10.46551/emd.v9n17a05 2526-6136

Educação Matemática Debate

Exploring the relationship between creativity and creative reasoning in the subject of Differential and Integral Calculus

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Recebido • 30/10/2024
Aceito • 12/02/2025
Publicado • 10/05/2025

Article

Abstract: The study investigates the relationship between creativity and creative reasoning in the context of working with mathematical tasks in the subject of Differential and Integral Calculus. The research explores how these concepts manifest themselves and influence mathematical problem solving, with a qualitative approach and detailed analysis of a task. It highlights creativity as a developable ability and creative reasoning as a process of novelty, flexibility and plausibility. Strategies to stimulate its development in teaching are also discussed. The analysis suggests that encouraging creative reasoning in students, motivating them to explore different approaches and question constraints, promotes a deeper and more innovative understanding of mathematical concepts.

Keywords: Creativity. Teaching Mathematics. Teaching Differential and Integral Calculus. Mathematics Education. Mathematical Tasks.

Explorar la relación entre creatividad y pensamiento creativo en la asignatura de Cálculo Diferencial e Integral

Resumen: El estudio investiga la relación entre creatividad y razonamiento creativo en el contexto del trabajo con tareas matemáticas en la asignatura de Cálculo Diferencial e Integral. La investigación explora cómo estos conceptos se manifiestan e influyen en la resolución de problemas matemáticos, con un enfoque cualitativo y el análisis detallado de una tarea. Enfatiza la creatividad como una habilidad desarrollable y el razonamiento creativo como un proceso de novedad, flexibilidad y plausibilidad. También se discuten estrategias para fomentar su desarrollo en la enseñanza. El análisis sugiere que fomentar el pensamiento creativo en los alumnos, motivándoles a explorar distintos enfoques y cuestionar las limitaciones, favorece una comprensión más profunda e innovadora de los conceptos matemáticos.

Palabras clave: Creatividad. Enseñanza de las Matemáticas. Enseñanza del Cálculo Diferencial e Integral. Educación Matemática. Tareas Matemáticas.

Explorando relações entre a criatividade e o raciocínio criativo na disciplina de Cálculo Diferencial e Integral

Resumo: O estudo investiga a relação entre criatividade e raciocínio criativo no contexto do trabalho com tarefas matemáticas, na disciplina de Cálculo Diferencial e Integral. A pesquisa explora como esses conceitos se manifestam e influenciam a resolução de problemas matemáticos, com uma abordagem qualitativa e análise detalhada de uma tarefa. Destaca-se a criatividade como competência desenvolvível e o raciocínio criativo como um processo de novidade, flexibilidade e plausibilidade. Além disso, discute-se estratégias para estimular seu desenvolvimento no ensino. A análise sugere que incentivar o raciocínio criativo nos estudantes, motivando-os a explorar abordagens diversas e questionar restrições, promove uma compreensão mais profunda e inovadora dos conceitos matemáticos.

Palavras-chave: Criatividade. Ensino de Matemática. Ensino de Cálculo Diferencial e Integral. Educação Matemática. Tarefas Matemáticas.

1 Introduction

Creativity and creative reasoning (RC) play crucial roles in the training of future engineers. In particular, the *Diretrizes Curriculares Nacionais para o Curso de Graduação em Engenharia* [National Curriculum Guidelines for Undergraduate Engineering Courses] (Brasil, 2019, p. 1) point out that the profile of the graduate of these courses should include, among other characteristics, the importance of "having a holistic and humanistic vision, being critical, reflective, creative, cooperative and ethical and with a strong technical background".

This work, which is the result of the first author's dissertation, under the guidance of the second authors, was motivated by the need to reflect on what characterizes this subject as *creative*, resulting in a theoretical deepening to understand how the literature discusses this expression cited in the guidelines.

Creativity has become a topic of widespread interest since the end of the 20th century, not only for those researching teaching and learning processes and human development, but for society as a whole (Craft, 1999; Gontijo *et al.*, 2021). For Gontijo *et al*. (2021), creativity is still a little-studied topic when compared to other themes, especially in the field of Mathematics Education. However, in recent decades, it has been consolidating and gaining more interest.

