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How are Students' Critical Thinking Skills When Solving Problems Collaboratively?

¿Cómo se manifiestan las habilidades de pensamiento crítico de los estudiantes al resolver problemas de forma colaborativa?

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Abstract ∞ Vygotsky's sociocultural theory states that interaction with peers will expand students' Zone of Proximal Development (ZPD) to think critically. Collaborative Problem Solving (CPS) is an activity that requires interaction between team members, and several studies state that CPS has an impact on increasing students' critical thinking abilities. This study describes students' critical thinking skills when solving collaborative math problems. Students' critical thinking skills when solving problems collaboratively appear in two conditions: when working independently (*individual space*) and when interacting with other team members (*collaborative space*). Students' critical thinking skills are triggered by the problems given in the individual space. In collaborative space, students' critical thinking skills emerge more because they are triggered by two things, namely the problem given and responses from other group members. A description of how students' critical thinking skills work when solving problems collaboratively is explained in more detail in the research results section.

Keywords ∞ Critical thinking skills; Collaborative problem solving; Mathematics; Collaborative learning

Resumen ∞ La teoría sociocultural de Vygotsky sostiene que la interacción con los pares expande la Zona de Desarrollo Próximo para pensar críticamente. La Resolución Colaborativa de Problemas (CPS) es una actividad que requiere la interacción entre miembros de un equipo, y los estudios han mostrado que tiene un impacto en el incremento de las habilidades de pensamiento crítico. Este estudio describe las habilidades de pensamiento crítico de los estudiantes al resolver problemas matemáticos colaborativamente. Dichas habilidades aparecen en dos contextos: cuando trabajan de manera independiente (espacio individual) y cuando interactúan con otros miembros del equipo (espacio colaborativo). Las habilidades de pensamiento crítico se desencadenan por los problemas planteados en el espacio individual. En el espacio colaborativo, dichas habilidades emergen en mayor medida porque son desencadenadas por el problema dado y las respuestas de otros miembros. En la sección de resultados se describen cómo funcionan las habilidades de pensamiento crítico de los alumnos cuando resuelven problemas de forma colaborativa.

Palabras clave ∞ Habilidades de pensamiento crítico; Resolución colaborativa de problemas; Matemáticas; Aprendizaje colaborativo

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1. INTRODUCTION

Critical thinking is a reflective and logical thinking activity in determining what to do and which things to use as a reference (Ennis, 2016). Critical thinking is a person's ability to formulate problems, see problems from various points of view, evaluate, and have a high level of sensitivity to a problem (Maričić et al., 2016). The increasingly rapid development of science and technology has made accessing various information or data easier. Critical thinking skills are skills needed to be able to identify the accuracy of the information or data that has been obtained (Arisoy & Aybek, 2021; Bezanilla et al., 2021). In receiving information, someone who has critical thinking skills will be able to carry out a series of intellectual processes to apply, analyze, synthesize and evaluate information that has been obtained from reflection, observation, communication, or experience as a reference to ensure the accuracy of the knowledge that has been obtained and determine following action.

Mathematics is a subject that is claimed to improve students' critical skills (Arisoy & Aybek, 2021; Xu et al., 2023). Currently, the focus of mathematics learning demands more conceptual understanding and the ability to justify rather than simply applying a set of mathematical rules (Mendezabal & Tindowen, 2018). Therefore, mathematics has the potential to develop students' critical thinking skills. Through mathematical problem-solving activities, critical thinking skills can be improved (Palinussa, 2013).

Critical thinking skills are skills that are acquired through continuous training (Ketabi et al., 2013). Critical thinking skills can be trained through activities in a social context. Ebiendele (2012) states that interaction will encourage someone to engage in deep thinking, which will trigger the use of critical thinking skills. Based on Vygotsky's sociocultural theory and ZPD model, Wass et al. (2011) stated that conversations with peers will expand students' ZPD to think critically. The results of Sun's research (2020) state that, during collaboration, there is exploratory talk, namely conversations between people considering each other's points of view constructively. If there is a difference in point of view, it must be accompanied by apparent alternatives or reasons. Exploratory talk will trigger someone to use critical thinking skills. In other words, an environment that supports interaction will trigger the emergence of students' critical thinking skills.

CPS is an activity that requires interaction between team members to solve problems. CPS contains two components, namely collaboration and problem-solving. Thus, the main characteristic of CPS is the interaction between individuals who collaborate to solve problems (Salminen-Saari et al., 2021). CPS emphasizes the existence of interdependent activities of group members to turn input into output in an effort to regulate task completion to achieve common goals through cognitive, verbal and behavioral activities (Hagemann & Kluge, 2017). In the context of mathematics, several research results state that CPS is effective in developing students' critical thinking skills (Jacob & Sam, 2010; Laal & Ghodsi, 2012; Sofroniou & Poutos, 2016; Sutama et al., 2022; Xu et al., 2023). However, research that explores students' critical thinking abilities when solving problems collaboratively is rare (Xu et al., 2023). Several studies examined students' critical thinking skills after collaborative problem-solving activities (Fung & Howe, 2012; Tang et al., 2020; Ugale & Shingan, 2018; Yin et al., 2011). In other words, students' critical thinking skills are still seen individually because they are seen after CPS activities. Of course, this cannot reveal how students' critical thinking skills are when they solve problems collaboratively. Students' critical thinking skills when solving problems collaboratively must be studied to ensure that CPS can trigger students' critical thinking skills based on existing research. Therefore, this research aims to describe students' critical thinking skills in collaborative mathematical problem-solving.

2. Literature review

2.1. Critical Thinking Skills

Thinking is a mental activity that can help determine a decision in solving a problem (Aldossari & Moh'd Ali Abu Jadou, 2021). Critical thinking is included in the category of higher-order thinking. Dewey (1910) stated that critical thinking is a process of considering beliefs or knowledge based on supporting evidence. Ennis (2016) states that critical thinking is a logical and reflective thinking activity with full consideration that focuses on deciding what to believe or do. Facione & Gittens (2016) states that critical thinking is a thinking activity that aims to prove, interpret, or solve a problem. Lou (2018) states that critical thinking is evaluating opinions, information, or resources based on logical and coherent information. From these opinions, the two main things in critical thinking are the accuracy of information and a clear understanding of the situation at hand.

