate and related components of forms of motion constitute the foundations of the subordinate and

related components of their corresponding sciences (inorganic science, organic science, social science)[36].

Having expressed his interest in mathematics while stressing the value of applying dialectical materialism to reform society and the sciences alike, Mao gave mathematicians both an incentive and a justification for studying the mathematical manuscripts of Karl Marx —for these in turn would serve to provide the perfect ideological rationale for their own work as pure mathematicians, despite the fact that pure, abstract mathematics was condemned during the Cultural Revolution[37]. Marx and the mathematical manuscripts offered a natural and acceptable route for the self-criticism of traditional mathematics and the rehabilitation of mathematicians themselves, especially how they taught mathematics. This would not only lead to the reform of mathematics in general, and to the revision of foundations for the calculus in particular, but in turn would help to justify interest in the development of two new fields in China, mathematical logic and nonstandard analysis.

IX. Chinese versions of the *Mathematical Manuscripts* of Karl Marx

There were two editorial groups working in the early 1970s on Chinese translations of Marx's *Mathematical Manuscripts*, one in Shanghai, the other in Beijing; the Shanghai group was the first to publish trial editions and then excerpts of Marx's *Mathematical Manuscripts*. Working initially from a Japanese translation, the *Fu Dan University Scientific Data Processing Group* (复旦大学理科资料组) completed a first draft which was circulated for discussion in 1971. Two years later, with a copy of the Russian-German edition in hand (which provided transcriptions of the original manuscripts in German), a revised trial edition was printed and in 1974, translations of Marx's essays on derivatives, differentials, and the history of the calculus were published in two successive issues of the Shanghai journal, *Dialectics of Nature* (自然辩证法)[38]. A year later, the entire translation appeared as a special edition (专辑) of the *Journal of Fu Dan University* (复旦学报), along with a brief *Remark on the Translation* (译者的话) [MARX, 1975a, p. i] [Figure 3].

Meanwhile, in the same year that the Shanghai edition of the manuscripts was printed, a study group at Beijing University published its own translation of three of Marx's essays on the history of the differential calculus, interpreted specifically within a Marxist framework as a *stage in the development of history*. When these appeared in the *Acta Mathematica Sinica* in 1975, they were preceded by a half-page of explanatory remarks from the *main editorial committee*, wherein it was emphasized that this was a proletarian work, published by the People's Press (人民出版社), and meant to contribute to the socialist revolution and to socialist reconstruction[39] [Figure 4].

The page reproduced here from the *Acta Mathematica Sinica* (数学学报), the most prestigious mathematical journal published in China, shows the first section of Marx's history of the theory of the differential as translated into Chinese. At the top of the page is the preface, which offers *An Explanation for Printing Excerpts from the 'Mathematical Manuscripts'* (转载«数学手稿» 的说明):

To promote the great campaign criticizing Lin Biao (林彪) and Confucius, the *Mathematical Manuscripts* of [Karl] Marx, who inspired the proletarian revolution, were translated and edited by the Mathematical Manuscripts Study Group of Beijing University, and published by the People's Press (人民出版社). This is a great event on our ideological battlefield.

Lenin pointed out that «with material dialectics to improve essentially the entire political economy, using dialectical materialism to elucidate history, natural science, philosophy, and the policies and strategies of the working class is the most important thing of concern to Marx and Engels, whereby they made their most important and novel contributions, and brilliantly took a giant step in revolutionary intellectual history»[40].

Marx, the preface points out, used dialectical materialism to evaluate the history of the calculus, and was especially critical of what he took to be its idealistic, metaphysical foundations. For the Chinese, Marx's mathematical manuscripts also served as a model for the way in which dialectical materialism should be used to evaluate critically the natural sciences, including mathematics:

In order to study political economy and philosophy, Marx began to study mathematics in the 1850s. From then on, mathematics was a subject Marx studied and about which he was always concerned. Over several decades, Marx kept a large number of reading notes and draft manuscripts, upon which the «Mathematical Manuscripts» are based. Marx traced the history of the differential and integral calculus, and analyzed the concept of differential and its operations in terms of dialectical materialism. He supported the new theories and criticized idealism, as well as metaphysics. Before the end of his life, Marx asked his daughter to edit his writings with Engels, paying particular attention to the publication of the second volume of «Das Kapital», as well as his «Mathematical Manuscripts». Engels said, «Marx knows mathematics very well [...] and has made some unique discoveries». Marx's «Mathematical Manuscripts» is a treasure trove with reference to Marxism, and a glorious model for how the natural sciences may be studied using the method of dialectical materialism [MARX, 1975b, p. 1].

This was little more than the party line, for Chairman Mao emphasized repeatedly that dialectics was the key to proper understanding of the sciences. Dialectical materialism was the weapon, literally, that Mao expected Chinese reformers to use —even mathematicians— to root out any bourgeois elements and advance mathematics down *Chairman Mao's revolutionary route.* Mathematicians thus took their publication of the mathematical manuscripts of Karl Marx as the perfect blueprint showing how their own criticism of mathematics should proceed:

The great leader, Chairman Mao, has written that «you who study the natural sciences should learn how to use dialectics». By studying Marx's «Mathematical Manuscripts», our theoretical understanding will reach a higher level, and will help us to take hold of the perfect weapons, advancing criticism of revisionism and of bourgeois world outlooks, [thereby] joining the battlefield with Marxism. People who study or teach mathematics should study and use dialectical materialism, which is clarified in the «Mathematical Manuscripts» of [Karl] Marx, to guide their practice and conscientiously improve their world outlooks, pushing the study of mathematics very quickly along Chairman Mao's revolutionary route, making a greater contribution to the socialist revolution and socialist construction [MARX, 1975b, p. 1].

Within months the Beijing University study group was satisfied that its entire translation was ready for publication, and in July of 1975 issued its definitive edition which included photocopies of several pages from Marx's original manuscripts [MARX, 1975c]. Part II reproduced *verbatim* the sections already issued previously that year [MARX, 1975b]. Although the Beijing translation differs in choice of words from time to time from the Shanghai translation, what sets the Beijing edition apart is its inclusion of explanatory terms from the original German version from which the Beijing translation was made. For example, terms like *Differentiation, abgeleitete Funktion,* and *Grenzwort* appear, parenthetically, to explain Chinese terminology when either new terms or characters are first introduced (for example, [MARX, 1975c, pp. 2, 4]) [Figure 5].

X. First Reactions to Publication of the Mathematical Manuscripts

No sooner had the first two parts of the translation of the manuscripts by the Shanghai group appeared in print than the editors of the *Journal of the Dialectics of Nature* (自然辩证法杂志) began to receive letters from a wide variety of readers. The next number of the journal to appear contained a selection of these letters in a section that was devoted to *Discussion of Problems Concerning Differentials and Limits* (关于微积分和极限问题的讨论). This began with a note from the explaining all of the mail the journal had received. Several letters were then published in their entirety, with excerpts from a number of others [Figure 6].

The first letter was from a second-year student at Beijing Middle School No. 144, He Fang (何放), who asked *How Should the Concept of Limit be Understood?* (应当怎样认识极限?). The next contribution was from a worker at Factory No. 5703 in Shanghai, Fu Xi-tao (傅锡涛): *Trying to Say Something Concerning my Feelings About Improving Teaching of the Calculus Using Dialectics* (试用辩证法 改革 微积分 教学的一点体会)

[FU XI-TAO, 1974, p. 147]. Fu Xi-tao was interested in explaining how dialectics could be applied to reform calculus teaching. A third letter came from Zheng Li-xing (郑礼星) of the Fujian Electrical Engineering School in Fuzhou, Fujian Province (福建机电学校), who took up one side of the debate over whether the differential *dx* was zero or not, arguing: *The Differential is Comparable to Zero* (微分是相对的零) [ZHENG LI-XING, 1974, p. 151].

Along with their publication of *Selections from Manuscripts Received* (来稿摘登), the editors of *Dialectics of Nature* included excerpts from letters by readers who had studied the translation of the mathematical manuscripts published in the preceding two issues of the journal [Figure 7].

The first excerpt was taken from a letter by Xu Ting-dong (徐庭栋), who identified himself as a young worker in the Qing-Hai Tractor Factory (西宁青海拖拉机厂). His comments were devoted to *The Differential is a Unity of Zero and Non-Zero* (微分是零和非零的统一), and drew on similar dialectical criticism of the foundations of the calculus already raised by Zheng Li-xing [XU TING-DONG, 1974]. But Xu Ting-dong also considered the calculus applied to motion, and was especially interested in discussing acceleration and the derivative. The next letter, attributed to Wu Guang-xia (吴光夏) of Bao Tou Teachers College (包头师范学校) in Inner Mongolia, was also concerned with the zero / non-zero aspect of the differential. Another letter along these same lines came from Chen Ke-jian (陈克艰), a *knowledgeable youth* (知识青年) from the Shang Shan countryside (上山下乡)[41]. Again, his analysis was devoted to considering the differential as *zero* and *non-zero*, interpreting the calculus as it applied to motion and the paradoxes that arise from trying to consider a moving point as being in any *one* place.

Ren Zheng-xing (任正兴) of the Shanghai Pu-Jing Middle School (上海浦泾中学) also wrote to agree that *The Differential is a Unity of Opposites* (微分是有和无的对立统一), where the phrase *Dui Li Tong Yi* (对立统一) is a Chinese Marxist expression of Hegelian inspiration, namely denoting the antithesis / synthesis duality [REN ZHENG-XING, 1974]. This was followed by an excerpt from Li Qin-nan (李钦南), who worked in a factory manufacturing electrical medical instruments (上海医用电子仪器厂). In contrast to the previous author who considered the differential as a synthesis of opposites, Li Qin-nan underscored the *opposites* themselves inherent in treating the differential as both zero and non-zero. In particular, he asserted that *The Differential is Both Zero and Not Zero* (微分既是零又不是零)[42].

From Harbin Industrial University (哈尔滨工业大学), Shen Tian-ji (沈天骥) wrote with arguments maintaining that *The Differential Reflects Quantitative Change from (Two) Different Points of View* (微分反映量变在不同层次的关节点). Here the two different points of view were of Dx versus dx, and the difference between non-zero and zero, as well as the meaning of $dy = f'(x) \cdot Dx$ [43]. The last letter in this collection of differing points of view prompted by publication of the *Mathematical Manuscripts* was from a young worker at a Shanghai machine packing plant, Chen Li-qin (陈利钦), who insisted that *The Differential Must be Considered as Zero* (微分应当归结为零). Chen's argument was based on his understanding of the limit:

$$\lim_{\Delta \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$$

XI. Mathematicians Begin to Respond

Thus in 1975, two definitive editions of Marx's *Mathematical Manuscripts* appeared in Chinese, translated in both Shanghai and Beijing. The Beijing edition differed only slightly from the Shanghai version, and in some cases they paralleled each other *verbatim* in the Chinese. But with the entire collection of Marx's *Mathematical Manuscripts* now at their disposal, it was not only high school students and factory workers who took an interest, but so too professional mathematicians. For example, writing in the *Journal of Beijing Normal University*, Zhi Zhou (郅洲), a member of the Philosophy Department, explained *How to Understand Derivatives* (导函数)—*Notes on studying Marx's Mathematical Manuscripts* (怎样理解异函数-学习马克思《数学手稿》笔记). In his introduction, Zhi Zhou explained that whereas the calculus as a scientific subject came into being at the end of the 17th century, it did not develop into a satisfactory theory until the middle of the 19th century. As a result of work done in the 17th and 18th centuries, when metaphysical concepts dominated the natural sciences, the fundamental concept of the calculus, namely the derivative, was also subjected to strong metaphysical influences. In the 1870s, the revolutionary teachings of Marx severely criticized such metaphysical foundations for the derivative, and advocated a correct interpretation on the basis of dialectical materialism.