According to Gontijo (2006), creativity is the ability to create and invent, referred to as the quality of someone who has original ideas or is capable of proposing new statements. It's not a matter of talent, but a ability which, as such, can be developed and perfected.

Lithner (2006) proposes a framework for analyzing RC and exemplifies it in the context of Differential and Integral Calculus (CDI). According to the author, this type of reasoning is mathematically based and has characteristics such as novelty, flexibility and plausibility, as opposed to *imitative* reasoning.

Based on the theoretical foundation proposed by Gontijo (2006) on creativity and Lithner's (2006) discussion on CR, this study aims to: (i) understand some interconnections between these two concepts; and, (ii) point out manifestations of students' RC when dealing with mathematical tasks in the context of the CDI discipline in Engineering courses.

2 Creativity

Although vast, the literature on creativity does not provide a single concept of its investigation, nor does it present a single definition of the term. One version states that for many years the phenomenon of creativity was seen as a mystical, incomprehensible and inexplicable act, attributing to human beings an almost divine connection through creation (Dollinger, 2007). According to Pinheiro (2009), researchers from a wide range of fields have questioned whether it is possible to explain creativity. Although there was consensus around the affirmative answer, attempts to express each person's point of view in an absolute way made disagreement the rule and unanimity the exception.

Another version of the first studies into creativity refers to the attribution of this characteristic to those considered geniuses. Lubart (2007, p. 12) states that "during the 18th century, philosophical debates arose about genius and, in particular, about the foundations of creative genius". These debates made creativity, once conceived as an almost divine gift, now "an exceptional form of genius, different from talent, and determined by genetic factors and environmental conditions" (Lubart, 2007, p. 12).

Currently, in relation to both versions, it is argued that creativity and intelligence have a close relationship, but that they are two different dimensions. For example, highly creative people are not necessarily more intelligent than less creative people, nor are people with a high intelligence quotient considered to be the most creative. In any case, part of the confusion about creativity is due to the fact that, for centuries, creativity was considered something mystical and religious. Therefore, until practically the 20th century, its study was not approached scientifically.

The various historical perspectives on creativity reflect the socio-cultural contexts and worldviews of the times in which they were developed, and are therefore more the result of transformations over time than complementary. The image of creativity as something mystical or related to a divine gift, for example, represents a dated understanding that has been re-signified with the advancement of science and social change.

In the 20th century, research began to emerge that approached creativity from a scientific perspective. Torrance (1965) *apud* Alencar and Fleith (2009, p. 14), for example, considers creativity "as a process of becoming sensitive to problems, deficiencies and gaps in knowledge; identifying the difficulty; seeking solutions by formulating hypotheses about the deficiencies; testing and retesting these hypotheses; and finally, communicating the results". Similarly, Ostrower (1977, p. 1) considers "creativity to be a potential inherent in man, and the realization of this potential one of his needs".

But it is in the 21st century that the topic has taken on greater force. Thus, authors such as Alencar and Fleith (2003, p. 13-14) define that "creativity implies the emergence of a new product, be it an original idea or invention, or the reworking and improvement of existing products or ideas". Martínez (2006, p. 70) argues that creativity "is expressed in the production of something that is considered both new and valuable in a given field of human action".

Based on research into creativity in mathematics, it can be seen that, in this area too, the definition of the term *creativity in mathematics* is not consensual. In view of this, different perspectives on this concept are presented in Table 1.

Table 1: Different definitions of creativity in mathematics

Author	**Definition**
Gontijo (2006, p. 4).	The ability to present numerous appropriate solution possibilities for a problem situation, so that they focus on different aspects of the problem and/or different ways of solving it, especially unusual ways (originality), both in situations that require problem solving and elaboration and in situations that require the classification or organization of mathematical objects and/or elements according to their properties and attributes, whether textually, numerically, graphically or in the form of a sequence of actions.
Makiewicz (2004, p. 2)	Mathematical creativity as a real version of scientific creativity is understood as the activity of constructing, modernizing and completing the system of knowledge through the observation of regularities, sensitivity to problems, exposing hypotheses and justifying propositions.
Krutetskii (1976, p. 68)	[...] the independent formulation of uncomplicated mathematical problems, finding ways and means of solving these problems, the invention of proofs and theorems, the independent deduction of formulas and finding original methods of solving non-standard problems.