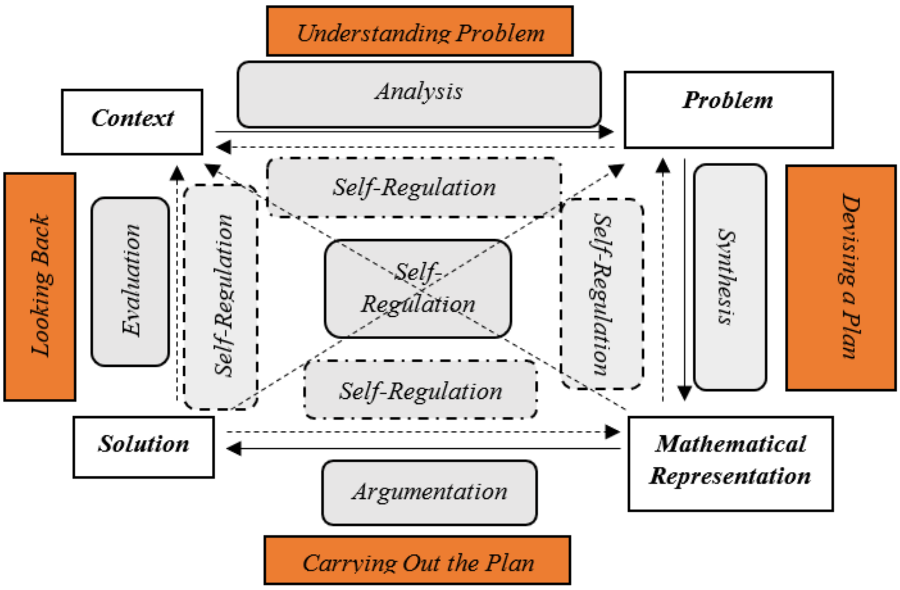
Critical thinking skills are a person's ability to be aware and organize their thinking process (Niu et al., 2013). Several indicators can demonstrate critical thinking behavior. Reynders et al. (2020) states four indicators of critical thinking skills: analyzing, synthesizing, forming arguments, and evaluating. Analyzing is one's ability to explore the meaning of information, synthesizing is identifying the relationship of several different information or concepts, forming arguments is the ability to produce well-structured arguments, and evaluating is the ability to determine the quality and accuracy of information. According to Facione & Gittens (2016), evaluation is divided into two, namely, examining statements from other people (evaluation) and examining one's thinking (self-regulation). Based on Reynders et al. (2020) and Facione & Gittens (2016), five indicators can be used to view critical thinking skills, namely: a) *analysis*, means describing and exploring the meaning of data based on existing knowledge; b) *synthesis*, namely making connections between some information or concepts; c) *argumentation*, namely providing a systematic explanation in responding to or providing some information; d) *evaluation*, namely assessing the credibility of the arguments that have been produced; and e) *self-regulation*, namely checking the quality of one's thinking.

2.2. Critical Thinking Skills in CPS

Problem-solving can be defined as the activity of carrying out non-routine tasks where the solver does not know the scheme or algorithm for solving it (Schoenfeld, 2016). CPS is a joint activity that requires a cooperative exchange of information to successfully solve problems as a common goal (Graesser et al., 2018). The characteristic of CPS is that the problem is problematic for all team members (Graesser et al., 2017; Westermann & Rummel, 2012). Individual problem-solving is the basis of CPS (Salminen-Saari et al., 2021). Hence, the stages of CPS refer to the stages of individual problem-solving built by Polya (1945), namely understanding the problem (UP), devising a plan (DP), carrying out the plan (CoP), and looking back (LB).

Lester (2013) states that mathematical problem-solving activities begin when individuals simplify complex problems by exploring information related to the problem, then selecting mathematical concepts to produce a mathematical representation that corresponds to the problem, manipulating mathematical representations, and examining the solutions found. The activities mentioned correspond to the problem-solving stages proposed by Polya. Furthermore, referring to the indicators of critical thinking skills mentioned by Reynders et al. (2020) and Facione & Gittens (2016), each mathematical problem-solving activity requires aspects of critical thinking skills, namely analysis is required when simplifying complex problems, synthesis is required to produce mathematical representations, argumentation is required when manipulating mathematical representations, evaluation is required when examining solutions found, and self-regulation is required in any mathematical problem-solving activity. Figure 1 shows the role of critical thinking skills in collaborative mathematical problem-solving.

Figure 1. The Role of Critical Thinking Skills in Mathematical Collaborative Problem Solving



The results of theoretical studies regarding the role of critical thinking in collaborative mathematical problem-solving and the characteristics of CPS that have been explained are the basis for forming five indicators of critical thinking skills in CPS. Table 1 shows the differences in indicators of critical thinking skills in individual and CPS.

Table 1. Indicators of Critical Thinking Skills

	Critical thinking skills in individual problem solving	Critical thinking skills in CPS
Analysis (An)	Describe and explore the meaning of data to understand the problem.	Describe (An1) and explore (An2) the meaning of data to understand problematic issues by communicating opinions or information.
Synthesis (Sy)	Connect some information or concepts to determine the idea of solving the problem.	Connect several pieces of information or concepts by accommodating various perspectives of team members to determine ideas that can support solving problematic problems.
Argumentation (Ar)	Provide a systematic explanation to apply the idea of problem-solving.	Provide systematic explanations to implement ideas for solving problematic problems that team members have agreed upon.
Evaluation (Ev)	Assess the argument's credibility to check the suitability of the solutions found with the problem.	Assess the credibility of the arguments generated to check the suitability of problematic problems with the solutions that team members have found.
Self-regulation (Sr)	Check the quality of one's thinking.	Check the quality of one's thinking during the problem-solving process.

Furthermore, the indicators of critical thinking used in this study are indicators of critical thinking skills in solving collaborative problems.

3. Methodology

This research is a qualitative research type case study. The selected case is a group that can solve the given problem collaboratively. Groups that can solve problems are characterized by all stages of problem-solving, namely understanding the problem, devising a plan, carrying out the plan, and looking back because indicators of critical thinking skills are found at each problem-solving stage.

