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The Positive Schur Property in Banach Lattices

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For notations and terminology concerning Banach lattices, we refer the reader to [1], [4] and [5].

For a Banach lattice E we denote its norm dual by E'. The absolute weak topology on E, $|\sigma|(E,E')$ is the locally convex–solid topology generated by the family of Riesz seminorms $\{p_{x'} : x' \in E'\}$, where $p_{x'}(x) = |x'|(|x|)$ for each $x \in E$.

Let E be a Banach lattice. Following [3], we say that a set $A \subset E$ is $L-$weakly compact if A is norm bounded and $\|y_n\| \longrightarrow 0$ as $n \longrightarrow \infty$ whenever $\{y_n\}_n$ is a disjoint sequence in the positive part of the solid hull of A. It is well known that every $L-$weakly compact subset of a Banach lattice is relatively weakly compact (see [3]), and it is also known that a norm bounded subset of an abstract $L-$space is relatively weakly compact if and only if is $L-$weakly compact (see, e.g. [1], Theorem 21.10).

Recall that a Banach space is a Schur space if weakly null sequences are norm null. In this note we give a characterization of Banach lattices on which every positive weakly null sequence is norm null.

Let E be a Banach lattice; we say that E has the positive Schur property if every positive weakly convergent sequence in E is norm convergent.

Every Schur lattice has the positive Schur property.

Let E be a Banach lattice E with the Dunford – Pettis property (i.e., $x_n \longrightarrow 0$ $\sigma(E,E')$ and $x_n' \longrightarrow 0$ $\sigma(E',E'')$ imply $x_n'(x_n) \longrightarrow 0$) and such that E' contains no lattice isomorph to ℓ_1. Then if (x_n) is a positive weakly convergent to zero sequence, by [2, Corollary 2.6] the sequence (x_n) is norm null provided $\lim u_n(x_n) = 0$ for each norm bounded disjoint sequence (u_n) in E'_+, now since E' contains no lattice isomorph to ℓ^1, norm bounded disjoint sequences in E' are weakly null, we conclude that E has the positive Schur property. In particular

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every abstract L-space lattice has the positive Schur property. Recently Wnuk [6] showed examples of Banach lattices with the positive Schur property and which are not isomorphic to any abstract L-space.

Also Wnuk noticed that, if E has the positive Schur property, then E is weakly sequentially complete and every infinite dimensional sublattice of E contains a lattice isomorph to ℓ_1.

THEOREM 1. *Let E be a Banach lattice. Then the following statements are equivalent.*
(a) E has the positive Schur property.
(b) Every weakly convergent sequence of pairwise disjoint elements of E is norm convergent.
(c) Every relatively weakly compact subset of E is L-weakly compact.

COROLLARY 2. *Let E be a Banach lattice. Then the following statements are equivalent.*
(a) E is a Schur lattice.
(b) E is a discrete Banach lattice with order-continuous norm and every relatively weakly compact subset of E, is L-weakly compact.
(c) E has the positive Schur property lattice and every $\sigma(E,E')$-converging sequence in E is $|\sigma|(E,E')$-converging.

Recall the following notions introduced in [3] : let E be a Banach lattice and X be a Banach space; an operator T from E into X is called M-weakly compact if T maps norm bounded disjoint sequences into norm convergent sequences; an operator $T: X \longrightarrow E$ is called L-weakly compact if T maps norm bounded sets of X into L-weakly compact subsets of E. It is known that T is M-weakly compact (resp. L-weakly compact) if and only if its adjoint T' is L-weakly compact (resp. M-weakly compact); moreover M-weakly compact operators are weakly compact. See [3]. Let E and F be two Banach spaces. Hence an operator $T: E \longrightarrow F$ is called a Dunford-Pettis operator if T carries weakly convergent sequences onto norm convergent sequences.

THEOREM 3. *Let E be a Banach lattice, then the following statements are equivalent:*
(a) E' has order-continuous norm.
(b) Every Dunford-Pettis operator T from E into an arbitrary Banach space is

$M-$weakly compact.
(c) Every compact operator T from E into an arbitrary Banach space is $M-$weakly compact.
(d) Every positive operator T from E into an abstract $L-$space is $M-$weakly compact.

The next result gives a simple characterization of Banach lattices for which weakly compact operator are $M-$weakly compact.

THEOREM 4. *Let E be a Banach lattice. Then the following statements are equivalent:*
(a) E' has the positive Schur property.
(b) For each Banach space F, every weakly compact operator $T: E \longrightarrow F$ is $M-$weakly compact.

$L-$weakly compact operators in Banach spaces are weakly compact. The next result shows that the converse is also true for Banach spaces X such that X' is a Schur space.

THEOREM 5. *Let X be a Banach space. The following statements are equivalent.*
(a) X' is a Schur space.
(b) For each Banach lattice E with order continuous norm, every weakly compact operator $T: X \longrightarrow E$ is $L-$weakly compact.

REFERENCES

1. ALIPRANTIS, CH., BURKINSHAW, O., "Locally Solid Riesz Spaces", Academic Press, New York, 1978.
2. DODDS, P., FREMLIN, H., Compact operators in Banach lattices, *Israel J. Math.* 34(4), (1979), 287–320.
3. MEYER-NIEBERG, P., Über Klassen schwach kompacter operatoren in Banachver bänden, *Math. Z.* 138 (1974), 145–159.
4. SCHAEFFER, H., "Banach Lattices and Positive Operators", Springer-Verlag, Berlin-Heidelberg-New York, 1974.
5. SÁNCHEZ, J., Operadores en retículos de Banach, Tesis, Universidad Complutense de Madrid, España, 1985.
6. WNUK, W., Some characterization of Banach lattices with the positive Schur property (preprint).