In the first section of his paper, Zhi Zhou examined the history of the derivative, and explained how Newton and Leibniz had introduced the concept as a ratio of differentials. He added that according to Bishop Berkeley, who he described as a representative of *English subjective idealism* (英国的主观唯心主义), the differential dx was literally, in Chinese, exactly what Berkeley had said it was in English, *the ghost of a departed quantity* (逝去量的鬼魂) [ZHI ZHOU, 1975, p. 19]. Zhi Zhou also considered both d'Alembert's approach to the calculus in terms of differences Dt and differentials dt, as well as Lagrange's approach expanding functions in terms of their Taylor series, for which the derivative was taken as the coefficient of the linear term of the infinitesimal h, i.e.:

$$f(x+h)= f(x) + hp_1(x) + h^2p_2(x)/2! + h^3p_3(x)/3! + ...,$$

where $p_1(x)$ was taken to define the derivative, i.e. $f'(x) = p_1(x)$. But whereas Marx stopped with his analysis of the historical development of the calculus at this point, Zhi Zhou went further to consider the contributions made by the French mathematician Cauchy (哥西), specifically the definition of the limit that Cauchy gave in his *Cours d'analyse* of 1821 (分析教程). This was all discussed expressly in terms that Engels had used in 1830 in his *Dialectics of Nature*. Although Zhi Zhou was aware of the fact that Marx never expressly mentioned Cauchy by name, he could not believe that Marx was unaware of the basic ideas used by Cauchy, since Cauchy's point of view was represented in many of the most popular scientific books of his day. Zhi Zhou, who asks why Cauchy did not permit the variable x to actually reach or attain the limit $x=0$, explained that it was because he feared this would lead to the *monster* (怪物) 0/0. He notes that later, in the 1850s, the e-d method of proof appeared, and a few decades later, in the 1870s, this was linked to a thorough critique of the real numbers (实数理论). Nevertheless, the first person to *strip away the appearances* and submit the concept of the derivative (导函数) to a thorough metaphysical analysis was, in Zhi Zhou's opinion, none other than Karl Marx [ZHI ZHOU, 1975, p. 21].

Zhi Zhou devoted the second part of his paper to describing Marx's analysis of the derivative, especially the differential quotient dy/dx in terms of the paradoxical nature of 0/0, which strictly speaking was undefined, or could represent any value at all. He concluded his essay by returning to the founders of the calculus, to Newton and Leibniz. The characterization was classic Marxism:

> Newton's and Leibniz's contributions to the calculus, are great pioneering works in the development of mathematics, but due to the constraints of metaphysical ideology, their works could not avoid being colored by mysticism (神秘主义) [ZHI ZHOU, 1975, p. 24].

From Newton until the time of Marx, although the calculus underwent considerable development, and despite the fact that the concept of the derivative also had a rich dialectical context, it was still trapped in a web of metaphysical ideology. Owing to the constraints of metaphysics, and even though they raised their voices against *old-fashioned orthodox schools of thought*, mathematicians could find no alternatives:

> [Our] revolutionary leader Marx, because of his deep grasp of the method of dialectical materialism, thus focused on the idea of the derivative and advanced a series of brilliant dialectical thoughts, even though over the past 200 years mathematicians have been working but have not yet been able to make a great contribution.

> Marx's «Mathematical Manuscripts» are one part of a brilliant, monumental mathematical work, and are a precious scientific legacy Marx has left to us. It is not only part of the

mathematical writings, but is also a part of his philosophy which uses the methods of dialectics as a model for studying mathematics. Teaching and studying the «mathematical manuscripts» is necessary for today's revolution in education, and is needed in the battle to conquer mathematics.

Scientists and especially mathematicians, from the study and research of the «Mathematical Manuscripts», [have] a powerful ideological weapon to transform directly the old mathematical system and to reform the study and teaching of mathematics [ZHI ZHOU, 1975, p. 24].

XII. Other Works of 1975

Wu Xie-he (吴燮和) and Zhang Hua-xia (张华夏) of Central China Workers College (华中工学院) published an article in *Science Bulletin* (科学通报), *Understanding Calculus From the Point of View of the Paradoxes of Motion,* (用运动的 矛盾 观点认识微分). The first part of the article argued that it was impossible to appreciate the true nature of the calculus from the point of view of metaphysics (形而上学), and covered the familiar arguments Marx had already introduced concerning *evanescent quantities* and the problem of interpreting *dy/dx* as equal to 0/0. Part two of the paper was devoted to arguing that only from the point of view of the paradoxes of motion was it possible to understand the true nature of the calculus [WU XIE-HE & ZHANG HUA-XIA, 1975, p. 7]. In this part of their paper the authors considered the calculus in terms of applications to motion and acceleration, concluding with a section devoted to understanding the differential as a synthesis of the opposites *zero* and *non-zero* [WU XIE-HE & ZHANG HUA-XIA, 1975, p. 10].

XIII. Journal of Fu Dan University (复旦学报) — 1975

Meanwhile, in Shanghai the editors of the *Journal of Fu Dan University* continued to publish new manuscripts submitted in the wake of their publication of the *Mathematical Manuscripts of Karl Marx.* In the second number of 1975, for example, Ou Yang Guang-zhong (欧阳光中) and Zhu Xue-yan (朱学炎) offered a *Discussion on some Ways of Looking at the Calculus of Functions of Several Variables* (谈谈对多元函数微积分的一些看法). Ou Yang was a prominent mathematician who published a considerable amount during the Cultural Revolution; the article, in fact, begins with strong praise for Marx:

One hundred years ago the great revolutionary teacher Marx wrote his mathematical manuscripts, and although in the course of these hundred years mathematics has undergone tremendous development, Marx's mathematical manuscripts nevertheless still shine with a brilliant radiance. Marx in his mathematical manuscripts used the special materialist dialectics of

Marxism to criticize every shade of idealist metaphysics, tearing the mysterious veil from the deceptive derivatives and differentials, and bringing to light their true essence, thereby setting a brilliant example for us [OU YANG GUANG-ZHONG & ZHU XUE-YAN, 1975a, p. 10].

In addition to political rhetoric, Ou Yang provided some sophisticated mathematics as well. This was more technical than anything written to this point in connection with the mathematical manuscripts of Karl Marx, for Ou Yang considered vector analysis, potential differences, gradients, the Poisson integral, triple integrals, and a host of related subjects.

On a more elementary level, in the next issue of the *Journal of Fu Dan University*, a mathematician by the name of Shu Zuo (舒左) offered a paper meant to serve as *A Starting Point for Calculating with Differentials*, (微分演算的出发). Here the name *Shu Zuo* was not only a pseudonym, but also a play on words and characters, for with a slightly different change in tone, the characters 数左(also *Shu Zuo)* mean *The Left Group of Mathematics*. Thus there are several layers of possible meaning here, of which readers would have been aware even before they began to examine the article itself, which began again with the familiar party line:

In the midst of the movement to study the theory of the dictatorship of the proletariat now surging forward with great momentum, publication of the translation of Marx's *Mathematical Manuscripts* is of great significance. «The proletariat must include in its superstructure (上层建筑) all areas of culture to exercise its dictatorship in every respect over the bourgeoisie». While in practice the domain of natural sciences is opposed to the universal dictatorship of the bourgeoisie, it is necessary to submit the development of the domain of the [sciences] to the great revolutionary criticism. Marx's mathematical manuscripts constitute a brilliant model for our great revolutionary criticism of the development of the domain of this subject [SHU ZUO, 1975a, p. 11].

After a study of the derivative, the very popular example $y=x^3$, and the problem of how to interpret dy/dx as 0/0, Shu Zuo's paper draws to a close with a citation from the well-known letter Engels wrote to Marx on November 21, 1882, in which he discussed the meaning of $x+h$ as a point moving from position x to x_1. The paper ends on a typically Marxist note:

We certainly must take this sharp weapon of dialectical materialism, to develop the great revolutionary criticism of the domain of our subject, dare to revolt, and know how to revolt! We are full of confidence that the mysterious veil of every shade enshrouding the natural sciences will certainly be torn away completely, and the domination of the natural sciences by idealism and metaphysics of the past systems will be thoroughly smashed, and the red flag of Marxism, Leninism and the thoughts of Mao Zedong will flutter high above the front position of the natural sciences. This is the universal truth that enlightens us by studying Marx's «Mathematical Manuscripts» [SHU ZUO, 1975a, p. 15].

Shu Zuo's paper was immediately followed by another concerned with the *Mathematical Manuscripts*, this one by Wu Wen-jing (吴文京) who identified himself as a worker at the *Birch Woods* rubber factory (牡丹江桦林橡胶厂) in Jiang Hua Lin, a town in the North-East of China. Wu Wen-jing's paper was devoted to *The Differential and Dialectics* (微分和辩证法), and interpreted dialectics in terms of change, translated into an analysis of the mathematics of motion, a favorite Maoist theme among Marxist mathematicians. The paper discusses velocity and acceleration in terms of derivatives. Through a proper application of dialectical materialism, Wu Wen-jing insisted that a critical evaluation of the calculus would reveal its true essence. He also introduced another familiar theme as well, that it was the forces of production in society that spurred development of the natural sciences, in the course of which mathematics changed from a study of constants to variables, from static situations to ones that were dynamic and constantly changing [Figure 8].

The last paper to be discussed here that was devoted to Marx's *Mathematical Manuscripts* in the 1975 issue of the *Journal of Fu Dan University* was a contribution by Yan Shao-zong (严绍宗), who presented his thoughts on *Basing the Concept of the Derivative on the Law of Opposites* (对立统一法则支配«导函数概念»的建立). Here '对立统一', the popular Maoist expression meaning *the unity of opposites*, was nothing other than the familiar Hegelian or Marxist doctrine of the dialectical polarities of antithesis / synthesis. Yan asked the usual question, *What is to be understood by dy/dx?* [YAN SHAO-ZONG, 1975, p. 7][44]. Yan also cited Marx and the problem of interpreting 0/0, and then took up the ubiquitous analysis of the equation $y=x^3$, in terms of which he discussed derivatives and distinguished between quotients of differences Dy/Dx and differentials dy/dx.

