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SIMPLE RECTANGULAR DISECTIONS

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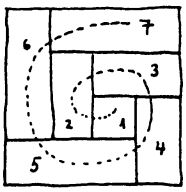
A *rectangular disection* (*RD*) *of order p* is a partition of a rectangle (*master rectangle*) in a finite number p of subrectangles, called *pieces* of the disection. A piece which shares no side with the border of the master rectangle is called an *inner piece*, otherwise it is called an *outer piece*. We say that a rectangular disection of order $p > 2$ is *simple* (*SRD*) if

(i) there is no proper subrectangle, i.e., no k pieces, $1 < k < p$, form a subrectangle and
(ii) there are no four pieces that share a common vertex.

A SRD can be drawn in the $\mathbb{N} \times \mathbb{N}$ lattice so that the *corners* of the pieces have integer coordinates, and the master rectangle has minimun area.
Two SRD are equal if and only if it is possible to transform one into the other through rigid motions.

The reader can check that there are no SRD's of order 3, 4 and 6, and there is only one SRD of order 5. Fig. 1 shows a SRD of order 7. We call it S_7 to remind the spiral arrangement of the pieces.



In 1960, Goldberg and Marsh [2] and independently in 1982 Chung et al. [3] proved that for all positive integer p, $p \geq 7$, there is a SRD of order p.

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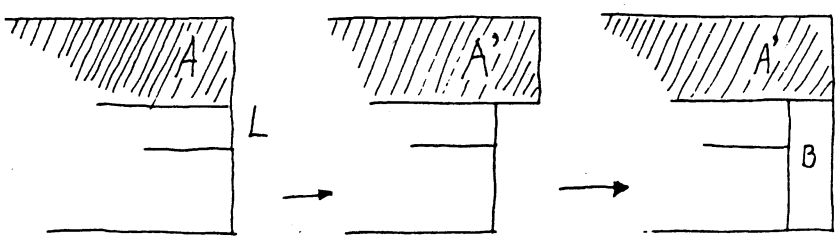
This note presents a simpler proof of this result and the technique used will allow us to tile whole plane with rectangles and satisfaying conditions (i) and (ii) as well.

Theorem. *For every integer p, $p \geq 7$, there is a SRD of order p.*

Proof: We note that any SRD of order p = 7 will have at least five outer pieces, so that there are three pieces bordering a common side L of the master rectangle. The proof wil proceed by induction on p. The inductive step is ilustrated in Fig. 2 and can be described as follow.



Suppose we have a SRD of order p = 7. Now choose a corner piece A touching L and pull it out to form another piece A'. This salient piece A' together with an extra piece B forms a RD of order p + 1. Clearly, if a proper subrectangle is formed, it would contain the piece B. But the existence of any proper subrectangle will imply that the original RD would have a proper subrectangle. But this is a contradiction.

If we start this construction with S_7, we obtain a sequence S_7, S_8, ... of SRD's. Since any circle of the plane can be covered by some S_n, n $\mathbb{N}$, it is clear that carrying out the former method to the limit, the plane can be tiled with subrectangles satisfaying conditions (i) and (ii).

In 1979, Bloch [1] published a catalogue with all the RD's of order p, $1 \leq p \leq 7$. As far as we know, a formula giving the number of RD's and SRD's with a given order has not been discovered. Nevertheless, it is interesting the following table.

Order p	1	2	3	4	5	6	7	8	9
Number of RD's of order p.	1	1	2	6	24	125	814	?	?
Number of SRD's of order p.	0	0	0	0	1	0	2	7	39

We propose the following two open problems:

a) To find the possible dimensions for the pieces in a SRD of order p.

b) To determine lower and upper bounds for the dimensions of the master rectangle of a SRD of order p.

REFERENCES

[1] C.J. Bloch, Catalogue of small rectangular plans, Environment and Planning, (series B) 6 (1960) 155 - 190.

[2] M. Goldberg and C.B. Marsh, Primitive descompositions, Ammer. Math. Monthly 67 (1960) 477 - 478.

[3] R.K. Chung, E.N. Gilbert, R.L. Graham, J.B. Shearer and J.H. van Lint, Tiling rectangles with rectangles, Math. Mag. 55 (1982) 286 - 291.