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SOME RESULTS ON THE FRAME BUNDLE OF AN ALMOST CONTACT METRIC MANIFOLD

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It is known that given an almost contact metric manifold (M,φ,ξ,η,g) a certain almost complex structure J is defined on it's frame bundle $\mathfrak{F}(M)$ ([2]),which results almost Hermitian when in $\mathfrak{F}(M)$ we consider the diagonal lift of g.

In this note we study the relationship among certain almost Hermitian structures on $\mathfrak{F}(M)$ and almost contact metric ones on M.

In the sequel,we shall suppose that M is a manifold endowed with an almost contact metric structure and verifying the $K_{1\varphi}$-curvature identity. We shall represent by ∇ the riemannian connection of g and the lifts will be done with respect to this connection.

Let J be the tensor field of type (1,1) on $\mathfrak{F}(M)$ defined in [2] :

$$J = \varphi^H + \sum_{\alpha=1}^{n} \eta^{H\alpha} \otimes \xi^{(\alpha+n)} - \sum_{\alpha=1}^{n} \eta^{H\alpha+n} \otimes \xi^{(\alpha)} + \eta^V \otimes \xi^{(2n+1)} - \eta^{H2n+1} \otimes \xi^H$$

The relationship between the normality of the almost-contact metric structure on M and the integrability of the almost-Hermitian structure on $\mathfrak{F}(M)$ is studied in [2]. We extend the study to some other different types of structures using the classifications of Gray-Hervella [5] and Oubiña [7] for almost Hermitian and almost contact manifolds respectively.

Some of the results we obtain are:

THEOREM 1.

If $(\mathfrak{F}(M),g^D,J)$ *is a* $\begin{cases} SK\text{-manifold} \\ G_2\text{-manifold} \\ Q\text{-}K\text{-manifold} \\ G_1\text{-manifold} \end{cases}$

then (M,φ,ξ,η,g) *is a* $\begin{cases} S\text{-}C\text{-manifold} \\ G_2\text{-}S\text{-manifold} \\ Q\text{-}K\text{-}C\text{-manifold} \\ Normal\text{-manifold} \end{cases}$

respectively.

THEOREM 2.

If $(\mathfrak{F}(M),g^D,J)$ *is a* $W_1 \oplus W_2 \oplus W_4$*-manifold then M is a Q-K-C-manifold if and only if* $\nabla_\xi \xi = 0$.

We have to consider the definition condition of the different types of structures and set the proper vector fields for substituing in the definition and the result follows,but the proofs are too long and we omit them.

We point out that there are some other types of structures that we are studying. We obtain interesting results if we consider certain almost contact metric structures on M and obtain a particular one on $\mathfrak{F}(M)$, but we have to impose strong conditions on M.

REFERENCES

[1] BLAIR,D. Contact manifolds in Riemannian Geometry.Lect. in Maths.509.New York. (1976).

[2] BONOME,A.,CASTRO,R., HERVELLA,L.; "Almost Complex Stucture on the frame bundle of an almost contact metric manifold". Math.Z. 193,431-440 (1986).

[3] CORDERO;L.A., DE LEON,M.; "Lifts of tensor fields to the frame bundle". Rend.Circ.Mat.Palermo(2) 32,236-271 (1983)

[4] CORDERO,L.A.,DE LEON, M.; " On the curvature of the induced Riemannian metric on the frame bundle of a riemannian manifold. J. Math. pures et appl. 65.81-91 (1986).

[5] GRAY,A., HERVELLA,L.; " The sixteen classes of almost Hermitian manifolds and their linear invariants ". Ann. Pur. Appl. (IV) Vol. CXXIII,35-38.(1980).

[6] KOBAYASHI;s:,NOMIZU,K.; Foundations of differential geometry Vols. I and II. New York.Interscience Publishers (1963).

[7] OUBIÑA,J.A.; " New classes of almost contact metric structures". Publ. Math. 32,187-193, (1985).

[8] YANO,K. ISHIHARA,S.; Tangent and cotangent bundles. New York. Marcel Dekker Inc. (1973).