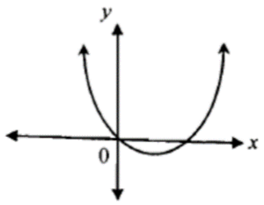
3.1. Participants and Data Collection Procedures

Participants of this research were 32 1st-grade high school students, consisting of 18 female and 14 male students. Participants were divided into sixteen groups. Each group consists of two students. Zuniga et al. (2021) stated that working in pairs can increase the activity of negotiating, interacting, reaching agreements, and evaluating between group members. Thus, CPS will likely run well. Grouping is based on mathematical ability obtained from test scores on quadratic function material reported by the subject teacher. The quadratic function test scores were chosen so that data about students' mathematical abilities was accurate and appropriate to the tasks given. Next, the pair composition covers all possible group combinations: two students with high mathematical ability (TT), two with medium mathematical ability SS, two with low mathematical ability (RR), one with high mathematical ability and one with medium mathematical ability (TS), one with high mathematical ability and one with low mathematical ability (TR), and one with medium mathematical ability and one with low mathematical ability (SR). Based on research results (Setiana et al., 2021; Ulfiana et al., 2019), students with high, medium, and low mathematical abilities could think critically. Apart from that, grouping is also done by paying attention to personal closeness between students to guarantee communication between students when collaboration runs smoothly. Communication within a group is fundamental to the success of CPS (Häkkinen et al., 2017; Hesse et al., 2015).

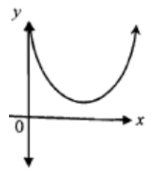
After the groups were determined, the researcher gave them a quadratic function task that experts had validated. Task validation criteria consist of three things, namely content, construction, and language. Content assesses assignments regarding their suitability for educational level and credibility to be completed. The construction of assessing whether or not a task can trigger students to think critically based on predetermined indicators. Language assesses tasks in terms of readability. Figure 2 is the problem of the quadratic function given to the group.

Figure 2. The Problem of The Quadratic Function

The following figure is the curve of function $f(x) = x^2 + kx$.



Based on this information, Saila was asked to draw the function curve of $f(x) = x^2 - kx + 5$. Next, Saila draws the following curve and states that the curve is a funcion of curve $f(x) = x^2 - kx + 5$.



Is Saila's statement true? Explain your reasons.

Groups are given a maximum work duration of 90 minutes. To support collaboration conditions, each group is only given one piece of paper to use together for the problem-solving process. During the problem-solving process, students cannot consult researchers or teachers. Apart from the fact that the questions have been validated in content and language, this is done so that students' critical thinking activities emerge naturally without influence from other parties. The activities of each group when solving collaborative problems were recorded using audio-visual material.

Three of the sixteen groups that carried out all stages of problem-solving (UP, DP, CoP, LB) were selected. This choice was made because the results of theoretical studies stated that indicators of critical thinking skills, namely *analysis, synthesis, argumentation, evaluation and self-regulation*, appear at each problem-solving stage (see Figure 1). Next, one group with the highest intensity of CPS was selected from the three groups. This selection was based on the research objective of describing critical thinking skills in CPS. In addition, the selected subjects have a recursive problem-solving flow, and each solving stage is carried out more than once, so this will enrich data about critical thinking skills in CPS. The subjects selected in this study were the T1T2 group, which consisted of students named T1 and T2 who both happened to have high mathematical abilities.

Next, interviews were conducted with the T1T2 group. The interview method used by researchers is a task-based semi-structured interview, namely questions related to the tasks that have been carried out. This method was chosen because interviews were conducted to explore students' critical thinking abilities, which are invisible in CPS transcripts. For example, researchers should explore whether new ideas emerge from identifying given information or from guesses.

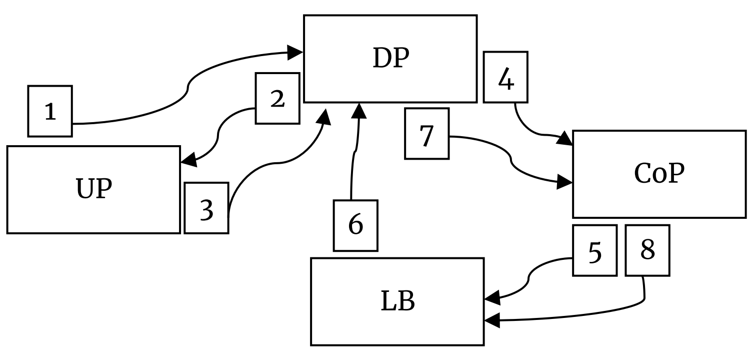
3.2. Data Analysis

The stages of data analysis refer to Merriam & Tisdell (2016), which consist of four stages, namely: 1) category construction is the coding of the transcription result; 2) sorting categories and data, namely removing categories that are not related to indicators of critical thinking skills; 3) naming the categories, namely grouping the selected categories based on indicators of critical thinking skills; and 4) theorizing, namely linking research and theories related to students' critical thinking skills in solving collaborative mathematical problems.

4. Results

In solving the problem, T1T2 carried out all stages of CPS recursively, and each stage was carried out more than once, namely UP 2 times, DP 3 times, CoP 2 times, and LB 2 times. Indicators of critical thinking skills for T1 and T2 were observed at each stage of CPS. Figure 3 represents the stages of CPS T1T2.

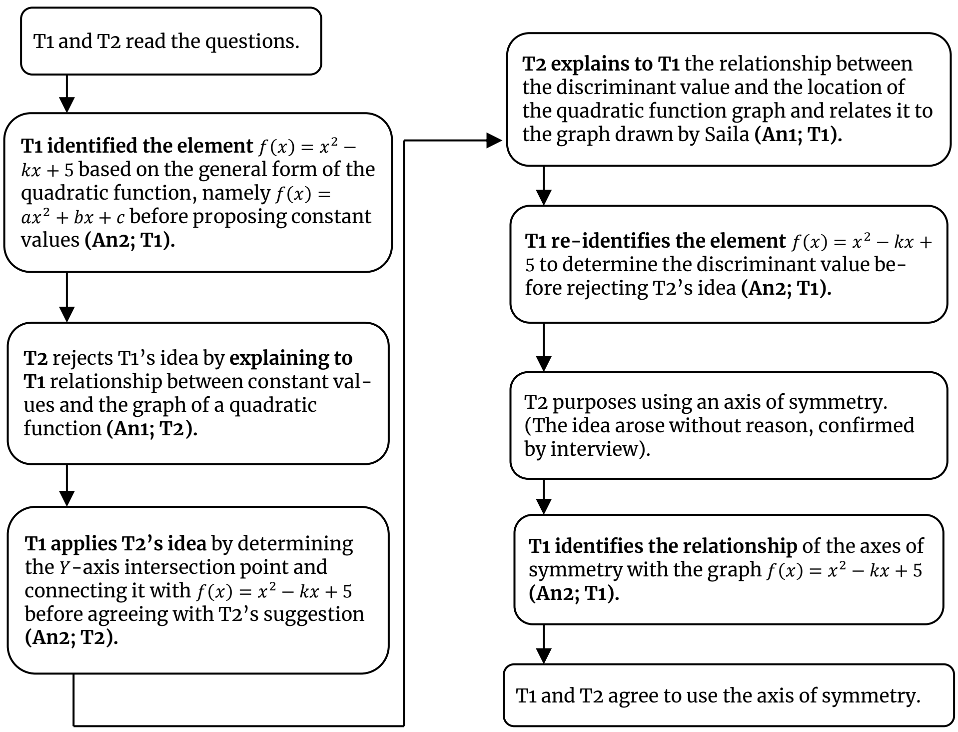
Figure 3. Problem-solving stages carried out by T1T2