Source: Own elaboration

When reflecting on the general characteristics of creativity, Vega (2003), Alencar (1990) and Guilford (1973) present converging ideas on the subject. While Guilford (1973) develops his work in a broader context of psychology, Alencar (1990) and Vega (2003) deal with creativity in the school environment. Although none of these authors deals specifically with creativity in mathematics, their conceptions present points of intersection with the definition proposed by researchers in the field.

Vega (2003, p. 27) argues that “fluency, flexibility and originality are three indicators of creativity”. According to the author, during the process of solving a problem, there is a preparatory phase in which it is natural for many ideas to be generated before selecting those that seem to lead to a solution. In this context, quantity can generate quality, since the first idea that emerges is not necessarily the best — a characteristic defined as fluency. In addition, the diversity of ideas allows the problem to be observed from different perspectives, reflecting in a flexible way. Finally, when proposed solutions are sought after, they are considered original.

Alencar (1990) argues that in order to stimulate the development of creativity, an environment must be created that allows students to show fluency, flexibility, originality and elaboration in their work. According to the author, fluency is given by the abundance or quantity of different ideas produced on the same subject, flexibility by the ability to alter thinking or conceive different categories of answers, originality by presenting infrequent or unusual answers and elaboration by presenting a large amount of detail in an idea (Alencar, 1990).

Guilford (1973), in turn, presents fifteen characteristics of a creative adult, which are based on

1. Flexibility: The ability to go beyond tradition, habits and the obvious. To transform ideas and materials for new, different and unusual uses.
2. Fluency: The ability to think of many ideas; many possible solutions to a problem.
3. Elaboration: The ability to work out the details of an idea or solution.
4. Tolerance of ambiguity: The ability to hold conflicting ideas and values and bring about a reconciliation without undue tension. The values of creative people, for example, seem to be both aesthetic and theoretical, two value systems that can be considered antithetical. The creative person seems to be interested not only in solutions to problems, but also in aesthetically satisfying, "elegant" solutions. Their goal seems to be truth and beauty.
5. Originality: Divergent rather than convergent reasoning, going beyond commonly accepted ideas to unusual forms, ideas, approaches and solutions.
6. Breadth of interest: Wide range of interests with much more concern for the “big ideas”, broad meanings and implications, rather than small details and facts for facts' sake.
7. Sensitivity: The ability to sense problems, to see shortcomings and needs in life, the challenge of finding solutions and fulfilling these needs. Sensitivity to our own inner life and the feelings, thoughts and feelings of others.
8. Curiosity: Openness to new ideas and experiences; the ability to be intrigued; active experimentation with ideas and the pleasure of seeking them out and discovering them.
9. Independence: thinking about things through our own confidence and strength.
10. Reflection: The ability to consider and reconsider, to evaluate one's own ideas as well as the ideas of others; to take time to gain understanding and insight, to look ahead and plan, to visualize the whole picture.
11. Action: The ability to put ideas into action; to start, to help, to shape ideas with great energy and enthusiasm.
12. Concentration and persistence: The ability to work hard, for a long time, consistently and persistently with extraordinary concentration.

> 13. Commitment: Deep involvement, intense commitment, deep care, almost metaphysical in nature.
> 14. Expression of the total personality: Expression of the masculine and feminine sides of nature, which sometimes leads to tension in our society. How the creative male shows supposedly feminine traits such as sensitivity, self-awareness and breadth of interests or how the female shows "masculine" traits such as independence, self-confidence and willpower.
> 15. Sense of humor: The ability to see and express the humor in life's contradictions and ambiguities. To maintain balance without losing commitment (Guilford, 1973, p. 3-4).