XIV. Chairman Mao Speaks!

A special issue of *Practice and Understanding of Mathematics* (数学的实践与认识) opened in 1975 with two slogans from *The Collected Sayings of Chairman Mao* (毛主席语录):

> From a certain point of view, the most talented and able soldier is one who has had the most practical experience.

> Our improvement is improvement on the basis of popularization; our popularization is popularization under the guidance of improvement [Figure 9].

These slogans were meant to reflect the ideology of the journal, as well as the articles in an issue devoted to popularizing mathematics while emphasizing its practical applications. The opening contribution was by the pseudonymous Shu Zuo, (舒左), who also contributed a paper that year to the *Journal of Fu Dan University* (see above). This time his article was devoted to a report of a meeting held to study the mathematical manuscripts of Karl Marx, in the spirit of popularization that Chairman Mao himself had admonished everyone to pursue, all of which was reflected directly in the aphorism at the head of the journal [SHU ZUO, 1975b, p. 1].

Another attempt to present the basic ideas found in Marx's *Mathematical Manuscripts* to a wider audience was a series of lectures devoted to *Studying Marx's 'Mathematical Manuscripts'* that appeared in the popular journal *Chinese Science* (中国科学). The first of these was written by Shu Li (舒立) from Beijing University, and was devoted to *Using Marxism to Conquer the Battlefield of Mathematics* (用马克思主义占领数学阵地). The allusion to conquering the battlefield was a rhetorical flourish drawing on language Mao himself often used in referring to the struggles China had to face on all fronts. In this case, the point was to advance the battle using dialectical materialism to criticize and revise the foundations of mathematics [SHU LI, 1975, p. 456].

XV. 1976 — Year of the Dragon

On January 8, 1976, Premier Zhou Enlai died. Six months later, on July 28, the industrial and mining city of T'ang-Shan was destroyed by a major earthquake, killing 655,000 people and leaving more than a million people homeless. The third cataclysmic event that year occurred on September 9, when Chairman Mao died.

The cover of the journal *Practice and Understanding of Mathematics* (数学的实践与认识), like every journal in China, immediately carried a portrait of the Chairman, adorned with the slogan *Eternal Glory to the Mighty Leader and Teacher Chairman Mao Ze-Dong!* (伟大的领袖和导师毛泽东主席永垂不朽) [Figure 10].

Inside, a message was prominently headlined from the Central Committee of the Chinese Communist Party, the General Committee of the PRC National Assembly, the PRC State Department, and the Central Military Committee of the Chinese Communist Party. Under their joint auspices, the decision was announced to erect a Memorial Hall for Chairman Mao Ze-dong in Beijing, wherein a *crystal coffin* would be placed so that all could come to view him. The message went on to describe plans to publish excerpts from the works of Chairman Mao, and to prepare and publish the entire series of his collected works. It concluded with a strong statement of the centrality of Marxism to the revolution in China, and the need to complete the democratic revolution Mao had begun to bring about the great socialist revolution that would overcome opposition from both the left and right. Whereas the Communist system of the Soviet Union was denounced, the Marxism of Chairman Mao was affirmed as critical to success of the revolution in China [Figure 11].

The opening paper in this memorial issue commemorating Chairman Mao was a joint work from the study group for Marx's *Mathematical Manuscripts* in the Department of Mathematics at Beijing Normal Teacher's College. The article, devoted to *Studying Different World Outlooks from Two Different Kinds of Mathematical Approaches* (从两种不同的数学推导看不同的世界观), contrasted d'Alembert's approach to the calculus with the foundations advocated by Marx. Admittedly a preliminary study, it was based on a *first reading* of the *Mathematical Manuscripts,* but nevertheless reflected a remarkably sophisticated view of the historical differences between d'Alembert's theory of limits and the critical views of foundations of the calculus held by Marx [Figure 12].

The same slogan —*Eternal Glory to the Mighty Leader and Teacher Chairman Mao Ze-Dong!*— also ran across the cover of the third number of the *Journal of Central China Political College*, atop its third issue for 1976, along with the same portrait of Chairman Mao that appeared virtually everywhere throughout China. Inside, however, a paper said to have been written by Shu Xuan (舒煊) in Wuhan was devoted to *Continuing to Use Marxism to Study Nonstandard Analysis* (坚持用马克思主义考察非标准分析) [SHU XUAN (ÊÏ"), 1976]. Although this article does not go into the technicalities of nonstandard analysis with actual applications to mathematics, pure or applied, it does try to develop the value of using nonstandard analysis in a spirit of evaluation and criticism of mathematics compatible with the views of Marx and Engels, both of whom are cited extensively in the article. The main point Shu Xuan makes here is that despite its suspect ideology, nonstandard analysis is nevertheless an important tool in reevaluating calculus along lines inspired by Marx and Engels.

The next issue of the *Journal of Central China Industrial College* also carried an article devoted to the ideology of mathematics, which appeared this time under the heading *Studying Dialectics of Nature—Pioneering Critical Analysis of this Subject Area* (学习自然辩证法开展学科领域的大批判). It began with an article by Wu Xie-he (吴燮和) and Zhang Hua-xia (张华夏) of Central China College of Technology. The two had collaborated on an earlier article, and were now publishing again, together, in the journal of their own institution. Their article, *Using the Point of View of Development and Transformation to Understand the Calculus* (用发展,转化的观点认识微分), began with a statement about the fundamental importance of calculus in practical applications for all of society. Wu Xie-he and Zhang Hua-xia extolled the calculus as one of the foundations for the development and production of major concepts throughout higher mathematics. From the 17th century on, as machines became increasingly central to daily life, they noted how study of motion and change became increasingly important, including all of the contradictions and paradoxes that arose with the study of motion, and correspondingly, with the calculus itself. In writing their preface, the authors emphasized not only the practical origins of the calculus, but its dialectical nature in the thesis-antithesis Hegelian setting that, typically, regarded the calculus in terms of motion:

> The differential arose and developed on the basis of the needs of society, and is the most important concept of higher mathematics. In the 17th century, the development of manufacturing and the application of machines required people to study mechanical motion. Therefore, the concept and method of differentials arose on the basis of experiment in order to describe and study mechanical motion. Due to the contradictory [nature] of motion itself, it is certain that the concept of differential is inevitably a contradictory concept as well. One must use the point of view of the contradiction of motion in order to understand it correctly [WU XIE-HE & ZHANG HUA-XIA, 1976, p. 13].

In the course of their article, Wu and Zhang noted the necessity of understanding the foundations of calculus from the point of view of the paradoxes of motion and the subsequent contradictions requiring analysis. The article ended with a quotation from Chairman Mao, and another from Marx's *Mathematical Manuscripts,* to the effect that only with the application of dialectical materialism could a proper understanding of transformation and change, of motion and the calculus, be accomplished [WU XIE-HE & ZHANG HUA-XIA, 1976, p. 26].

In a similar spirit of reforming mathematics based upon a careful reading of Marxist principles, the *Journal of Beijing Normal University* (北京师范大学学报) published an article by Huang Shun-ji (黄顺基) and Wu Yan-fu (吴延涪) of the Philosophy Department who wrote about *A Brilliant Model Using Dialectical Materialism to Transform Mathematics* (用唯物辩证法改造数学的光辉典范). This article appeared in a section of the journal devoted to *Studying Marxism—Criticizing Revisionism* (学习马克思主义—批判修正主义), and was a continuation of an earlier article devoted to *Lessons from Studying the 'Mathematical Manuscripts' of Karl Marx* (学习马克思«数学手稿»的体会).

The first section of this paper was devoted to *Critical Analysis of the Mysterious Methods* (神秘方法) *of Newton and Leibniz,* and considered technical differences between Newton's fluxions (and dot notation) as opposed to Leibniz's differentials (and dx notation). Part two continued with a *Critical Analysis of the Rational Method* (理性方法) *of D'Alembert,* in the course of which Huang and Wu introduced d'Alembert's approach to the calculus in terms of limits. The next section of the article offered further *Critical Analysis of the Algebraic Method* (纯代数方法) *of Lagrange,* and described

Lagrange's method of defining the derivative of a given function in terms of the linear coefficient of its corresponding power series [HUANG SHUN-JI & WU YAN-FU, 1976, pp. 14-22].

The historical discussion presented by Huang and Wu followed in general along the same lines already outlined by Marx himself in considering the different approaches taken by Newton, Leibniz, d'Alembert and Lagrange to the calculus. The second half of their paper, however, was devoted to more philosophical considerations, but again followed strictly Marxist lines. This part was entitled *The Problems of Dialectics are Merely Problems of Contradictions* (矛盾). This opened with the assertion that *The Calculus is a unity of the opposites (antithesis / synthesis) of infinite divisibility and infinite aggregation* (微积分是无穷分割和无穷求和的对立统一), which considered both differentiation, integration and the fundamental theorem of the calculus [HUANG SHUN-JI & WU YAN-FU, 1976, pp. 22-24]. This began with a famous quotation from Chairman Mao: *Everything divides into two* (事物都是一分为二的), along with another of Mao's sayings that *From the contradiction of thesis and antithesis, of struggle, the motion and change of all things proceed* (矛盾着的对立面又统一，又斗争，由此推动事物的运动和变化)[45]. The authors went on to maintain that *Derivatives are the unity of the opposites of the process of motion and the result of motion* (导函数是运动过程和运动结果的对立统一) [HUANG SHUN-JI & WU YAN-FU, 1976, pp. 24-27]. This part of their paper again took up familiar themes, arguing for example that motion necessarily entailed the contradictory conclusion that a point in continuous motion was both in a definite position, and was not in any definite position at the same moment. This in turn led to discussion of the contradictions inherent in deciding how to interpret 0/0, and the distinctions to be drawn between Dy/Dx and dy/dx. The article closes with a section devoted to a familiar topic: *The differential is a unity of the opposites of the infinitesimal as zero and not-zero* (微分，即无穷小是 0 和非 0 的对立统一) [HUANG SHUN-JI & WU YAN-FU, 1976, pp. 27-31].

XVI. Symposium on the *Mathematical Manuscripts* of Karl Marx

In 1975, to mark publication of the *Mathematical Manuscripts,* a symposium was held involving about twenty scholars drawn from Beijing University, Qinghua University, Beijing Normal University, Beijing Teachers College, Tianjin Nankai University, and various Institutes of the Chinese Academy of Sciences, including the Institutes for Mathematics and Computing. In an article published a year later in *Practice and Understanding of Mathematics* (数学的实践与认识), Qiao Chong-qi presented a summary of ideas raised during a first-semester course led by the Qinghua University study group on Marx's *Mathematical Manuscripts,* [QIAO CHONG-QI, 1976, pp. 4-11].