4.1. Understanding the Problem (UP) 1

UP 1 started when T1T2 read problem. After T1T2 read the problem, T1 proposed to look at the function $f(x) = x^2 - kx + 5$'s constant value. Before proposing, T1 identified the element of $f(x) = x^2 - kx + 5$ based on the general form of the quadratic function, namely $f(x) = ax^2 + bx + c$. Next, T2 rejected T1's idea. T2 rejected this idea by explaining to T1 the relationship between constant values and quadratic function graphs, namely that constant values are used to see the intersection point with the Y axis. T1 then agreed to T2's idea by first applying T2's idea to the problem, namely by determining the intersection point $f(x) = x^2 - kx + 5$ on the Y axis and checking its suitability with the constant value at $f(x) = x^2 - kx + 5$. After T1 received a suggestion from T2, then T2 proposed to find the discriminant value (D) of the function $f(x) = x^2 - kx + 5$. T2 explains the relationship between D and the location of a curve to T1. If $D > 0$, then the curve intersects on the X axis; if $D < 0$, then the curve will not intersect on the X axis; and if $D = 0$, then the curve touches the X axis. The idea of using this D came from T2's initiative. Responding to T2's idea, T1 re-identifies the element $f(x) = x^2 - kx + 5$ to determine the D. T1 finds that the D of $f(x) = x^2 - kx + 5$ cannot be determined because it contains an unknown element, namely k. Responding to the findings from T1, T2 then looked for another alternative, namely by examining the location of the symmetry axis of the function $f(x) = x^2 - kx + 5$. Based on the results of the alternative interviews proposed by T2, this is the result of trial and error. T1 accepted T2's idea after T1 identified the relationship between the axis of symmetry and the location of the quadratic function graph. According to T1, the axis of symmetry can be used to determine the location of the graph $f(x) = x^2 - kx + 5$; that is, if the axis of symmetry $f(x) = x^2 - kx + 5$ matches the Saila drawing, there is a possibility that the Saila drawing is correct. Figure 4 is a T1 and T2 activity chart in the UP 1 stage.

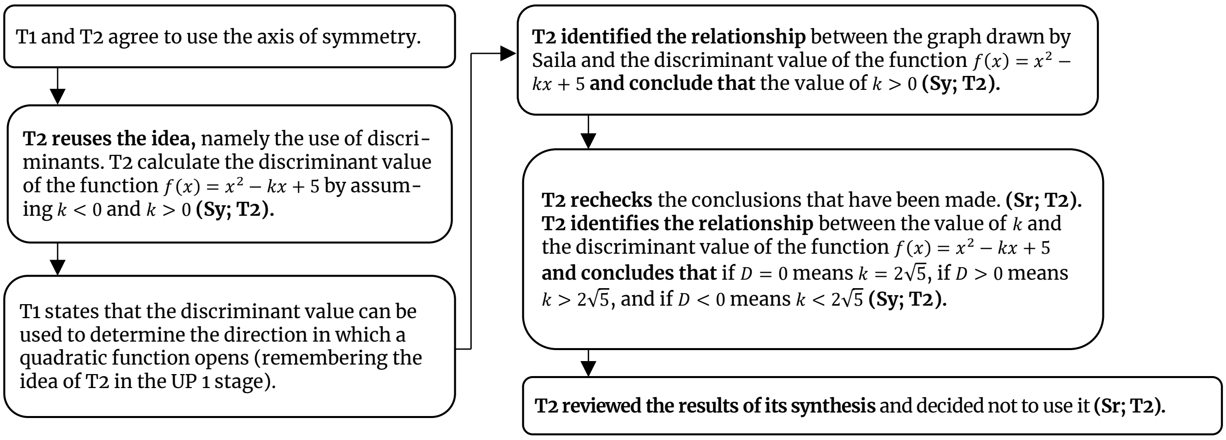
Figure 4. Activities in UP Stage 1



4.2. Devising a Plan (DP) 1

Stage DP 1 began when T2 took the initiative to refocus on the initial idea, namely using D. According to T2, the D makes it easier to check the correctness of the graph drawn by Saila than the symmetry axis value. T2 tries to calculate D from the function by assuming all possible values of k namely $k < 0$ and $k > 0$. Then, T2 concludes that the D when $k < 0$ or $k > 0$ will be the same because of k^2. Explanation of T2 related to the D triggers T1 to remember the idea of T2 that relates the D to the graph of the quadratic function (in stage UP 1). T1 states that the D can be used to determine the direction in which a quadratic function opens. Statement T1 triggers T2 to evaluate T1's opinion by stating that the D is used to determine the location of the graph of the quadratic function based on the X-axis. This evaluation by T2 triggers T2 to identify the relationship between the graph drawn by Saila and the D of the function $f(x) = x^2 - kx + 5$. According to T2, this results in $k > 0$. Next, T2 re-examines the conclusion $k > 0$ by identifying the relationship between the value of k and the D of the function $f(x) = x^2 - kx + 5$. The identification result of T2 is that if $D = 0$, it means $k = 2\sqrt{5}$; if $D > 0$, it means $k > 2\sqrt{5}$; and if $D < 0$, it means $k < 2\sqrt{5}$. After examining the synthesis results, T2 decided not to use them because, according to T2, these results were unrelated to problem-solving. Figure 5 is an activity chart of T1 and T2 activities at the DP 1 stage.