Kneller *apud* Guilford (1973, p. 53) highlights relevant reflections on productive capacities and related types of reasoning. Productive capacities can be classified into two distinct categories: convergent and divergent.

> The first is triggered by thinking that moves after a certain or conventional response. The second is triggered by reasoning that moves in various directions in search of a given answer. We can therefore conclude that convergent reasoning occurs where the problem is offered, where there is a standard method for solving it, known to the thinker, and where a solution can be guaranteed within a finite number of steps. Divergent reasoning tends to occur where the problem is yet to be discovered and where there is no settled way of solving it. Convergent reasoning implies a single correct solution, whereas divergent reasoning can produce a range of appropriate solutions.

In summary, Vega (2003), Alencar (1990) and Guilford (1973) have convergent conceptions, among which flexibility, fluency and originality stand out. This approach is not only due to the use of the same words, but also to the characterization attributed to each of these aspects.

Regarding the development of creativity, Gontijo (2007) mentions that the literature brings together various strategies for stimulating creative reasoning, focusing specifically on teaching strategies. According to the author, these strategies, or categories, include: a) appreciation; b) animation; c) association; d) alteration; e) abdication. According to Gontijo (2007, p. 67), appreciation includes

> techniques used to get to know one or more aspects or attributes of a situation, product or problem under consideration. These techniques can be used to help students focus on important features of the problem, notice patterns and outline a variety of possible solutions.

In animation, "the techniques related to this category can be used in activities to involve students interactively with problems, situations or products" (Gontijo, 2007, p. 68). In turn, the use of association techniques

> can help students make comparisons and establish connections between a problem that they don't immediately have a method for solving with concepts, algorithms and strategies they already know. These creativity techniques help by focusing attention on making these connections (Gontijo, 2007, p. 69).

With alteration techniques

> students systematically change parts of a product, situation or problem. “What if...” questions are present in most mathematical investigations and insights. These techniques make it possible to deepen mathematical conceptions by systematically changing parts of the problem or its solution, leading to new and interesting questions or problems to explore (Gontijo, 2007, p. 70).

Finally, “abdication techniques are intended to allow the subconscious to reflect on the problem when you're not actively working on it” (Gontijo, 2007, p. 71).

3 Creative Reasoning

Lithner (2006) contrasts RC with imitative reasoning, defined as the mere reproduction of a model or example, without any attempt at originality. RC, on the other hand, is characterized by its unusual nature, going beyond the mechanical application of basic algorithms and routine procedures. In addition, according to Lithner (2006, p. 6), RC can be understood as a form of justification, “of a different character in different series, and that the specific format of the justification is less important than a clear communication of mathematical ideas”.

The RC proposed by Lithner (2006) meets the following criteria: (i) novelty, with a new sequence of reasoning being created or a forgotten sequence being recreated; (ii) flexibility, fluently admitting different approaches and adaptations to the situation, without fixing the universe of content or seeking memorized or algorithmic solutions; (iii) plausibility, considering arguments that support the choice and/or execution of the strategy, and that allow consideration of whether the conclusions are true or plausible; and (iv) mathematical foundations, with the arguments anchored in the intrinsic mathematical properties of the components involved in the reasoning.

According to the author, in order for students to develop RC when solving tasks, it is essential that they initially try to come up with their own solutions. If they are unsuccessful, it is necessary to identify the specific difficulties they face and offer targeted feedback to help them construct more elaborate and innovative responses.

According to Lithner (2017), when designing a task designed to promote RC, it is imperative to ensure that students do not know the solution method in advance. However, it is also necessary that the task is not difficult for the student. The author emphasizes that the solution of the task must be linked to the construction of certain knowledge, which makes this process even more challenging.

In addition, Lithner (2017) emphasizes that the task should provide students with the possibility of constructing arguments anchored in mathematics, so that these arguments support the reasoning in the task-solution. If students don't remember or don't have access to a solution method, there are two possibilities for solving the task. The first is guesswork, which, although it can be a constructive part of problem solving, is rarely enough to solve the task on its own. The second possibility is constructing the solution, which requires guidance or some kind of argument (explicit or implicit) to support the students' choices and conclusions.