XVII. Serious Notice of Nonstandard Analysis

1976, the Year of the Dragon, was also the first in which a serious attempt was made in China to relate the technical details of Abraham Robinson's nonstandard analysis to a technically proper understanding of the calculus. Written under a pseudonym, Shu Ji (舒基), an article appeared in the *Journal of North-West University* (西北大学学报) devoted to *Discussing the Physical Origins of the Mathematical Structure of* *R (谈谈数学结构*R 的现实原型)[46]. The major point of this paper was to introduce the nonstandard continuum *R, which included both infinitesimals and transfinite numbers as legitimate real numbers. Shu Ji sought to justify these, as well as nonstandard analysis in general, in terms of Marxist dialectical materialism. Once the theory was on firm ideological ground, the article proceed with deeper technical discussion of nonstandard analysis on its own terms. The article itself, and the views it introduced concerning nonstandard analysis, were prompted, Shu Ji notes, by opinions formed *after studying the dialectics of nature and Marx's mathematical manuscripts* [SHU JI, 1976, p. 1] [Figure 13].

Shu Ji (舒基) devotes an entire section of his article to arguing that *the infinitely small (large) really are real numbers* (无限小(大)量是现在的数), with the *really are real numbers* meaning here that they are ontologically real, concrete —in physical, material terms. After quoting from Marx's *Mathematical Manuscripts,* Engels' *Dialectics of Nature,* and Chairman Mao's *On the Correct Handling of Contradictions Among the People* (关于正确处理人民内部矛盾的问题) [SHU JI, 1976, p. 2][47], Shu Ji claims that Robinson himself recognized that nonstandard analysis was grounded in a concrete, material way in so far grounded in a concrete, material way in so far as the usefulness of infinitesimals was best seen in applications to real-world problems.

XVIII. 1977

In 1977 the first draft of a course of lectures given at Beijing Normal University were published by Huang Shun-Ji (黄顺基) and Wu Yan-Fu (吴延涪) in the journal *Understanding and Practice of Mathematics.* The opening lecture began with an introduction to studying the *Mathematical Manuscripts,* noting that these constituted a *brilliant document* using dialectical materialism to analyze mathematics, and were a *treasure trove* (宝库) of dialectics [HUANG SHUN-JI & WU YAN-FU, 1977, p. 5]. The first lecture follows Marx very closely in offering a critical analysis of the foundations of the calculus through its historical development. The authors point out that studying the *Mathematical Manuscripts* confirms what Engels said at Marx's graveside: that Marx had a special interest in mathematics and made fundamental contributions of his own to the subject. The contributions were primarily in applications to Marx's theory of surplus value, and in applications revealing the special laws of change underlying the evolution of capitalism and patterns of development reflected in modern society.

As Huang and Wu emphasized in their introduction:

The times we are facing today «are times when everything is turned upside down, to which nothing in past history can compare». To strengthen and reinforce the dictatorship of the proletariat, using Marxist-Leninism, the thoughts of Mao Ze-dong have taken command of every position, pioneered study of the manuscripts and research of very important practical significance[48].

The authors' introductory lecture is divided into four parts, the first devoted to describing the aims Marx had in mind when he wrote the manuscripts. Then comes a section devoted to the major contents and basic ideas of the manuscripts, followed by a third section explaining the process of writing and publishing the manuscripts. The last and most interesting part of this introduction to Marx's mathematical manuscripts considers their practical significance [HUANG SHUN-JI & WU YAN-FU, 1977, p. 5]. Here Huang and Wu list a number of major practical results that follow from study of the *Mathematical Manuscripts*. Above all, they note that in every branch of science the manuscripts may be used as *a pioneering weapon of revolutionary criticism*:

As Chairman Mao said: «the dictatorship of the proletariat must build atop the accomplishments, including all aspects of culture produced by the bourgeoisie». Using dialectical materialism, this proletarian world view and methodology will guide scientific research, continue to eliminate idealism, mysticism and every kind of bourgeois thought, and is certainly an important aspect of furthering the dictatorship of the proletariat on the battlefront of science and technology. In the «Manuscripts», Marx insists that two philosophical approaches are competing in the domain of science, and for the foundational theory (基本理论) of the calculus there is the clear distinction between materialism and idealism, where the boundary between dialectics and metaphysics certainly sets an example for how we may initiate the revolutionary critical analysis of mathematics and the natural sciences. In our implementing and carrying out what Chairman Mao indicated about the importance of theoretical issues, the revolutionary spirit of the «manuscripts» provides a powerful ideological weapon for the critical revolutionary analysis of every subject related to the relentless development of idealism, metaphysics and all reactionary bourgeois thoughts [HUANG SHUN-JI & WU YAN-FU, 1977, p. 13].

Second, study of the «Manuscripts» must encourage increased awareness of technical workers and the vast revolutionary masses in the conscious study and application of dialectical materialism. As Chairman Mao said, «Those of us who study the natural sciences, must study the application of dialectics». In the «Manuscripts», Marx's concrete applications of dialectics to mathematics leads to a series of original discoveries, and with respect to the systematic dialectical interpretation and study of the calculus and explanation of the sciences, this is the beginning of a new area of conscious application of dialectics to the natural sciences. Therefore, profound study of the «manuscripts» should inspire great interest in our conscious study and enthusiastic application of dialectics. The mind can transform matter, vast numbers of specialist workers and the revolutionary masses can master dialectical materialism in a single day, [therefore] it is certain that the revolution in science and engineering education can be advanced as powerfully as a thunderbolt, and the natural sciences can be advanced swiftly and dramatically.

Study of the «Manuscripts» should also promote the reform of teaching, lead to a critical analysis of both the history of mathematics and the sciences, as well as the accomplishments of modern mathematics and everything that has the «ten kinds of significance». In the «[Mathematical] Manuscripts», Marx understood the methods of derivatives (代数) and of differentials as reflecting a mutual positive-negative dialectical relationship [...]. In the «Manuscripts», Marx established the theory of infinitesimals on the basis of the unity of the opposites zero and not-zero, and criticized the one-sided nature of the theory of limits, [...]. Our critical analysis of modern mathematics [amalgamates] two positive results — that of Cauchy (哥西) whose theory of limits provides a foundation for standard analysis, and that of Leibniz (莱布尼茨) in terms of nonstandard analysis due to Abraham Robinson (罗宾孙)[49]. Their dialectic synthesis results in the sharpest ideological weapon (锐利的思想武器).

[...] The radiance of dialectics sparkles throughout the «Mathematical Manuscripts», and their publication is an epoch-making event in the history of mathematics. After continuous investigation and extensive development of the study and research of the «Manuscripts», the application of Marxism has captured the front positions of mathematics and the sciences, certainly having far-reaching influence [HUANG SHUN-JI & WU YAN-FU, 1977, p. 14].

Among later lectures in the series, the third devoted to derivatives made explicit use of the latest results of nonstandard analysis. The article, by Xue Shu (薛舒), was part of the series devoted to *Studying Marx's 'Mathematical Manuscripts'*. Xue Shu's contribution was an article titled *On Problems of the Theoretical Foundations of Calculus: the Marxist Position, Viewpoint and Method* (在微分学 理论 基础 问题上，马克思的立场，观点和方法), and was published in the first issue of the *Journal of Beijing Normal University* (北京师范大学学报) for 1977[50]. The author began with a lengthy discussion of the merits of using the symbol *dy/dx* in place of 0/0, and cited Engels' *Dialectics of Nature* to the effect that *Mathematics is the science of quantity* (数学是数量的科学)[51].

The next issue of the *Journal of Beijing Normal University* (北京师范大学学报) carried the continuation of the previous issue's series of lectures, again authored by

Xue Shu (薛舒) and constituting further discussion (part 3) on the subject of differentiation. This was entitled *The major significance of Marx's Discussion of Differentiation* (马克思关于导函数的论述的重要意义), and began by noting that it was already 100 years since Marx wrote his manuscripts. Since then, mathematics had undergone significant development, especially in analysis, whereupon Xue Shu identifies four major stages in its development. This lecture provided the most sophisticated analysis yet, at least historically, of the calculus, and did not stop at the usual mention of Newton and Leibniz, but went on to discuss the contributions of two of the brothers Bernoulli, Jakob I (贝努里) (1654-1705) and Johann I (1667-1748), and work they published in 1694 and 1691-92, respectively [BERNOULLI, 1694 and 1691-92]. Mention was also made of the contribution of the French mathematician the Marquis de l'Hôpital (洛必达) (1661-1704), specifically his *Analyse des infiniment petits*[52].

The second stage Xue Shu singles out in the history of the calculus ran from the 18th through the early 19th century. This was a period when mathematicians and philosophers alike were especially concerned with foundations. In response to the calculus based upon infinitesimals (无穷小方法) pioneered by Leibniz and Newton, he mentions the work of Brooke Taylor (泰勒), who used a *method of increments* (增量方法), as well as the method of *fluxions* (流数论) used by Colin Maclaurin (马克劳林) (1742), both of whom were basically Newtonians. On the continent, d'Alembert (达兰贝尔) undertook to supply a more satisfactory version of the calculus based on limits (极限方法), which he advanced in works of his published in 1748 and 1755, both of which Xue Shu mentions. D'Alembert also wrote two articles related to the fundamental ideas of the calculus with Denis Diderot (狄德罗) for the French *Encyclopédie* (百科全书). There was the algebraic method (代数方法) due to Lagrange (拉格朗日), and Euler, who used a method of infinitesimals, both of which are discussed, along with a number of Euler's works including two in particular, his *Introductio in analysin infinitorum* (1748), and *Institutiones calculi differentialis* (1755).

The third period Xue Shu considers covered the 19th century to about 1870. This was the period dominated by A.L. Cauchy (哥西), especially his works of 1821, 1823, and 1826-28, all of which Xue Shu rightly appreciates as being of extreme importance to the historical development of analysis. The fourth and last period Xue Shu considers here runs from the 1870s to the present. This is the period beginning with Karl Weierstrass (外尔斯特拉斯), Richard Dedekind (狄德金) and Georg Cantor (康托)[53].

With the widespread availability of the *Mathematical Manuscripts* of Karl Marx in Chinese, not only mathematicians, but as the above has made clear, students, soldiers, farmers, and workers —literally everyone— seemed interested in saying something about mathematics from the perspective of dialectical materialism. Among these was

Xue Yu-chuan (薛雨川), a graduate from Beijing Normal University in 1976, who identified himself as a member of the Department of Mathematics, *Worker-Farmer-Soldier Class of '73*. Xue Yu-chuan wrote about *Variables and Differentials* (变量和微分), arguing that according to the principles of dialectical materialism, *the objective world is a world of motion, and motion is a form of matter. There is neither motion without matter, nor matter without motion.* With this in mind, Xue analyzes various aspects of the calculus, providing liberal doses of Marx and Engels as he does so [XUE YU-CHUAN, 1977.]

The final article to be considered here, because it was devoted not only to the importance of Marx's mathematical manuscripts, but appealed to nonstandard analysis in order to do so, was published by Zhou Guan-xiong (周冠雄) in 1977: *Using the Philosophy of Marxism to Evaluate Nonstandard Analysis* (以马克思 主义 哲学 为 指导评价非标准分析). This appeared in the *Journal of Central China Industrial College*, and summarized its main argument as follows:

The study and discussion of Marx's «Mathematical Manuscripts» are of real and profound value in helping us to understand dialectical materialism, and in studying mathematics using Marxism.