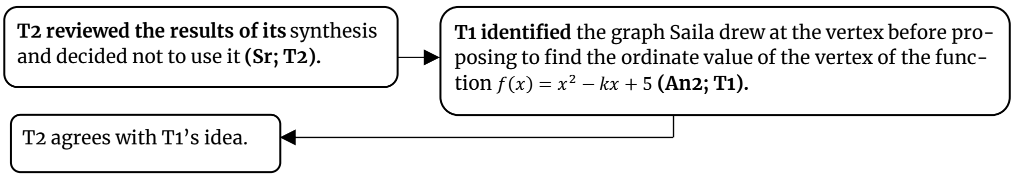
Figure 5. Activities at the DP Stage 1



4.3. Understanding the Problem (UP) 2

T1T2 started re-analyzing the problem after not getting an idea for a solution at DP stage 1. At this stage, T1 identified the graph Saila drew at the peak point. According to T1, the peak point can be used to determine whether the graph is below or above the X-axis. Therefore, T1 proposes finding the ordinate value of the peak point of the graph of the function $f(x) = x^2 - kx + 5$. T2 approved T1's idea. Figure 6 is an activity chart of T1 and T2 activities at the UP 2 stage.

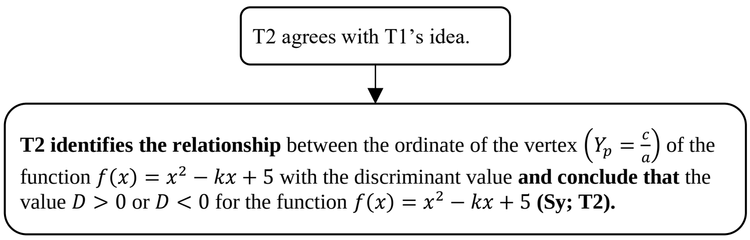
Figure 6. Activities in UP Stage 2



4.4. Devising a Plan (DP) 2

T2 agrees with T1's idea to use the ordinate value of the vertex of the function $f(x) = x^2 - kx + 5$ graph. Therefore, T2 starts to plan a problem-solving using the ordinate value. At this stage, T2 identifies the relationship between the ordinate of the peak point of the function $f(x) = x^2 - kx + 5$ and the D. T2 determines the ordinate value $f(x) = x^2 - kx + 5$ using the formula $y = \frac{c}{a}$, so that $y = 5 > 0$ is obtained. Based on this, T2 determines the location of the peak point $f(x) = x^2 - kx + 5$ above the X axis. T2 then draws a parabolic curve, which is a characteristic of the graph of a quadratic function. A total of two parabolic curves are depicted by T2, namely one parabolic curve facing upwards and one parabolic curve facing downwards. T2 next investigates the D of the function $f(x) = x^2 - kx + 5$ based on the two parabolic curves that have been generated. Based on this, T2 concludes that there are two possible values for the discriminant of the function $f(x) = x^2 - kx + 5$ namely negative or positive. Figure 7 is an activity chart for T1 and T2 activities at DP stage 2.

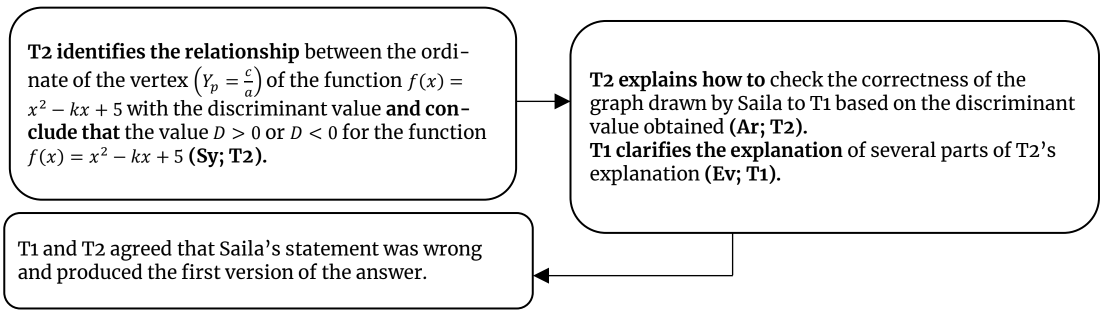
Figure 7. Activities in DP Stage 2



4.5. Carrying Out the Plan (CoP) 1 and Looking Back (LB) 1

After T1T2 found the possible D of the function $f(x) = x^2 - kx + 5$, T1T2 used these possible D to check the correctness of the graph Saila drew. The CoP 1 stage begins when T1 proposes to substitute the value $k = 4$ in the function $f(x) = x^2 - kx + 5$. T2 then carried out the idea from T1. T2 then explained how to check the correctness of the graphs drawn by Saila to T1 based on the D obtained in the DP 2 stage. The systematic explanation of T2 to T1 shows that T2 has good argumentation skills. Explanation of T2 begins with selecting $k = 4$ to be substitued for the function $f(x) = x^2 - kx + 5$. Then determine the peak point (X_p, Y_p) from $f(x) = x^2 - 4x + 5$ using the formula $X_p = \frac{-b}{2a}$ and $Y_p = \frac{-D}{4a}$ to produce the peak point $(2,5)$. The results of this peak point were then clarified by T1 because they did not match the results of his calculations, so in this activity, T1T2 entered the LB 1 stage. This clarification made by T1 showed that T1 carried out a credibility assessment of the claims stated by T2. Then, T2 accepts the evaluation results of T1 by recalculating the peak point $f(x) = x^2 - 4x + 5$ and producing the peak point $(2,1)$. T2 continues his explanation by focusing on the discriminant value $f(x) = x^2 - 4x + 5$, namely $D = -4$. T2 concludes the first explanation: if $k = 4$, then $D = -4 < 0$. In the same way, T2 explains to T1 that if $k = 6$, then $D = 16 > 0$. Based on these results, T2 concludes that for the function $f(x) = x^2 - kx + 5$ graph, because the value of k is not known with certainty if any value of k is taken, it will produce values $D < 0$ and $D > 0$. The graph drawn by Saila does not intersect the X axis, meaning that the graph drawn by Saila only satisfies one possibility, namely $D < 0$. Based on this, T2 concluded that Saila's answer was not quite right because it did not fulfil $D > 0$. T2's explanation is approved by T1, resulting in the first answer. Figure 8 is a chart showing the activity of T1 and T2 activity in the CoP 1 and LB 1 stages.