In addition, active learning methodologies, such as Project-Based Learning, reinforce this approach by giving students the opportunity to explore, create and collaborate in real and challenging contexts. As Azevedo and Maltempi (2019) point out, these practices not only motivate students, but also expand the possibilities for building mathematical knowledge, connecting it to a training space focused on autonomy and creativity. This perspective complements creative reasoning by allowing students to develop flexibility and plausibility in their solutions.

4 Creativity and RC

From the research carried out on the topics of creativity and creative reasoning (RC), it was possible to identify some similarities, which will be presented in this section of the text. Approximations refer to both the characteristics of creativity and the aspects of RC as described by Lithner (2006). Although some of the aspects highlighted are identical in name and concept, there are also important distinctions. The approximations between these concepts are discussed below.

- Novelty (Originality): in Lithner (2006), the *novelty* is associated with something new or recreated, which is in line with the idea of originality in Vega (2003), Alencar (1990) and Guilford (1973), where *originality* also implies something beyond the ordinary, in other words, something new.
- Flexibility: the *flexibility* in Lithner (2006) seems to encompass the definitions of flexibility and fluency found in Alencar (1990), Guilford (1973) and Vega (2003). For these authors, this aspect refers to the ability to adapt strategies, arguments or approaches to different situations.
- Plausibility and Divergent Reasoning: the *plausibility* in Lithner (2006) is related to the idea of *Divergent Reasoning* according to Kneller (1973), which involves generating multiple ideas and choosing the best solution from among several options. This concept is also present in Vega's fluency (2003), which focuses on the quality of the choice or execution of strategies.
- Mathematical Foundations: the *Mathematical Foundations* in Lithner (2006) are related to Gontijo's (2007) definition of mathematical creativity, which highlights the importance of classifying mathematical elements based on their properties and attributes.

Based on these approximations, it was concluded that the four criteria proposed by Lithner (2006) on RC are directly linked to the characteristics of creativity found in the literature. Each of the criteria is associated with at least one characteristic of creativity, which justifies the use of the term RC in this part of the article, especially when recognizing its manifestations in the analysis of a task proposed to CDI students.Table 2 summarizes the relationships between the aspects of creativity and RC.

Table 2: Approximation and relationship between aspects of creativity

Aspects of Creativity according to Creativity literature in general	Aspects of RC according to Lithner (2006)
Originality according to Vega (2003)	Novelty
Originality according to Alencar (1990)	
Originality according to Guilford (1973)	
Flexibility according to Vega (2003)	Flexibility
Flexibility according to Alencar (1990)	
Flexibility according to Guilford (1973)	

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Fluency according to Alencar (1990)	
Fluency according to Guilford (1973)	
Fluency according to Vega (2003)	Plausibility
Divergent reasoning according to Kneller (1973)	
Creativity according to Gontijo (2007)	Mathematical Foundations

Source: Own elaboration

5 Manifestations of RC: analysis of a mathematical task

To investigate the relationship between creativity and RC in the context of mathematical tasks, this research adopted a qualitative approach. This methodology consists of exploring phenomena in their natural contexts, focusing on detailed descriptions and meanings constructed by the participants. This approach allows for an in-depth understanding of subjective and complex processes, valuing the perspective of the subjects and detailed analysis of interactions and experiences (Bogdan and Biklen, 1994). The methodology was designed with the aim of analyzing how these concepts manifest and interconnect in educational practices, specifically in a challenging mathematical task.

The participants involved in this analysis were students from the Mechanical Engineering and Materials Engineering courses at a federal university in the state of Paraná, who took the subject Calculus of more than one variable (CDI 2) in the second semester of 2022. This subject, taught by one of the authors, covers a variety of mathematical concepts, including real functions of several variables, limit and continuity of functions, differentiability, multiple integration and their applications.

