Universidad Rey Juan Carlos

TESIS DOCTORAL

Mathematical optimisation applied to complex industrial problems

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A mis abuelas, Pina y Sofía

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Resumen

Antecedentes

Las empresas necesitan responder a los nuevos retos que surgen en sus procesos productivos, y para ello requieren hacer un uso eficiente de los recursos de los que disponen. Los cada vez más complejos procesos productivos hacen que no sea posible llevar a cabo una optimización de los recursos de forma manual y sea necesario emplear sistemas expertos de ayuda a la toma de decisiones.

Para solventar esta necesidad, las empresas recurren a consultoras con gran experiencia para construir los sistemas expertos que les permitirán tomar decisiones más precisas y con menos errores. En este contexto, la consultora española *IDOM Consulting, Engineering, Architecture* (IDOM), líder en el sector industrial con presencia en más de 125 países y más de 3.000 empleados, utiliza diferentes tecnologías para abordar los problemas de sus clientes. Entre estas tecnologías, se encuentran complejos sistemas de simulación y algunos modelos de optimización.

El desarrollo de esta tesis doctoral viene motivado por la necesidad de dotar a IDOM de las herramientas necesarias para el desarrollo de modelos y métodos de optimización matemática complejos que ayuden al proceso de toma de decisiones de sus clientes. La investigación se ha desarrollado en el marco del programa de doctorado industrial del Gobierno de la Comunidad de Madrid IND2018/TIC-9614 entre la consultora IDOM y la Universidad Rey Juan Carlos.

Objetivos

El objetivo principal de esta tesis doctoral es desarrollar herramientas que ayuden a las empresas a abordar problemas complejos que aparecen en la toma de decisiones de sus procesos productivos y así mejorar su eficiencia operativa. En concreto, se han desarrollado modelos matemáticos para la toma de decisiones eficientes en el proceso de planificación de dos conocidas empresas internacionales. Este objetivo general se plasma en los siguientes objetivos parciales:

- El desarrollo de un modelo de planificación estratégica para determinar la política de gestión adecuada en la red de puntos de venta de una compañía en el sector del *retail*. El objetivo consiste en decidir qué puntos de venta deben seguir formando parte de la red y cuáles deben ser cerrados; además, para los que permanecen abiertos, se decidirá qué tipo de gestión es el más adecuado: gestión propia; gestión externa, pero dentro de la cadena; sólo suministro de producto, pero sin pertenecer a la cadena o venta a un competidor.

- El desarrollo de un modelo de planificación operativa para una empresa que se dedica al corte de bobinas de acero. Entre sus procesos se encuentra el corte longitudinal de bobinas de acero que consiste en desenrollar y cortar a lo largo las bobinas de gran tamaño para obtener bobinas más estrechas denominadas flejes. El objetivo que se persigue es la obtención de un plan de corte que indique qué bobinas se deben seleccionar del *stock* y cómo tienen que ser los patrones de corte para satisfacer la demanda de flejes y minimizar el sobrante generado. Es necesario tener en cuenta las restricciones de los clientes y de la operativa a la hora de definir los patrones de corte, por lo que resulta muy complejo realizar la planificación de forma manual. Una característica determinante de este problema es que la demanda viene dada en peso de producto y este puede servirse en uno o más flejes no necesariamente iguales. Cada fleje tiene que respetar los límites inferiores y superiores de peso y de diámetro impuestos por el cliente. Esto obliga a incluir cortes transversales en las bobinas.

- La integración de la fase de asignación de máquinas de corte, la selección de bobinas y el diseño de patrones de corte para la misma empresa del punto anterior. A partir de las limitaciones observadas en el modelo de planificación operativo para el corte de bobinas de acero (la imposibilidad de cortar algunos patrones en las máquinas disponibles y el desequilibrio en la carga de trabajo de las máquinas), se estudia una ampliación del problema para integrar la siguiente fase del proceso productivo: la asignación de bobinas a las líneas de corte. La empresa dispone de varias máquinas o líneas de corte con diferentes características (velocidad de procesamiento, configuración de cuchillas, ancho mínimo de corte, etc.) que imponen diferentes condiciones a los patrones de corte que se pueden obtener. Además de los objetivos considerados en el problema anterior, la empresa desea equilibrar la carga de trabajo entre las distintas líneas de corte para mejorar la productividad del proceso.