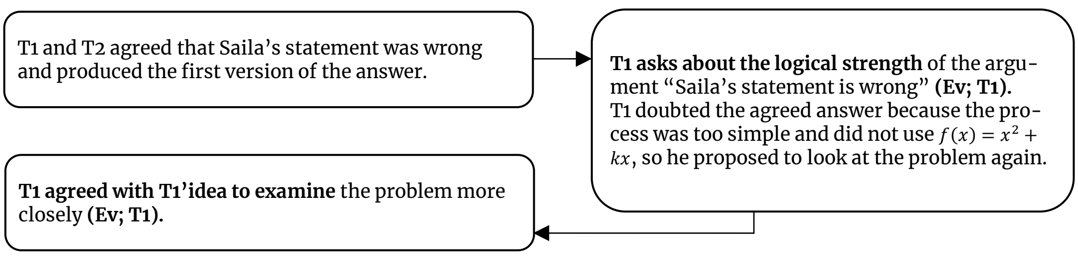
Figure 8. Activities at CoP 1 and LB 1 Stages



4.6. Looking Back (LB) 2

After producing the first version of the answer, T1 examined by asking T2 the logical strength of the argument, which stated that Saila's statement was wrong. T1 asked if Saila's statement was wrong; what should the graph drawn by Saila be like? Responding to T1's question, T2 changed the agreed answer: Saila's statement was correct but needed to be completed. T2's answer did not satisfy T1, so T1 stated that T1 doubted the truth of the first version of the answer for two reasons, namely: 1) feel that the settlement process is too easy; (2) there is information in the problem that is not used, namely the function $f(x) = x^2 + kx$. Therefore, T1 suggested that T2 reread the questions carefully. Figure 9 is an activity chart for T1 and T2 activities at LB stage 2.

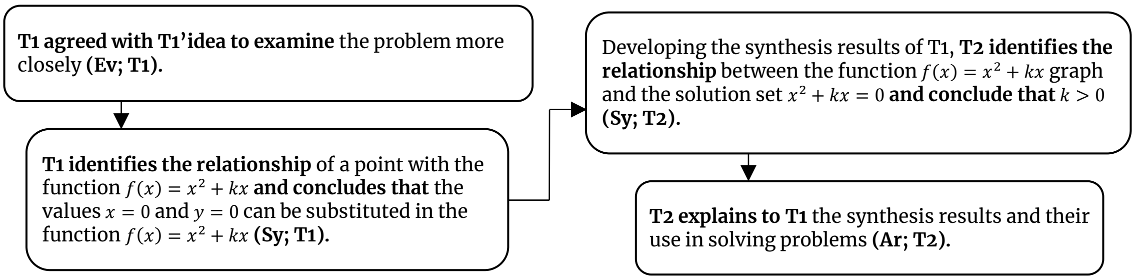
Figure 9. Activities at LB Stage 2



4.7. Devising a Plan (DP) 3 and Carrying Out the Plan (CoP) 2

The third DP stage begins when T1 identifies a point's relationship on the function $f(x) = x^2 + kx$ graph and the function $f(x) = x^2 + kx$. T1 finds a point that passes through the graph $f(x) = x^2 + kx$, namely $(0,0)$, so T1 proposes substituting the values $x = 0$ and $y = 0$ in the function $f(x) = x^2 + kx$. T2 then developed T1's identification results to find the k value. T2 identifies the value of k by connecting the solution set $x^2 + kx = 0$ with the known graph $f(x) = x^2 + kx$. The T2 identification results produce $x = 0$ or $x = -k$. In other words, the graph has two intersection points, namely $(0,0)$ and $(k, 0)$. Next, T2 explains to T1 the location of the point $(k, 0)$, based on the graph $f(x) = x^2 + kx$ provided. In the end, T1T2 agreed that the value of $k > 0$. Figure 10 is a chart showing the activity of T1 and T2 activity in the DP 3 and CoP 2 stages.

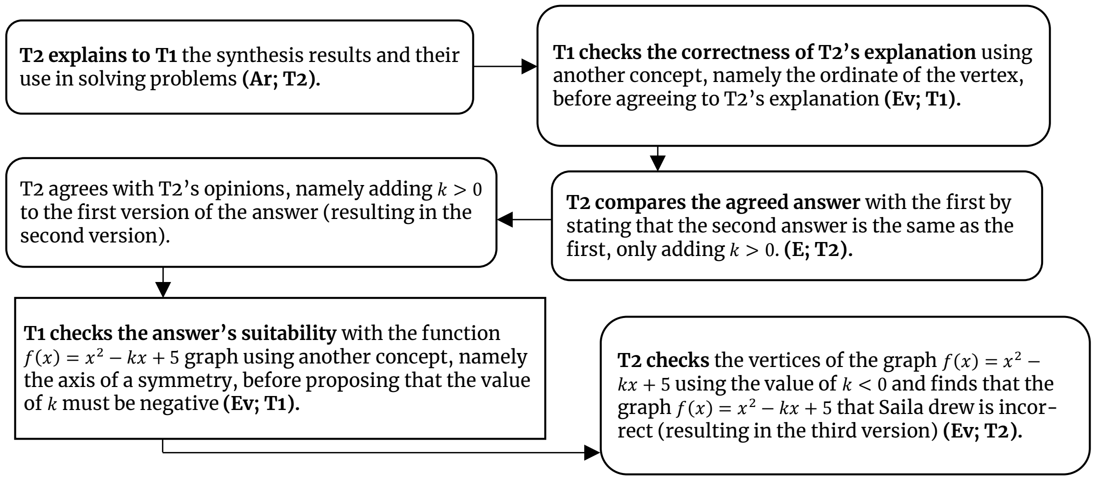
Figure 10. Activities at DP Stage 3 and CoP 2