It is important to note that the typical profile of students attending engineering courses often includes a lack of previous experience with investigative tasks, expectations of lectures and difficulties in working in groups and expressing their ideas. It should also be noted that this is a context in which there is a strong relationship between dropping out of the course and failing the CDI subject (Godoy and Almeida, 2017).

The material collected throughout the semester included written records, images, videos and audios of the students as they worked in groups (with three or four members) on various mathematical tasks with the potential to develop RC. It should be noted that although not all of the productions presented the four RC criteria — novelty, flexibility, plausibility and mathematical foundations — in the analysis presented below we chose to consider a task in which it was possible to identify these four criteria.

The mathematical task selected for this study was adapted from Stewart (2013) and applied on August 18, 2022, at the very beginning of the course. The aim was to introduce the content of functions of two or more variables, and it was given two lessons of 50 minutes each.

Table 3: Implemented task

You want to build an open box, and the base material costs R$ 10.00 per square meter and the side material costs R$ 6.00 per square meter.
a) What do you imagine this box would look like, so that the cost of producing it would be minimal?
b) Construct a mathematical expression for the total cost of the material used to make the box.
c) What changes (or not) in your previous answers if we consider that the box should have a volume of 10 m^3?
d) What if we consider that, in addition to the volume of 10 m^3, the base of the box must be

rectangular, with the measurement of one side equal to twice the measurement of the other??

Source: Adapted from Stewart (2013)

The task was to determine the ideal characteristics for building an open box, considering different costs for the base and sides. It was applied before the content was introduced to the students, so they did not yet know a formal definition of a function of two or more variables. The questions asked were: a) imagine the appearance of the box that would result in the minimum production cost; b) develop a mathematical expression that represents the total cost of the material to make the box; c) evaluate the changes in the previous answers when considering that the box should have a volume of 10 m^3; d) analyze the implications of having the base of the box as a rectangle, where one side is double the other, in addition to meeting the volume of 10 m^3.

In order to point out some manifestations of students' creativity and RC when dealing with this task, we analyzed four excerpts from the discussion between students A, J and T about what happened when they solved the task.

In excerpt 1, the students discuss item a of the task, which asks what the box should look like so that the cost of producing it is minimal.

A: The side material is cheaper than the base material.

A: In letter a, in order for the cost to be minimal, we have to think about the following, the square meter of the side material is 40% cheaper than the base material, but think about, like, those ballad glasses, they make a tall glass with a bigger side and a small base and the volume of it is small, it fits almost nothing.

The other members agree.

A: On the other hand, we have to think that in order to have the lowest possible cost this box has to have a small base, I don't know 1 m² and depending on the volume we want we can vary the height of the side.

T: I'd keep the base the same and increase the side.

A: So a simpler square box, with a base of 1 m² and a height of 1 m² of material.

A: Wow, but look, we only have one base, we have four sides. In fact, it makes more sense to use more material on the base than on the sides.

J: So the base has to be bigger than the side.

A: Exactly.

T: So a rectangle, not a square.

A: Yeah, because what's going to happen? The material on the side won't have such an impact on the volume.

A: So it would be a box in which most of the material is proportionally concentrated at the base.

A: It would look like a box with low sides and a larger base.

During the discussion, criteria (ii) flexibility and (iii) plausibility were identified. Flexibility is recognized when they see two different shapes for the box (square and rectangular) and plausibility when they justify and argue that the best option would be the rectangular box, since, regardless of the fact that the sides are cheaper, there are four sides and only one base. Thus, the students conclude that the best option in terms of the shape of the box to keep the cost to a minimum would be a box with a larger base and smaller sides.

In excerpt 2, the students discuss item *b* of the task, which asked them to construct a mathematical expression for the total cost of the material used to make the box. This resolution could involve mathematical concepts such as algebraic expressions.

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J: In "b", there would be two things for the expression, the side and the base.

A: Wow, would that be a two-variable expression? Because we have to add up the amount of material used on the sides multiplied by six, and the amount of material used on the base multiplied by ten.