As early as his «Address at the Symposium on Literature and Art in Yan An (延安)», Chairman Mao pointed out «We cannot ignore the heritage of the ancients and foreigners, or even those things from feudalism and capitalism», but should «accept those useful things and ideas with a critical eye or from a critical point of view», although we should not «accept them without criticism». [In his essay] «Discussing ten great relationships», [Mao] also pointed out clearly: «We lag behind foreign countries in the natural sciences; we should learn from them, but with a critical eye; but we cannot accept them without criticism [...]. [Nevertheless] those things that are [already] familiar to us, we should not copy». Chairman Mao's directive identifies how we should approach our study of foreign things, how the accounts of Marx, Engels, Lenin and Mao of the infinite and of higher mathematics supply theoretical weapons for evaluating nonstandard analysis. In his «Mathematical Manuscripts», Marx traced the history of the calculus from Newton to Lagrange, acknowledging their contributions and pointing out their idealistic and metaphysical errors.

Marx also analyzed the concepts of derivative, differential, differential operations, etc. Using his own philosophy, Marx outlines a series of very important results, which constitute a glorious model for examining nonstandard analysis. We must follow the advice and ideas of [our] revolutionary superiors. Consequently, we should strive to examine everything from two points of view rather than from one — from history [in the long run] versus the moment now; from studying with critical analysis versus emotions and intuition.

The core of nonstandard analysis provides a foundation for higher mathematics [with] infinitesimals. In [his] nonstandard analysis, [Abraham] Robinson (鲁滨逊) shows there is a certain infinitesimal between zero and any positive number using the methods of mathematical logic. The entire theory of nonstandard analysis constructs a mathematical system based on infinitesimals.

The system provides another interpretation for the [viability] of the calculus, and another (mathematical) method distinct from the method of limits. We should accept the contributions Robinson has made, but object to the influence of Robinson's formalism, which in a system of natural science has its limitations. We must criticize Robinson's idealism as it appears in his works.

Robinson's philosophy is a mixture of logical positivism and pragmatism. He called himself a logical positivist [actually, this is a conclusion drawn by the author based on Robinson's statement that «It is meaningless to discuss ontology of any kind with respect to the concept of the infinite in mathematics». In nonstandard analysis he introduces the infinite and the infinitesimal axiomatically, and denies any objective reality for infinitesimals [...].

Pragmatism is a popular philosophy in the West [...]. Mathematical logic achieves a great deal of its success under the influence of reactionary philosophies (idealism, formalism), and the advanced development of the computer. The achievements [of mathematical logicians] were made in spite of their faulty philosophies, especially the wide-spread acceptance of positivism in varying degrees. Therefore, criticism of positivism is an important subject in the field of mathematics, reflecting the struggle of materialism against idealism.

But in evaluating nonstandard analysis, we should acknowledge Robinsons' mathematical work, but must seriously criticize his philosophy, which is a serious and important task facing [both] scholars of mathematics and philosophy [ZHOU GUAN-XIONG, 1977, pp. 115-122] [Figure 14].

XIX. Conclusion

Since the founding of the People's Republic of China in 1949, Chinese scholars have produced a series of studies meant to explain, popularize and establish the methods and philosophy of dialectical materialism in virtually every field of study. In the sciences this has led to criticism, if not condemnation, of Mendelian genetics, of physics in both its Newtonian and Einsteinian interpretations, and in mathematics, of Euclidean geometry and —as has been described in some detail here— of the infinitesimal calculus. But unlike many of their colleagues in the Soviet Union, the Chinese avoided the disastrous consequences of Lysenko's triumph over Mendel by allowing that successful scientists, despite faulty philosophies, nevertheless unconsciously must have used dialectical materialism in guiding their research [Figure 15].

Throughout the Cultural Revolution (1966-76), Mao Ze-dong promoted Marxism and dialectics to encourage reforms in all fields of endeavor, including the sciences. In mathematics, this encouraged, as it had Marx, an appreciation (with criticism) of the infinitesimal calculus. For Chinese mathematicians, application of Abraham Robinson's newly created nonstandard analysis not only rehabilitated infinitesimals in a technical sense, but (when understood within an appropriate materialist framework), could be used to justify and promote two new fields of study in China —model theory and nonstandard analysis.

NOTES

1. DAUBEN [1984, p. 86], and DAUBEN [1992, p. 54]. Translation from the *Scholium* to EUCLID [1888, X, I, p. 417]. For other accounts of the drowning episode, see IAMBLICHUS [1891, XXV, pp. 76-78], and IAMBLICHUS [1937, XVIII, 88, p. 52; XXXIV, 247, p. 132]. BURKERT [1972, p. 455] writes that "the tradition of secrecy, betrayal, and divine punishment provided the occasion for the reconstruction of a veritable melodrama in intellectual history".
2. For Pythagoreanism and its political dimensions, see BURKERT [1972], FOWLER [1987], MINAR [1942], PHILIP [1966], RAVEN [1948] and DE VOGEL [1966]. Hippasus was noted among Pythagoreans for his political literalness.
3. See KNORR [1975], WASCHKIES [1977] and FOWLER [1987] for detailed accounts of the significance of the discovery of incommensurability in the context of ancient mathematics, and Plato's treatment of same in his dialogue, the *Theatetus*. Plato condemned the ignorance of those who did not understand the nature of incommensurability as a "disgrace to human nature" [*Laws*, VII, 819-820].
4. See the preface to ALEXANDER [1956], especially Part II, "The Argument of the Correspondence", pp. xiii-xxxii.
5. See, among others, GRABINER [1981], BELHOSTE [1985 and 1991].
6. See DAUBEN [1979/1990, pp. 66-69, 133-138, 160-168, 280-281], EDWARDS [1988], MESCHKOWSKI [1967, pp. 134-139], and CHARRAUD [1994, pp. 34-37, 196-199].
7. See also STRUIK [1948] and KENNEDY [1977].
8. See VOGT [1983, p. 56, note 32]. As Annette Vogt stresses, Marx studied mathematics in England, where the subject was significantly behind the levels of research published in either France or Germany. Only with the so-called *Analytical Society* comprised of Charles Babbage, George Peacock and John Herschel, did British mathematics begin to catch up with the continent, in part by translating Lacroix's *Elementary Treatise on the Differential and Integral Calculus* into English in 1816 [VOGT, 1983, p. 55]. Apparently, this was the most modern text on calculus Marx ever read. Among English authors, Babbage was especially

influential on Marx, in particular his *On the Economy of Machinery and Manufactures* [VOGT, 1983, p. 56]. Lagrange's *Theorie der analytischen Funktionen* (which Marx read in a German translation) was also a major influence. Marx, who was interested in proof theory and mathematical methods, wrote on philosophy, history of mathematics and mechanics, and was especially interested in the problem of where mathematical ideas come from.

According to Annette Vogt, Marx studied, among others, works by Boucharlat, D'Alembert, Euler, Hall, Hegel, Hemming, Lacroix, Lagrange, Landen, MacLaurin, Moigno, Newton and Taylor. Most of his reading seems to have been directed by references to mathematicians he encountered in his study of Hegel [VOGT, 1995, pp. 39-40].

9. ENGELS [1894]. Karl Eugen Dühring (1833-1921) was a philosopher and political economist who posited a primordial human being from which all else had evolved, a theory he called "the law of determinate number". He was a vigorous proponent of capitalism and nationalism, he attacked religion, and was an outspoken anti-Semite. Engels objected especially to Dühring's *a priori* approach to mathematics, and held that the most basic concepts (like number) were not arbitrary creations of the mind, but were rooted in experience and the material world. For more on Dühring see HERMANN [1979].

10. See also ENGELS [1940, p. 314], and KENNEDY [1977, p. 311].

11. LOMBARDO RADICE [1972, p. 275]; quoted from KENNEDY [1977, p. 316].

12. Much of the information supplied here about the *Mathematical Manuscripts* and their history in the Soviet Union was kindly provided to me by my colleague, Serguei Demidov, Editor of *Istoriko-Matematicheskie Issledovaniya*, and Director of the Department of History of Mathematics, Institute of the History of Natural Science and Technology, Russian Academy of Sciences, Moscow, in a letter of September, 1996.

13. Kol'man concluded: "So kämpft Marx als wahrer Dialektiker sowohl gegen die rein analytische Zurückführung des Neuen zum Alten, die so charakteristisch für die Methodologie des «mechanistischen» Materialismus des XVIIII. Jahrhunderts war, als auch gegen die rein synthetische Einführung des Neuen von außen her, was nicht nur für den Hegelischen Standpunkt, sondern auch für den heutigen Intuitionismus bezeichnend ist, der das Prinzip der mathematischen Induktion für dasjenige Neue hält, das von außenher kommt, und der auf diese Weise den Uebergang zwischen der Logik und der Mathematik vernichtet" [KOL'MAN, 1932, p. 351].

14. Despite the delay in publication, the work of this group finally appeared on the occasion of Marx's 150th birthday. The *Mathematical Manuscripts* were published in a dual-language edition, the original German and Russian translations side-by-side. See VOGT [1983, p. 55, note 25]. Study of Marx's *Mathematical Manuscripts* had a major impact on Soviet research in the history and philosophy of mathematics, beginning in the 1930s. This was especially true in philosophy of mathematics, where virtually all of the work published between 1930 and 1950 dealt with the manuscripts. The history of mathematics, however, also received considerable stimulation due to what Marx had written. For example, among Marx's manuscripts there is an essay on the history of the foundations of mathematical analysis (from its origins to Lacroix). This essay by Marx was in fact the ideological pretext on

which A.P. Youschkevitch launched a detailed historical study of the calculus, which became an important focus of his work and that of his students.

Thus the significance of the discovery and study of the mathematical papers of Karl Marx in the Soviet Union may be assessed in several different ways. To the extent that editorial work on the manuscripts promoted study in the 1930s of the history of mathematics, its effect was positive. In particular, the manuscripts provided a strong rationale for serious examination of the history of analysis. It also followed that to appreciate Marx fully, it was necessary to study the history of mathematics in general. Unfortunately, where foundations of mathematics are concerned, Marx and the manuscripts have had a largely negative impact. This has been due primarily to the tendency of foundational research to focus almost exclusively on dialectical interpretations of mathematics according to Marx's fundamental doctrines.

As for the technical, internal development of mathematics itself, Marx's manuscripts do not seem to have played any appreciable role, positive or negative. The reason for this, according to Serguei Demidov, is due in part to the powerful mathematical community in the USSR, which was interested above all in pure and applied mathematics rather than ideology. Although prominent members of this community endorsed the importance of the manuscripts for the history of mathematics, more important for their eventual fate was the fact that analysis of the manuscripts was in the hands of a well-trained scholar, Professor S. Yanovskaya, who was not only a Marxist but a dedicated mathematician [Serguei Demidov, in a letter to JWD, September, 1996].