4.8. Looking Back (LB) 3

In the third LB stage, T1 checks the correctness of T2's explanation using another alternative: the ordinate of the peak point of the function $f(x) = x^2 + kx$. The results of the T1 calculation show that the ordinate is negative, which is by the graph $f(x) = x^2 + kx$. Thus, T1 agrees with T2's explanation. Next, T2 compares the agreed answer with the first version of the answer produced. T2 states that the first version of the answer is the same as the first answer. The difference lies in the k value. In the first version of the answer, the value of k cannot be confirmed as positive or negative. In the second version of the answer, the k value can be confirmed that the k value is positive. T1 agrees with T2's opinion, resulting in the second version of the answer. On T1's initiative, the second version of the answer was then checked by T1. T1 rechecks the correspondence between the value $k > 0$ that has been obtained and the graph $f(x) = x^2 + kx$ by observing the location of the peak point of the graph $f(x) = x^2 + kx$. T1 finds that the peak point of the graph $f(x) = x^2 + kx$ is to the right of the X-axis. This means the abscissa value of the graph peak point $f(x) = x^2 + kx$, namely $x = -\frac{b}{2a}$, must be positive. Therefore, according to T1, the value of k must be negative. T2 agreed with the results of T1's evaluation. Next, T2 checked the peak point of the graph of the function $f(x) = x^2 - kx + 5$ using the value $k < 0$ and found that the graph of the function $f(x) = x^2 - kx + 5$ that Saila drew was indeed incorrect. Thus, at the end of T1T2's answer, they gave three possible forms of the graph $f(x) = x^2 - kx + 5$, which T1T2 had agreed on, namely cutting the X axis, touching the X axis, and not cutting the X axis (third version of the answer). Figure 11 is a chart showing the activity of T1 and T2 activity in the LB stage 3.

Figure 11. Activities at LB Stage 3



5. DISCUSSION

Based on the research results, critical thinking skills of T1 and T2 appear at each problem-solving stage (see the sentences in bold on the chart). Critical thinking skills of T1 and T2 appear in two conditions: when working independently (*individual space*) and when interacting with other team members (*collaborative space*). This case is similar to that found in Schindler & Lilienthal (2022) study, which found that there is a condition of individual space in a collaborative environment. Individual space generates new ideas to be brought to the collaborative space. The results showed that the individual spaces (rectangular charts) carried out by T1 and T2 produced new ideas, which were then offered to group members. In the research results, the new ideas resulted from students' critical thinking activities. It is called a "new idea" because it emerged on its initiative. Thus, critical thinking skills emerge when individual spaces are triggered by the problem given. After a new idea is brought into the collaborative space triggers other group members to respond, reject, and evaluate to produce a mutual agreement. Critical thinking skills of T1 and T2 become more visible in a collaborative space because there are two triggers: the problems given and responses from other group members. Construction of shared knowledge, sharing of shared understanding, and holding on to maintain shared understanding triggers students to debate, study, and reflect on their own and other people's thoughts so that they can develop critical thinking skills (Häkkinen et al., 2017; Yang et al., 2008). This indirectly shows that CPS can trigger students' critical thinking skills. Table 2 is a description of T1 and T2 critical thinking skills in individual space and collaborative space.

Table 2. Critical Thinking Skills of T1 and T2 on CPS

T1		T2
Individual space	Collaborative space	Individual space
Analysis		
1. Identify known data elements based on definitions before conveying the idea to T2. 2. Identify data that is known to use other concepts (which have never been used) before conveying the idea at T2.	T1: 1. Apply the concepts presented by T2 to the problem before accepting T2's ideas. 2. Identify the completeness of the known data before accepting the idea from T2. 3. Identify the usefulness of the idea proposed by T2 in solving the problem before accepting T2's idea. T2: 1. Explain the use of the ideas proposed by T1 in solving problems before rejecting T1's ideas.	1. Relate the known concepts with the information in the question before explaining in T1.
Synthesis		
1. Connect some known information to the problem based on known concepts.	T2: 1. Connect T1's ideas with concepts that T2 already knows. 2.Connect T1's ideas with the information contained in the question. Connect the information in the question with the concepts T2 already knows.	1. Connect the concept with the information known in the problem.
Argumentation		
	T2: 1. Give a coherent explanation based on the facts contained in the problem. 2. Provide a coherent explanation based on the facts found during the problem-solving process.	
Evaluation		
1. Asking about the logical strength of the arguments that have been produced. 2. Check the answer using another point of view.	T1: 1. Asking the origin of the arguments generated by T2. 2. Using other concepts to check the truth of the statements conveyed by T2. T2: 1. Checking the suitability between the facts and arguments presented by T1. 2. Checking the adequacy of information processing according to the evaluation results submitted by T1.	1. It compares the answers generated with previous answers to look for differences in the completion process and the conclusions obtained.
Self-regulation		
LB: 1. Pay attention to information on questions not used during problem-solving. 2. Pay attention to the complexity of the problem-solving process that has been carried out.		*DP:* 1. Comparing the potential effectiveness of new ideas and old ideas in solving problems. 2. Observe the relationship between the results of the synthesis that has been produced and the solution to the problem.

Furthermore, there are findings that CPS can give rise to cognitive conflict situations. Cognitive conflict is when someone is aware of a mismatch between their understanding and the environment (external information) (Meissner, 1986; Zorica & Cindrić, 2012). External information in CPS is in the form of information or concepts that other people understand. The research results show that cognitive conflict arises when one individual experiences a difference in understanding the meaning of a problem with another individual, which results in doubts about the answer produced. This cognitive conflict triggers other individuals to carry out evaluations (see Figure LB 2). In a collaborative environment, increased critical thinking of students can arise through cognitive conflict with other students (Yang et al., 2008).

Another finding is the link between conceptual knowledge, critical thinking skills and problem-solving processes. The results of critical thinking in students with good conceptual knowledge will impact the problem-solving process. Students who have good conceptual knowledge will produce ideas that are close to solutions, ensure that the ideas being executed are ideas that are close to solutions and be able to develop other people's ideas, combine them with the knowledge they have to produce a synthesis that is useful in planning the completion process problem. Meanwhile, for students with less conceptual knowledge, the results of their critical thinking will not impact the problem-solving process as T1 did in LB 3. T1 clarified T2's explanation by stating that the value of k does not have to be positive. The idea conveyed by T1 keeps away from solving the problem because this idea will result in the first version of the answer (returning to the initial answer, which needs to be corrected). Yu et al. (2015) stated that incomplete conceptual knowledge will affect the results of problem identification and interpretation of information carried out, impacting the problem-solving process.