T: That makes sense, but it would still be four more sides.

A: Yes, but then we use the direct material, like 10 m^2 to make the sides times 4.

A: $z = x.6 + y. 10$

T and J: That's right

The discussion identified the criteria (i) novelty and (iv) mathematical foundations. Novelty is evidenced in the questions about whether it is an expression of two variables in relation to the cost of the sides and the base, given that the student seems to be surprised to note that two variables would be needed to form the expression, the sides and the base.

And the mathematical reasoning is manifested when the students use algebraic expressions to conclude the appropriate expression for the situation, i.e. they use letters, numbers and symbols of the mathematical operations, addition and multiplication, to make the expression for calculating the total cost of the material used in the box and name x as the material used in the sides and y as the material used in the base.

In section 3, the students discussed item *c* of the task, in which they were expected to justify whether or not they would change their previous answers if the box had a volume of 10 m^3. To do this, the students were able to go back to their previous answers and analyze whether they would need to change any aspect of their solutions or whether they would keep the same characteristics in light of this new condition.

A: What's going to change?

J: That's the question, if we take it.

Interaction with other trios (random conversations)

A: So, what will change if the box has a volume of 10 m^3?

A: A box with a volume of 10 m^3 is a box that multiplies its length, width and height.

T: That's 10 m^3.

A: The length is the base, the width is also the base, the only factor that influences the sides is the height, so the base influences the total volume twice as much as the height.

J: So we could make a base of 3 lengths and widths and 4 heights.

A: But is it addition or multiplication? I think it's the base times.

T: It's times.

A: To make a 10 m^3 box, it has to be 2 meters long, 5 meters wide and 1 meter high.

J: Or 4; 2.5 and 1.

T: Or 2; 2; and 2.5.

A: What changes is this, like in our answer "a" it's still true, but it's not the minimum amount of material on the sides, because sometimes it's better to throw a bit on the side than on the base, even if the base has more influence, right?

J and T: Right.

J: So it doesn't change anything, because for example here we're using 5 m^2 at the base, but we're using 12 on the side and the side is still low.

T: There's another thing, the side is four times.

A: That's right, the side is four times, so it doesn't change anything. It's still a box with low sides and a larger base.

A: So "a" doesn't change anything and "b" doesn't either, the expression still applies.

T: What changes then is nothing.

A: With a volume of 10 m³, the material used for the base will have more influence on the volume for less money, even though it's more expensive, 66% more expensive.

In this excerpt, criteria (iii) plausibility and (iv) mathematical foundations were identified. Plausibility was recognized when they returned to items *a* and *b* to argue and validate their choices. Mathematical reasoning appeared when they discussed the operation and the possible measurements for the volume to be 10 m³, taking into account the three dimensions: height, length and width. In the last section, the students discuss item d of the task, in which, in addition to the volume of 10 m³, the base of the box should be rectangular, with one side being twice the other.

J: Now, in addition to the volume of 10 m³, the base of the box must be rectangular, with one side equal to twice the other.

A: Then things change, because if we have a box that has 8 m² of material at the base, there's no way we can make it 10m³ at the top.

T: 1.5 and 3.

A: Yes, but 1.5 and 3 is 4.5 times 2 is 9.

J: The base is 2 by 4 and 1.25.

A: Is that the best possible configuration?

A: Does it change anything? Because 4 times 1.25 is 5 and 5 times 4 is 20.

J: Times 2, that's 10.

T: Why is it times 2?

A: Because it's a rectangle, the sides are different.

A: Yeah, I think that's the best possible configuration.

J: Because it's double, right?

During the discussion, criterion (iii) plausibility was observed, when the students discussed arguments and validity tests in order to confirm the trio's choice of strategy in relation to the shape of the box and the restrictions of alternative d. In other words, they evaluated whether, according to the chosen strategy, the volume of 10 m³ would be compatible with the base of the rectangular box, with one side being twice the other, in order to reach a joint conclusion.

To summarize the analysis of this task, Table 4 highlights some aspects of the RC expressed by the students, associating them with the characteristics of the task.