Among Yanovskaya's historical publications, see YANOVSKAYA [1947 and 1950], the first devoted to Michel Rolle as a critic of the infinitesimal calculus, the other to Lobachevskii, whose work she describes as "a combat weapon against idealism in mathematics". For an appreciation of Yanovskaya's life and work, see YANOVSKAYA [1956].

15. Note that throughout this essay, Chinese names are usually given in both their Pin-Yin and Wade-Giles (or other alternative) orthographies, since the spelling of Chinese names is by no means uniform and, depending upon the sources in question, the same name may occur in various forms, few of which have become standard. There is the further complication that in order to promote literacy in China, simplified characters have been introduced for many traditional characters. Thus in the case of Mao (above), Mao Ze-Dong is the Pin-Yin, Mao Tse-tung the Wade-Giles spelling of his name. Similarly, 毛澤東 gives the version of his name in traditional characters, whereas 毛泽东 are the corresponding simplified characters. In general, simplified characters are given in what follows only for names or titles occurring in material published since the founding of the PRC in 1949.

16. Chen Du-xiu 1920, essay of October 1, 1920; quoted from BRIÈRE [1956, p. 24]. D.W.Y. Kwok devotes an entire chapter to Chen (as Ch'en Tu-hsiu) [KWOK, 1965, pp. 59-81]. Kwok points out that Chen insisted that democracy and science were the "two most precious possessions of modern civilization" [KWOK, 1965, p. 63]. See also CHEN DU-XIU [1915 and 1917a].

17. CHEN DU-XIU [1917]; quoted from BRIÈRE [1956, p. 23].

18. CHEN DU-XIU [1923, p. 36], as quoted in BRIÈRE [1956, p. 23].

19. CHEN DU-XIU [1916], as quoted in KWOK [1965, pp. 72-73].

20. WU ZHI-HUI [1925], quoted from KWOK [1965, p. 48].

21. Guo Mo-ruo, quoted by VAN BOVEN [1946, p. 72], and BRIÈRE [1956, p. 33]. According to Brière, Guo was "the first of the writers to formulate the principles of the dialectical materialism to which he has been devoted since 1925. In the light of his faith he scrutinizes the antiquity of China, and draws from it entirely new conclusions [...]. It goes without saying that his conclusions do not at all match with those of other specialists on antiquity" [BRIÈRE, 1956, p. 33]. See also the discussion of Guo Mo-ruo in KWOK [1965, pp. 166-167].

22. REN HONG-JUN [1919, p. 192]; quoted from KWOK [1965, p. 116].

23. GUO ZHAN-BO [1935, p. 314], as quoted in BRIÈRE [1956, p. 86].

24. From the advertising prospectus (1931) announcing publication of *Twentieth Century* (二十世纪), quoted as well (with slight variations) in BRIÈRE [1956, p. 84].

25. WU ZHI-HUI [1933], quoted from KWOK [1965, p. 41]. For a special issue of *New China* in 1933, an entire issue was devoted to Wu's idea of *Saving the Country with Motors.*

26. AI SI-QI [1934, p. 9]; quoted in BRIÈRE [1956, p. 79].

27. DING WEN-JIANG [1923, esp. pp. 1, 4-9]; quoted from BRIÈRE [1956, p. 30].

28. ZHANG JUN-MAI [1923], quoted from GUO ZHAN-BO [1935, p. 322]. Zhang was better known under his political name of Carson Chang. As a young professor, he had studied in both Japan and Germany [BRIÈRE, 1956, p. 29]. The same passage is translated slightly differently in KWOK [1965, pp. 141-142]. For details of the extensive debate over the issue of scientism versus traditional philosophy, see the discussion in KWOK [1965], as well as *Science and the Philosophy of Life* (科學與人生觀), a two-volume collection of articles devoted to the debate (1923).

29. MAO ZE-DONG [1937a]; quoted with slight variations in MAO ZE-DONG [1965, vol. 1, pp. 296-297], and in KNIGHT [1990, p. 133].

30. MAO ZE-DONG [1938]; quoted from KNIGHT [1990, p. 93].

31. MAO ZE-DONG [1937a]; quoted with slight differences in MAO ZE-DONG, [1965, vol. 1, pp. 303-304], and in KNIGHT [1990, pp. 141-142].

32. MAO ZE-DONG [1957]; quoted from MAO ZE-DONG [1967, p. 358].

33. MAO ZE-DONG [1938]; quoted from the section of Mao's *Lecture Notes on Dialectical Materialism* devoted to "On Space and Time" [KNIGHT, 1990, pp. 111-112].

34. MAO ZE-DONG [1938]; quoted from KNIGHT [1990, p. 108].

35. "The Universality of Contradiction", from MAO ZE-DONG [1937b]; quoted from KNIGHT [1990, p. 165]. Note that this text was left out of the official edition when *The Law of the Unity of Contradictions* was published as part of Mao's essay "On Contradiction" in the official text in MAO ZE-DONG [1965, 1, pp. 311-347]. For details, see KNIGHT [1990, p. 154].

36. "On Motion (On Development)" [MAO ZE-DONG, 1938]; quoted from KNIGHT [1990, pp. 109- 110].

37. Deng Xiao-ping acknowledged the widespread persecution of intellectuals in general, and of scientists in particular, during the Cultural Revolution. As he noted in a speech at the opening ceremony of the National Conference on Science held in Beijing on March 18, 1978: "The very fact that today we are holding this grand gathering, unparalleled in the history of science in China, clearly indicates that the days are gone forever when the Gang of Four —Wang Hongwen, Zhang Chunqiao, Jiang Qing and Yao Wenyuan— could want only sabotage the cause of science and persecute intellectuals" [DENG XIAO-PING, 1978, p. 40].

38. *Journal of the Dialectics of Nature* (自然辩证法杂志), *(2, 3)* (1974).

39. The material in question was translated from the dual-language Russian-German edition of Marx's mathematical manuscripts published in Moscow [MARX, 1968b, pp. 137-189].

40. MARX [1975b, p. 1]. The Board of Editors noted that "Here our journal reprints parts of the *Mathematical Manuscripts* [of Karl Marx]. Papers discussing this topic will be forthcoming." Indeed, the journal published an extraordinary array of reflections by students, factory workers, school teachers and university mathematicians, on various implications to be drawn from study of Marx's *Mathematical Manuscripts.* See the discussion below.

    As for Lin Biao / Lin Piao (林彪), he was the general who masterminded the defeat of Chiang Kai-shek in Manchuria, thereby insuring Communist victory over the Mainland in 1949. As a strong supporter of Mao, he was not only named Minister of Defense in 1959, Lin Piao oversaw the editing of the famous *little red book* used to disseminate the *Sayings of Chairman Mao* to the People's Liberation Army. Although he was officially designated as Mao's successor in 1969, in 1972 the government announced his death in a plane crash as he was supposedly fleeing the country for the Soviet Union, having tried to overthrow the government the previous year. For details of Lin Piao's life and works, see EBON [1970] and VAN GINNEKEN [1976].

41. Chen Ke-jian is doubtless a pseudonym, *Ke-jian* meaning *overcome difficulties.* The fact that he is said to be "knowledgeable of the mountains above and the countryside below" may also refer to his life during the Cultural Revolution. See *Journal of the Dialectics of Nature* (自然辩证法杂志), *(4)* (1974), p. 157.

42. Li asks the same question as did Engels in a letter to Marx in August of 1881, both of which are reminiscent of Berkeley's criticism of the calculus, namely what to make of $dy/dx=0/0$? See LI QIN-NAN [1974, p. 160], as well as Ou Yang Guang-zhoung, who also analyzes Engels' letter [OU YANG, 1975b, p. 6].

43. Here Chen also drew on Marx's letter to Engels in the *Collected Works* [MARX & ENGELS, 1962, vol. 35, p. 21; vol. 20, p. 604].

44. Yan refers to the German edition of the *Collected Works* [MARX-ENGELS, 1962, vol. 35, p. 22].

45. MAO ZE-DONG, *Problems of the Proper Treatment of Internal Contradictions of the People* (关于正确处理人民内部矛盾的问题), quoted by HUANG SHUN-JI & WU YAN-FU [1976, p. 22].

46. SHU JI [1976, p. 1]. This is doubtless another pseudonym, for Shu Ji, with different characters (数基) but with basically the same pronunciation, also means *foundations of mathematics.*

47. Quoting from MAO ZE-DONG [1967].

48. HUANG SHUN-JI & WU YAN-FU [1977, p. 5]. Although not identified by Huang and Wu, the quotation is highlighted in their manuscript, and therefore must be a quotation from either Marx, Engels or Mao.

49. Note that here Robinson's name appears in different Chinese characters from those adopted by Shu Xuan (舒煊) [1976], where Robinson's name was given as 鲁滨逊 = Lu-Bin-Sun. The name conferred on Robinson by Huang and Wu is slightly different in terms of pronunciation, and quite different in terms of the characters: 罗宾孙 = Luo-Bin-Sun.

50. Several things are curious about the appearance of Part III of this series of lectures devoted to the study of Marx's *Mathematical Manuscripts.* This part is published under a pseudonym – Xue Shu written as characters 学数 (rather than 薛舒) means, literally, *study mathematics!* Moreover, Part I was printed with the names of Huang Shun-Ji (黄顺基) and Wu Yan-Fu (吴延涪), and while Huang and Wu originally published their Part I of the lectures in another journal, *Practice and Understanding of Mathematics* (数学的实践与认识) HUANG SHUN-JI & WU YAN-FU [1977], they had also published together, jointly, in the *Journal of Beijing Normal University* (北京师范大学学报), namely [HUANG SHUN-JI & WU YAN-FU, 1976]. The fact that the lecture series continued to appear, but now anonymously and in another journal, but one with which Huang and Wu were also associated, suggests that they at least had a hand in writing the article, if they were not indeed its authors.

51. ENGELS [1962a, p. 235]; quoted from XUE SHU (薛舒) [1977, p. 18].

52. L'HÔPITAL [1696]. For the historical material included here, Xue Shu seems to have relied primarily on SCOTT [1958] and WIELEITNER [1924-25], although Xue Shu only mentions Wieleitner by name and does not cite any specific work of his in particular.

53. Xue Yu-chuan mentioned the - method championed by Weierstrass, cited Dedekind's monographs of 1872 and 1888, and apparently relied upon P.E.B. Jourdain's English translation of Cantor's *Beiträge zur Begründung der transfiniten Mengenlehre* of 1895 and 1897 [XUE YU-CHUAN, 1977, p. 24]. Following Xue Shu's own analysis, a second part followed, entitled *On the History of the Calculus* (关于微积分的历史), which offered a set of reference materials for studying the mathematical manuscripts compiled by the study group that had been working on the manuscripts in the department of mathematics. This was divided into six sections, dealing with background on the creation of the calculus, its historical roots, an entire section devoted to Newton and Leibniz, another to the debates over the calculus in the 18th century, the 19th century, emphasizing both the formulation and development of the systems due to Cauchy and Weierstrass, and finally, closing remarks.