Metodología

Para llevar a cabo los objetivos propuestos anteriormente se utiliza la optimización matemática. Se trata de una disciplina muy potente para optimizar los recursos y ayudar a las empresas a tomar decisiones mejores y más informadas.

Los modelos de optimización matemática son representaciones de sistemas reales que permiten abordar estos problemas y encontrar buenas configuraciones de los mismos. Mediante el uso de ecuaciones e inecuaciones (preferiblemente lineales) se pueden representar las relaciones entre los elementos del sistema y encontrar el mejor valor para las variables de decisión a partir de una función objetivo que permite evaluar cada posible solución.

Se considera que la optimización matemática tiene su origen en 1947, cuando George Dantzig publicó el algoritmo del Simplex[1] (Dantzig, 1947) y desde ese momento se ha convertido en una herramienta esencial en la planificación y gestión óptima de recursos. El algoritmo del Simplex permite resolver problemas en los que todas las variables son continuas y tanto la función objetivo como las restricciones son lineales. El desarrollo de versiones mejoradas de este algoritmo, incluso de alternativas de complejidad polinomial (como los algoritmos de punto interior (Karmarkar, 1984)), y la potencia actual de los ordenadores hacen posible que hoy por hoy se pueda resolver prácticamente cualquier problema de optimización lineal continua (la limitación suele venir dada por la memoria del ordenador y no por su capacidad de cómputo). Esto permite abordar problemas complejos, aunque es el uso de variables enteras (en particular, binarias, que permiten representar decisiones dicotómicas) lo que ha mostrado el verdadero potencial de la optimización matemática.

Los modelos matemáticos de optimización lineal entera son aquellos en los que algunas de las variables solo pueden tomar valores enteros, mientras que tanto la función objetivo como las restricciones son expresiones lineales. La inclusión de variables enteras en el problema introduce un grado de complejidad muy alto, lo que hace que incluso problemas de pequeña dimensión (cientos de variables o restricciones) sean difíciles de resolver en tiempo de computación razonable.

Los métodos clásicos de resolución de este tipo de problemas parten de una relajación del problema original: destacan el algoritmo *Branch and Bound*, introducido por Land and Doig (1960), y los métodos basados en planos de corte, introducidos en Gomory (1958) y Gomory (1960). En 1991, se propusieron los métodos de *Branch and Cut* (Padberg and Rinaldi, 1991), que combinan los dos anteriores y que son los que se utilizan en las librerías de optimización comerciales.

[1] Elegido como uno de los 10 algoritmos más importantes del siglo XX por la ACM (Cipra, 2000)

Sin embargo, incluso con los métodos más eficientes, no siempre es posible encontrar una solución factible al problema o una buena solución en tiempo razonable. Por ello, en los últimos tiempos el desarrollo de métodos aproximados (no exactos) ha tenido un gran desarrollo. Estos procedimientos no exactos, las metaheurísticas o matheurísticas, son útiles para obtener soluciones en tiempos de computación razonables.

Esta tesis doctoral se centra en la construcción de modelos de optimización matemática entera mixta para abordar los tres problemas planteados. Para el primero de ellos, la gestión de la red de puntos de venta, se desarrolla inicialmente un modelo de optimización entera no lineal para, a continuación, plantear una reformulación de las expresiones no lineales, obteniendo un modelo lineal. Para el segundo problema, la obtención del plan de corte diario para el proceso de corte longitudinal de bobinas de acero, se plantea un modelo de optimización lineal entera mixta. Por último, el problema de corte y asignación de máquinas se resuelve mediante una extensión de este último modelo. Todos los modelos se formulan mediante el uso del lenguaje algebraico AMPL (Fourer et al., 1990) y se resuelven con la librería de optimización Gurobi (Gurobi Optimization, 2020).