Table 4: Table of RC criteria expressed by students in task 1

Aspect	Callsigns	Task characteristics
Novelty	Function of two variables (Excerpt 2)	A family context that involved the sum of two variables.
Flexibility	Two different approaches to solving the question (Excerpt 1)	Exploratory task, with various possibilities regarding the shape of the box.
Plausibility	Different approaches (Excerpt 1) Arguments supporting the choice (Excerpts 3 and 4)	Questions about the shape of the box. Questions about adding restriction.

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Mathematical foundation	Using properties to calculate volume (Excerpt 3) Using properties of algebraic expressions (Excerpt 2)	Known context about volume. Context known from the final years of elementary school (representation and calculation of an algebraic expression)

Source: Adapted from Stewart (2013)

The analysis of this task reveals the importance of RC in solving complex mathematical problems. Creative reasoning, as described by Lithner (2006), involves generating innovative and mathematically based strategies to solve problems. During the task, the students demonstrated creativity by proposing different configurations for the box, considering the restrictions imposed. Creativity is evidenced in the ability to think outside traditional patterns, imagining new ways and strategies to achieve the most efficient solution.

On the other hand, RC is present when these innovative ideas are evaluated and tested according to mathematical criteria, such as plausibility and reasoning. This validation process is essential to ensure that the proposed solutions are not only original, but also mathematically feasible. Thus, creativity and RC are not isolated processes, but interdependent ones, in which creativity provides the initial ideas and RC refines and validates them.

Based on the analysis of the task, it can be inferred that, in general, encouraging students' creative reasoning, stimulating them to explore multiple approaches and question the restrictions imposed, can result in a deeper and more innovative understanding of mathematical concepts.

6 Final considerations

This study has deepened our understanding of the interconnections between creativity and RC in the context of mathematical tasks, highlighting their implications for teaching and learning. It was observed that the four RC criteria proposed by Lithner (2006), novelty, flexibility, plausibility and mathematical foundations, are deeply aligned with the characteristics of creativity described by authors such as Gontijo (2006) and Alencar (1990).

The analysis revealed that creativity in mathematical reasoning is not just a matter of generating new ideas, but also of applying these ideas in a flexible and well-founded way. The ability to formulate original solutions, adapt strategies in an innovative way and justify these approaches with sound mathematical arguments represents the essence of creative reasoning. Thus, creativity and RC emerge as complementary processes, where creativity provides the initial fuel for ideas and RC channels this potential into viable and effective solutions.

In addition to the ideas already mentioned, it is essential to understand the relationship between creativity and RC. Creativity in the mathematical context involves generating new and innovative ideas, but creative reasoning goes further, requiring the evaluation and justification of these ideas with solid mathematical foundations. RC functions as a structure that organizes and supports creative solutions, ensuring their applicability in a mathematical context. The interaction between these two aspects enables not only the exploration of new approaches, but also the guarantee of their viability. Therefore, by integrating both creativity and creative reasoning into teaching, an environment is fostered in which students are challenged to think in an innovative yet grounded way, which contributes to more meaningful and effective learning.

The results of this study suggest that an integrated approach to teaching mathematics, especially CDI, which values both originality and conceptual grounding, can significantly enrich students' problem-solving process. Fostering an environment in which the fluency of ideas, the ability to adapt and the construction of sound mathematical arguments are encouraged not only stimulates innovation, but also strengthens the deep understanding of mathematical concepts. This makes learning more dynamic and relevant, encouraging students to adopt a new attitude to tackling complex challenges creatively and efficiently.

Thus, by recognizing and promoting creativity and RC in mathematics teaching, we contribute to the formation of individuals who are better prepared to face more complex and dynamic challenges. The combination of creativity and RC is therefore not only a pedagogical goal, but a fundamental need for education.

Note

The translation of this paper from Portuguese into English was funded by the Minas Gerais State Research Foundation (Fapemig — Fundação de Amparo à Pesquisa do Estado de Minas Gerais), under Call for Proposals 8/2023.

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