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Characterizations of Algebras whose Modules with Fitting's Property are of Finite Lenght

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Let R be a ring. A left $R-$module M is said to satisfy property (I) (resp. (S)) if every injective (resp. surjective) endomorphism of M is an automorphism, M is said to satisfy Fitting's property (for short Property (F)) if for every endomorphism f of M there exists $n \in \mathbb{N}$ such that :

$M = \mathrm{Im} f^n \oplus \mathrm{Ker} f^n$. The object of this paper is to report the following result.

THEOREM. *For a finite dimensional algebra R over an algebraically closed field $\mathbb{K}$, the following conditions are equivalent.*

1) Every left R*−module with property* (I) *is Artinian.*

2) Every left R*−module with property* (S) *is Noetherian.*

3) Every left R*−module with property* (F) *is of finite lenght.*

4) Every left R*−module is a direct sum of finitely generated left $R-$modules.*

Proof. We show that if one of the conditions 1), 2) or 3) holds then every indecomposable left $R-$module M of infinite lenght is also of infinite lenght, considered as right $\mathrm{End}_R M-$module, and this last condition implies that R is of finite representation type (see [3]). Thus we obtain $1) \Rightarrow 4)$, $2) \Rightarrow 4)$, $3) \Rightarrow 4)$. The implications $4) \Rightarrow 1)$, $4) \Rightarrow 2)$ and $4) \Rightarrow 3)$ result from the fact that, on condition 4) R is of finite representation type (see [3]). For more informations on rings with one of the four conditions of the above theorem one may see [5], [6], [8] and [9]. ∎

COROLLARY 1. *Let R be a finite dimensional algebra over an algebraically closed field $\mathbb{K}$. The following conditions are equivalent.*

a) Every left $R-$module with property (I) *(resp.* (S)*, resp.* (F)*) is Artinian (resp. Noetherian, resp. of finite lenght).*

b) Every right R−module with property (I) (*resp.* (S), *resp.* (F)) *is Artinian (resp. Noetherian, resp. of finite lenght).*

COROLLARY 2. *Let $\mathbb{K}$ be an algebraically closed field of characteristic $P \neq 0$, and G a finite group. The following conditions are equivalent.*

1) Every left $\mathbb{K}[G]$−module with property (I) (*resp.* (S), *resp.* (F)) *is Artinian (resp. Noetherian, resp. of finite lenght).*

2) The sylow p−subgroups of G are cyclic.

3) Every left R−module is a direct sum of finitely generated left R−modules.

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