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FIGURES

FIGURE 1.- I see a vestige of man! [GREGORY, 1703, frontispiece], reproduced by permission of the Science, Industry and Business Library, The New York Public Library, Astor, Lenox and Tilden Foundations, New York City, USA.

FIGURE 2.- Traditional seal characters: «Saving the Country with Motors,» from *New China*, 1933; reproduced in [KWOK, 1965, p. 40].

译 者 的 话

对于"马克思所遗留下来的极其重要的数学手稿"(《反杜林论》)，恩格斯极为重视。但在马克思和恩格斯生前，都未能发表。

在马克思和恩格斯的书信中，也包含与数学手稿有关的内容。1881年8月18日恩格斯在给马克思的信中说："研究了你的数学手稿"，"我向你祝贺"。1882年11月21日和22日恩格斯和马克思的相互通信也都对微分演算作了精辟的论述。上述书信均见《马克思恩格斯全集》第35卷。

1933年，苏联《在马克思主义旗帜下》杂志第一次刊登数学手稿部分内容的俄译文。

1971年，我们曾按这部分内容的日文本译成中文，印了试译本。1973年，我们根据德文原文对手稿的整个论文部分重新进行翻译，印了第二个试译本。接着，在上海《自然辩证法》杂志1974年第2、3两期上刊登了"导函数的概念"，"论微分"和"微分演算的历史发展过程"等主要论文。

这次刊出的译本，又对前两次的译稿进行了全面的修订和补充，目的是进一步征求对于译文的意见。这个译本收集了我们所见到的比较完整的手稿内容，整理成六个部分。前面五个部分都是马克思在八十年代(1880—1883)的手稿，最后第六部分标明是"七十年代的几份手稿"。这样，可以更好地领会马克思关于微分演算本质的思想发展和理解手稿的精神实质。

关于译文，有几点说明：

1. 在马克思的手稿中，变数 x，y 变化以后写为 x'，y'，译文按现在通常的写法改为 x_1，y_1。

2. 凡有方括号"[ ]"的标题是译者所加的，译文中方括号内的文字也是译者所加的。

3. 译文下边的注都是译者加的。

参加翻译工作的有(按姓氏笔划为序)：复旦大学王福山、谷超豪、苏步青、陈少新、陈国亮、张开明、李立康、金若水、秦曾复、舒五昌，上海师范大学吕[illegible]刚、茆诗松、周宝熙、翟其襄等同志。

复旦大学理科资料组

一九七五年二月

FIGURE 3.- «Remarks on the Translation,» in the Shanghai edition of *Marx's Mathematical Manuscripts* [MARX, 1975a, p. i].

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转载《数学手稿》的说明

在伟大的批林批孔运动推动下，无产阶级革命导师马克思的《数学手稿》由北京大学《数学手稿》编译组编译，人民出版社正式出版了，这是我国思想战线上的一件大事。

列宁指出："用唯物辩证法从根本上来改造全部政治经济学，把唯物辩证法应用于历史、自然科学、哲学以及工人阶级的政策和策略——这就是马克思和恩格斯最为注意的事情，这就是他们做了最重要最新颖的贡献的地方，这就是他们在革命思想史上英明地迈进的一步。"由于研究政治经济学、哲学的需要，自十九世纪五十年代起，马克思就开始钻研数学。此后，数学始终是马克思经常关心和研究的一个专门领域。几十年间，马克思写下了许多读书笔记和研究手稿，其中对于微积分，特别是对微分学的发展过程、微分概念和运算作了辩证唯物主义的精辟分析，热情支持新生事物，批判唯心主义和形而上学。马克思逝世之前，还曾嘱咐他的女儿爱琳娜，要他和恩格斯一起整理他的全部文稿，并特别关心《资本论》第二卷和数学论著的出版工作。恩格斯指出："马克思是精通数学的"，并且"有独到的发现"。马克思的《数学手稿》是马克思主义的宝贵文献，是用唯物辩证法研究自然科学的光辉典范。

伟大领袖毛主席指出："你们学自然科学的，要学会用辩证法。"认真学习马克思的《数学手稿》，必将帮助我们提高理论水平，掌握思想武器，深入批判修正主义和资产阶级世界观，坚持用马克思主义占领自然科学阵地。对于从事数学研究和教学工作的同志来说，更应学习和运用马克思在《数学手稿》中阐述的辩证唯物主义思想来指导自己的工作实践，自觉地改造世界观，以促进数学科学沿着毛主席的革命路线飞速发展，为社会主义革命和社会主义建设作出更大贡献。

现本刊转载部分内容，并将陆续发表学习体会文章。

本刊编辑部

马克思的数学手稿（摘载）

三　关于微分学的历史

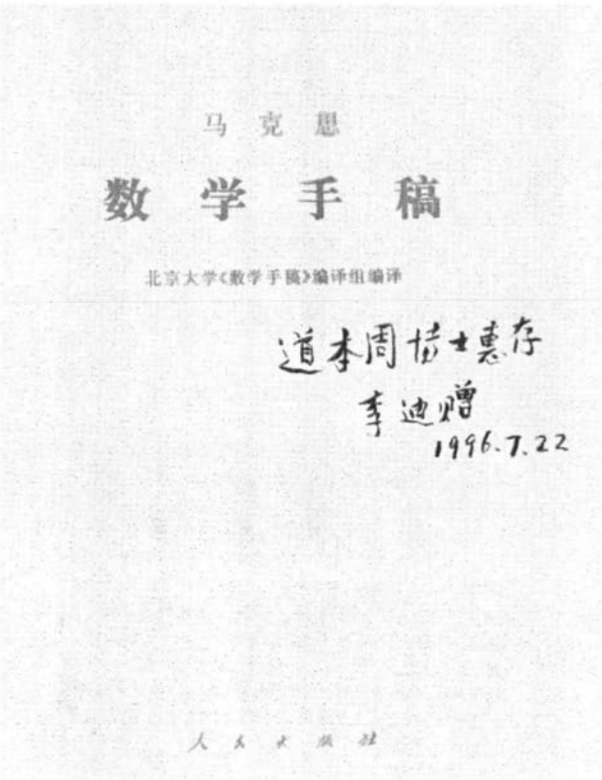
历史的发展过程

1）神秘的微分学

$x_1 = x + \Delta x$ 从一开始就变成 $x_1 = x + dx$ 或 $x + \dot{x}$，这里 dx 是通过形而上学的[illegible]定的。它首先存在，然后加以解释。

尔后 $y_1 = y + dy$ 或 $y_1 = y + \dot{y}$ 也是如此。从随意的假设产生的后果是：为了得正确的结果，在展开二项式 $x + \Delta x$ 或 $x + \dot{x}$ 时，必须用魔术变掉例如所得出的除一阶

FIGURE 4.- Preface and first page of the translation by mathematicians at Beijing University, of excerpts from *Marx's Mathematical Manuscripts*, published in *Acta Mathematica Sinica* [MARX, 1975b, p. 1].



马 克 思

数 学 手 稿

北京大学《数学手稿》编译组编译

道本周博士惠存
李迪赠
1996.7.22

人 民 出 版 社

FIGURE 5.- Title page from the Beijing edition of *Marx's Mathematical Manuscripts* [MARX, 1975c].

关于微积分和极限问题的讨论

〔编者按〕 本刊一九七四年第二、三两期发表了马克思的《数学手稿》以后，收到了很多读者的来信来稿，他们谈了自己的学习体会，还针对微积分和极限问题提出了不同的看法，展开了讨论。这里发表的是其中一部分。

应当怎样认识极限？

北京 144 中学高二(2)班学生 何 放

最近，我从《自然辩证法》杂志上看到马克思的《数学手稿》，很受启发。联想我们课本上讲的极限概念，觉得和《手稿》的神是不一致的。

在《手稿》中，马克思直截了当地写上 $dx=0$，$dy=0$，把分——扬弃了的差值归结为零。这使我对极限的概念有了进步认识。原来我读的《数学》教材，给极限作了这样一个定义“无论预先给定怎样小的正数，在数列里都能找到一项，从项起，以后所有各项与 A 的差的绝对值，都小于预先给定数，那么，这个常数 A 就叫做这个无穷数列的极限。……记 $\lim_{n\to\infty} a_n = A$。”这个定义和它所提供的证明方法以及对它的解告诉我们：即使 n 无限地增加下去，a_n 与它的极限——常

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FIGURE 6.- First page of an issue of the *Journal of the Dialectics of Nature* in which eight letters to the journal were printed in response to earlier publication of parts of *Marx's Mathematical Manuscripts.*

辩证法，歪曲了微积分的本来面目。我们今后还要继续开
对这个领域中唯心主义和形而上学的批判，坚持实践第一观
，坚持一分为二的辩证法，不断改进我们的数学教学。

微分是相对的零

福建机电学校 郑礼星

马克思在《数学手稿》里指出：微分"作为扬弃了的或消失了的差值"，写上 $dx=0$、$dy=0$。这里的"消失"应理解为相对的"消失"，"0"应理解为相对的"0"。

恩格斯说："物质是按质量的相对的大小分成一系列较大的、容易分清的组，使每一组的各个组成部分互相间在质量方面具有确定的、有限的比值，但对于邻近的组的各个组成部分则具有在数学意义下的无限大或无限小的比值。"（《自然辩证法》）并指出，现实世界为数学的无限提供了原型。dx 与 dy 属于同一组，dx 的变化与 dy 的变化紧密联系着。跨"组"而言，dx、dy 对于 x、y，可以被扬弃；微分 dx、dy 对于变数 x、y 只是"消失了的差值"，都归结为 0 来处理，"而无需任何只是无限接近之类的遁辞"。（《数学手稿》）计算 y' 的时候，数学家们实际上也是这样处理的。如果他们固执地、"严格"地坚持 dx 只是"无限接近"于 0，那怎么能把含 dx 的项一笔勾掉呢？可见他们也是自相矛盾的。如果认为在计算 y' 时 dx 不能作为"消失了的差值"，不能是相对的 0，因而也不能勾掉的话，那就象对物体作力学计算时，还固执地将物体上多了一个分子还是少了一个分子计算在内

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FIGURE 7.- From the *Journal of the Dialectics of Nature*, selections from letters received by the editors after preliminary publication of *Marx's Mathematical Manuscripts* in 1974.