In the end, there are several things that researchers need to pay attention to in revealing students' critical thinking skills in the CPS context. Researchers must carefully group activities based on problem-solving stages (UP, DP, CoP, LB). Furthermore, researchers must carefully determine which activities are included in critical thinking and which are not. In this case, the researcher must directly observe and record the ideas or responses expressed by participants during problem-solving. Interviews were conducted after the observation activities to ascertain the source of the ideas or responses submitted by the participants. Audiovisual material is used to reaffirm the researcher's understanding of the activities carried out by the participants.

This research is limited to only one group, which is representative of the cases. In future research, the representative group can be expanded, for example, by considering numbers. Paying attention to the research results, more group members may influence on improving students' critical thinking skills because there will be more other points of view. Future research can also pay attention to group composition based on mathematical ability because this research is limited to groups with high mathematical ability composition. Several studies (Setiana et al., 2021; Ulfiana et al., 2019) state that students with moderate and low mathematics abilities have critical thinking skills at several levels. This will be useful for educators who support collaborative learning in determining the composition of study groups to maximize students' critical thinking skills.

Despite the limitations of the research, it is hoped that the findings of this research will provide valuable insights for researchers and teachers who choose collaboration as the learning method used. The following suggestions are offered for learning critical thinking in collaborative problem-solving. *First*, teachers must pay attention to two things, namely collaboration and problem-solving, in designing tasks based on collaborative situations. The research results show CPS can trigger students to think critically. Designing problems that give rise to cognitive conflict will encourage students to discuss, negotiate and debate so that students' critical thinking skills will develop. Yang et al. (2008) stated that increasing students' critical thinking can be realized through cognitive conflict with other students in problem situations. *Second*, apart from the assignments given, ideas, responses, arguments, or evaluations made by other people can trigger students to think critically. These findings can be used as a basis for the idea that in selecting group members, apart from paying attention to mathematical abilities, you must also pay attention to students' interpersonal closeness. Students' interpersonal closeness is important to pay attention to because it influences their interactions during the problem-solving process. Good interaction provides an excellent opportunity for students to criticize each other's activities carried out by other team members.

6. Conclusion

Students' critical thinking skills when solving problems collaboratively appear in two conditions: when working independently (*individual space*) and when interacting with other team members (*collaborative space*). Students' critical thinking skills in *individual space* conditions are identifying known information, connecting several known data, checking and comparing agreed answers, and comparing the effectiveness of ideas that have been proposed based on their knowledge. Students' critical thinking skills in *individual space* arise after students read the questions, pay close attention to the problem-solving process that has been carried out, or recheck the agreed answers. Students' critical thinking skills in *individual spaces* produce new ideas in the form of concepts or ways of solving problems that have never been presented in group forums. The new ideas are then brought to the *collaborative space* and trigger other group members to criticize each other's ideas, responses, arguments, or evaluations submitted by other group members. Thus, in a *collaborative space*, the data that students criticize is not only data originating from the problem given, but also data produced by other group members. Therefore, students' critical thinking skills in *collaborative space* conditions are identifying ideas, concepts, and uses of ideas conveyed by other group members; connecting other group members' ideas with the information contained in the problem; systematically explaining the problem-solving process to group members based on facts found; examining arguments and evaluation results submitted by other group members.

Students' critical thinking skills are triggered by the problems given in the *individual space*. In *collaborative space*, students' critical thinking skills emerge more because they are triggered by two things, namely the problem given and responses from other group members. In future research, representative groups can be expanded, for example, by considering the number or composition of group members based on mathematical ability.

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¿Cómo se manifiestan las habilidades de pensamiento crítico de los estudiantes al resolver problemas de forma colaborativa?

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La teoría sociocultural de Vygotsky sostiene que la interacción con los pares expande la Zona de Desarrollo Próximo (ZDP) de los estudiantes para pensar críticamente. La Resolución Colaborativa de Problemas (CPS) es una actividad que requiere la interacción entre los miembros del equipo, y los estudios han mostrado que tiene un impacto en el incremento de las habilidades de pensamiento crítico de los estudiantes. Sin embargo, son escasas las investigaciones que exploran las habilidades de pensamiento crítico de los estudiantes al resolver problemas de forma colaborativa. Este estudio describe las habilidades de pensamiento crítico de los estudiantes al abordar problemas matemáticos colaborativamente. Los participantes fueron un grupo de dos estudiantes de segundo de bachillerato, quienes intentaron resolver problemas relacionados con funciones cuadráticas. Se evidencian las habilidades de pensamiento crítico individuales cuando estos estudiantes resuelven problemas de manera colaborativa. Dichas habilidades aparecen en dos contextos: cuando trabajan de manera independiente (espacio individual) y cuando interactúan con otros miembros del equipo (espacio colaborativo). Las habilidades de pensamiento crítico se desencadenan por los problemas planteados en el espacio individual. En el espacio colaborativo, dichas habilidades emergen en mayor medida debido a que se desencadenan por dos elementos: el problema planteado y las respuestas de los demás miembros del grupo. En el espacio individual las habilidades de pensamiento crítico de los estudiantes consisten en identificar información conocida, conectar diversos datos conocidos, comprobar y comparar respuestas consensuadas, y comparar la efectividad de las ideas propuestas en función de sus conocimientos. Dichas habilidades surgen tras la lectura de las preguntas, la atención minuciosa al proceso de resolución de problemas llevado a cabo o la comprobación de las respuestas acordadas. Además, en el espacio individual, las habilidades de pensamiento crítico generan nuevas ideas en forma de conceptos o métodos de resolución de problemas que nunca se han presentado en foros grupales. Estas nuevas ideas se trasladan al espacio colaborativo y desencadenan que otros miembros del grupo critiquen las ideas, respuestas, argumentos o evaluaciones presentadas por otros miembros del grupo. Así, en el espacio colaborativo, los datos que critican los estudiantes no provienen únicamente del problema planteado, sino también de la información generada por otros miembros del grupo. En consecuencia, las habilidades de pensamiento crítico de los estudiantes en espacios colaborativos comprenden la identificación de ideas, conceptos y usos de las ideas transmitidas por otros miembros del grupo; la conexión de las ideas de otros miembros con la información contenida en el problema; la explicación sistemática del proceso de resolución de problemas a los miembros del grupo basándose en hechos; y el examen de los argumentos y resultados de la evaluación presentados por los demás miembros del grupo.