微 分 和 辩 证 法

牡丹江桦林橡胶厂 吴 文 京

社会生产的发展推动了自然科学的发展，使数学从研究常数到研究变数。恩格斯说：“数学中的转折点是笛卡儿的变数。有了变数，运动进入了数学，有了变数，辩证法进入了数学，有了变数，微分和积分也就立刻成为必要的了”。(《自然辩证法》第236页)微积分就是这样建立在变数的基础上，是辩证法在数学上的运用。

马克思在《数学手稿》中用辩证观点对微分的本质作了深刻的论述。只有坚持辩证法的观点，才能正确认识微分的本质，离开了辩证法，就不可避免地陷于一团迷雾之中。微积分的发明到现在已有二、三百年了，并且已被广泛地采用，但数学家们并没有真正弄清微积分的本质。马克思在上一世纪八十年代所写的《数学手稿》不仅指导我们正确认识微分的本质，而且是在自然科学领域内坚持辩证唯物主义，用辩证唯物主义指导自然科学研究的光辉典范。

微分是建立在变数的基础上的

微积分的发明不是“人类精神的最高胜利”，而是自然科学发展的必然结果，微积分也不是人类精神的纯粹的“自由创造物和想象物”，而是实践的产物。

我们先从物体的最简单的机械运动来加以考察。如一物体运动的路程 s 与时间 t 的函数关系为 $s=s(t)$，求物体的运动速度。大家知道，在匀速运动时速度等于单位时间内通过的路程，即 $v=\frac{\Delta s}{\Delta t}$。但在变速运动中，速度不再是常数了，利用初等数学只能求出物体运动的平均速度，即 Δt 时间内的平均速度 $\bar{v}=\frac{\Delta s}{\Delta t}$。平均速度 $\bar{v}$ 是掩盖了 Δt 时间内速度的差异而得出来的平均值，它只考虑了物体运动在时刻 t 和 t_1 两点的情形，描写的只是物体运动在 Δt 时间内的平均状况，而不能反映出 Δt 时间内每一时刻的速度，即瞬时速度。瞬时速度是物体运动在某一时刻的速度，显然必须取 $\Delta t=0$，这时 $\Delta s=0$，瞬时速度 $v=\frac{0}{0}$，因而什么结果也没有得到。初等数学走到了它的尽头，再也无法前进一步了。问题的根源在于初等数学静止地、孤立地，即形而上学地研究问题的结果。它只孤立地考虑了物体在时刻 t 一点的情况。而物体运动在某一时刻的速度是不能和它前后时刻隔裂开来

FIGURE 8.- From the *Journal of Fu Dan University,* the first page of a contribution by WU Wen-jing on «The Differential and Dialectics» [WU WEN-JING, 1975, p. 16].

毛 主 席 语 录

在某种意义上说，最聪明、最有才能的，是最有实践经验的战士。

我们的提高，是在普及基础上的提高；我们的普及，是在提高指导下的普及。

FIGURE 9.- Slogans from *The Collected Sayings of Chairman Mao*, quoted on the opening page of the journal *Practice and Understanding of Mathematics* (1975).

FIGURE 10.- Chairman Mao Ze-dong, on the cover of *Practice and Understanding of Mathematics* (4) (1975).

中　国　共　产　党　中　央　委　员　会
中华人民共和国全国人民代表大会常务委员会
中　华　人　民　共　和　国　国　务　院
中　国　共　产　党　中　央　军　事　委　员　会

关于建立伟大的领袖和导师
毛泽东主席纪念堂的决定

（一九七六年十月八日）

为了永远纪念我党我军和我国各族人民的伟大领袖、国际无产阶级和被压迫民族被压迫人民的伟大导师毛泽东主席，教育和鼓舞工农兵和其他劳动群众继承毛主席的遗志，坚持马克思主义、列宁主义、毛泽东思想，把无产阶级革命事业进行到底，决定：

（一）在首都北京建立伟大的领袖和导师毛泽东主席纪念堂。

（二）在纪念堂建成以后，即将安放毛泽东主席遗体的水晶棺移入堂内，让广大人民群众瞻仰遗容。

中共中央关于出版《毛泽东选集》
和筹备出版《毛泽东全集》的决定

半个多世纪以来，伟大的领袖和导师毛泽东主席根据马克思列宁主义的普遍真理和革命具体实践相结合的原则，在领导我国完成新民主主义革命和进行社会主义革命、社会主义建设的伟大斗争中，

FIGURE 11.- Official government proclamation announcing construction of a Memorial Hall and plans to exhibit the body of Chairman Mao in Beijing, dated October 8, 1976, in *Practice and Understanding of Mathematics* (4) (1975).

从两种不同的数学推导看不同的世界观*

——初学《数学手稿》的一点体会

北京师范学院数学系《数学手稿》学习小组

伟大导师马克思在《数学手稿》中，按照辩证唯物主义观点，深刻地阐述了微积分的第一个基本概念——导函数，不仅有独到的发现，并且全面地揭示了他的方法同牛顿、莱布尼茨的"神秘的微分学"和达兰贝尔的"理性的微分学"的根本差别，为我们树立了运用唯物辩证法研究数学的光辉典范. 但是，直到今天，在绝大多数微积分教材中，导函数概念的建立基本上还是沿袭达兰贝尔的方法. 因此，我们特别有必要通过学习马克思的《数学手稿》，并通过马克思的方法与达兰贝尔方法的比较，来深刻领会马克思的关于导函数概念的辩证实质，从而划清唯物辩证法和形而上学的界限. 这对彻底批判旧微积分的学科体系，做到用马克思主义占领数学阵地，有着极为重大的指导意义.

马克思指出：达兰贝尔用严格的代数方法推导出了导函数概念，他"**脱下了微分学的神秘外衣，取得了很大的进步**"，给予了肯定的评价. 但是，在数学上推导出了导函数概念并不意味着就揭示了导函数概念的实质，数学方法的严格和推导无误并不等于能够正确深刻地反映了客观现实. 所以，马克思在肯定达兰贝尔方法的同时又指出，他的推导实质上同牛顿和莱布尼茨的推导相同，是预先承认现成的导函数存在，只不过把镇压改成了严格的推导，因而是解脱的方法，不是发展的方法，"**又引起过某些形而上学的恐怖**". 马克思的方法与此根本不同，是从运动出发，在运动和变化中，最后发展出导函数. 马克思的方法同样是严格的数学方法，而这种严格却是建立在唯物辩证法的基础之上，因而深刻地揭示了导函数概念的辩证实质. 他指出："**首先取差（Differentiation），然后再把它扬弃，这样在字面上就导致无. 理解微分运算时的全部困难（正像理解否定之否定本身时那样），恰恰在于要看到微分运算是怎样区别于这样的简单手续并因此导出实际结果的.**"

马克思对此已经做了全面的阐述. 下面仅就我们领会到的，谈一点体会. 为了比较和论述方便，把达兰贝尔的方法和马克思的方法以函数 $y = x^3$ 为例并列于下：

达兰贝尔方法	马克思方法
$f(x)$ 或 $y = x^3$	$f(x)$ 或 $y = x^3$
a) $f(x + h)$ 或 $y_1 = (x + h)^3$ $= x^3 + 3x^2h + 3xh^2 + h^3$	a) $f(x_1)$ 或 $y_1 = x_1^3$
b) $f(x + h) - f(x)$ 或 $y_1 - y = 3x^2h + 3xh^2 + h^3$	b) $f(x_1) - f(x)$ 或 $y_1 - y = x_1^3 - x^3$

* 1975 年 6 月收到.

FIGURE 12.- First page of an article devoted to exploring the implications of two different approaches to the foundations of mathematics; at the bottom of the page, on the left, the approach of d'Alembert is contrasted with that taken by Marx, on the right. From *Practice and Understanding of Mathematics* (4) (1975), p. 1.

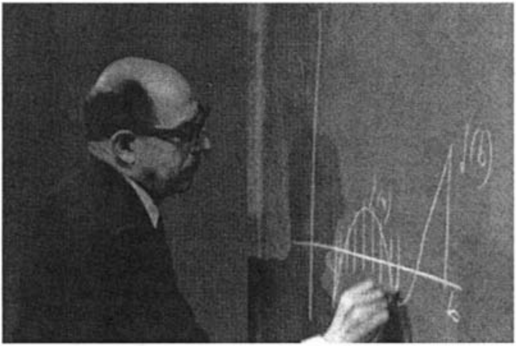

FIGURE 13.- Abraham Robinson, lecturing on nonstandard analysis [DAUBEN, 1995, p. 423].

以马克思主义哲学为指导评价非标准分析

周冠雄

学习马克思《数学手稿》，开展对非标准分析的讨论，对于我们掌握唯物辩证法，坚持用马克思主义指导数学研究，更好地推动数学的发展，有着现实的和深远的意义。

毛主席早在《在延安文艺座谈会上的讲话》中就指出："我们决不可拒绝继承和借鉴古人和外国人，那怕是封建阶级和资产阶级的东西"，应当"批判地吸收其中一切有益的东西"，不能当作"毫无批判的硬搬和模仿"。《论十大关系》则更明确地指出："自然科学方面，我们比较落后，特别要努力向外国学习。但是也要有批判地学，不可盲目地学。……已经清楚的那一部分，就不要事事照办了。"毛主席的这些重要指示，为我们研究外国的东西，规定了马克思列宁主义的路线、方针和政策。马克思、恩格斯、列宁、毛主席关于无限性的论述，关于高等数学的论述，为我们用马克思主义考察非标准分析提供了理论武器。马克思的《数学手稿》，研究了微积分学从牛顿到拉格朗日的历史发展，肯定了他们各自的贡献，指出了其中存在的[illegible]科学的剖析，对微积分学的一系列重大成果作出了马克思主义的哲学概括，贯穿了辩证唯物主义和历史唯物主义的观点，为我们用马克思主义考察非标准分析提供了光辉典范。革命导师的上述论述和著作的光辉思想，都是我们考察非标准分析时所必须坚持的马克思列宁主义的指导原则。根据这些原则，对于非标准分析的评价，应该提倡两点论，反对一点论；提倡历史地考察，反对割断历史的联系；提倡批判地分析和研究，反对不加批判地肯定和照搬。

非标准分析的实质，就是运用实在无限小量为高等数学（首先是微积分学）重新奠基。在非标准分析中，鲁滨逊用数理逻辑的方法推导出存在着大于零而小于一切正数的实在无限小量。整个非标准分析的工作，就是要建立一个以实在无限小量为基础的数学体系。这个体系，给出了区别于极限理论的微积分基础的另一种说明，提出了区别于极限方法的无限小量运算的另一种方法。对此，应该予以肯定，并继续考察它在自然科学领域中获得应用的前景。但是，由于鲁滨逊形式主义思想的影响，它作为自然科学的理论体系，是有它的局限性的，而对于其中散布的资产阶级唯心主义的观点，则必须加以批判。

（一）

非标准分析体系的基础——实在无限小量概念，源于莱布尼兹早期著作的观点。莱布尼兹是一个不自觉的辩证的数学家。实在无限小量的提出，说明他猜到了有限和无限是一个互相联系的统一体，有限包含着无限，有限可以反映无限。这种辩证法的思想，使他突破了初等数学的绝对固定的界限，把实在无限小概念应用于研究无限的数学，创立了微积

FIGURE 14.- A page from the article by Zhou Guan-xiong, «Using the Philosophy of Marxism to Evaluate Nonstandard Analysis,» which appeared in the *Journal of Central China Industrial College* in 1977.

FIGURE 15.- The first all-China Symposium on Mathematical Logic and Nonstandard Analysis, meeting at Xinxiang in Henan province.