Resultados

Los modelos planteados se han desarrollado en colaboración con dos empresas internacionales, una en el sector del *retail* y otra en el sector del acero. En el primer caso, el modelo ha permitido mejorar el conocimiento del que la empresa disponía sobre la red de ventas; mientras que en el segundo se ha logrado una transferencia directa de los resultados gracias a la implantación de los modelos propuestos en el sistema de planificación de la empresa. Se detallan a continuación los resultados obtenidos:

- Deslocalización de servicios en el sector *retail*: Se ha estudiado el problema del rediseño de la red de puntos de venta de una compañía en el sector del *retail*, considerando posibles cierres o cambios de gestión de los puntos de venta. Para abordar este problema se ha propuesto un modelo de optimización matemática que permite decidir cambios en el tipo de gestión o el cierre de los puntos de venta, con el objetivo de mejorar el funcionamiento de la red. El tipo de decisiones que se extraen de este modelo no se aplican directamente sobre la estructura de la red, ya que afectan otros criterios que no siempre es posible tener en cuenta (política de empresa, competencia, etc.). Sin embargo, el modelo permite conocer cuál es la mejor estructura para maximizar el beneficio y poder tomar las decisiones de una manera más informada. Los resultados obtenidos se reflejan en el siguiente artículo que ha sido publicado:

> **Sierra-Paradinas, M.**, Alonso-Ayuso, A., Martín-Campo, F.J., Rodríguez-Calo, F. & Lasso, E. (2020), 'Facilities Delocation in the Retail Sector: A Mixed 0-1 Nonlinear Optimization Model and Its Linear Reformulation', *Mathematics* **8**(11):1986. doi:10.3390/math8111986.

- Planificación del corte longitudinal de bobinas de acero: Se ha desarrollado un modelo de optimización lineal entera que permite obtener un plan de corte para el proceso de corte longitudinal de bobinas de acero con el fin de satisfacer la demanda. El plan de corte define qué bobinas serán seleccionadas del *stock* y los patrones de corte para cada una de ellas. El modelo desarrollado ha permitido dotar a la empresa de una herramienta de planificación que mejora su operativa actual reduciendo los tiempos invertidos en la planificación del proceso y mejorando en varios aspectos operativos. Por un lado, se mejora sensiblemente el uso de las bobinas respecto de la operativa actual, pasando de utilizar el 50 % del material para servir los pedidos de los clientes a utilizar el 80 %. Esto supone una reducción considerable en el sobrante generado, tanto reutilizable como no reutilizable. Por otro lado, el modelo propone soluciones en las que se seleccionan bobinas más pequeñas del *stock* que corresponden con restos de procesos de corte anteriores. Esto, unido a la menor generación de sobrante, supondrá a medio plazo una reducción del *stock* procesado con la consiguiente mejora de la gestión del *stock*. Por último, el modelo permite ajustar mejor el peso servido al realmente demandado, por lo que se consigue mejorar los beneficios de la empresa al reducir las penalizaciones o descuentos debidos a estas desviaciones. Los resultados obtenidos han sido publicados en el siguiente artículo:

  > **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (2021), 'An exact model for a slitting problem in the steel industry', *European Journal of Operational Research* **295**(1), 336-347. doi:10.1016/j.ejor.2021.02.048.

- Planificación del corte longitudinal de bobinas de acero incluyendo la asignación a la máquina de corte: Se ha desarrollado un modelo que integra el proceso de selección de bobinas y diseño del patrón de corte con el proceso de asignación de máquinas de corte a cada bobina. Aunque este modelo todavía no se encuentra en producción, si la empresa finalmente lo implanta, permitirá una mejor planificación del proceso de corte longitudinal, ya que, sin empeorar significativamente los objetivos considerados en el problema anterior, se consigue mejorar la distribución de la carga de trabajo entre las distintas máquinas y se resuelven algunos problemas de infactibilidad que aparecían en el modelo anterior. Por último, permite incluir en la planificación pedidos que se pueden servir en varios días, lo que obliga a finalizar solo aquellos pedidos que deben servirse en un determinado periodo, pero se puede iniciar el corte de futuros pedidos, siempre y cuando se consiga una mejora en el aprovechamiento de las bobinas. Los resultados obtenidos se detallan en el siguiente artículo, enviado para su publicación:

Sierra-Paradinas, M., Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (submitted March 2022), 'An exact model for the 1.5-dimensional cutting *stock* problem in the steel industry with heterogeneous parallel slitting lines allocation', *European Journal of Operational Research*

Conclusiones

Se ha logrado el principal objetivo de esta investigación, que era desarrollar herramientas que ayuden a las empresas a abordar problemas complejos que aparecen en la toma de decisiones de sus procesos productivos. Esto se ha llevado a cabo mediante el desarrollo de un modelo de planificación estratégica para determinar la política de gestión adecuada en la red de puntos de venta de una compañía en el sector del *retail*, el desarrollo de un modelo de planificación operativa para una empresa que se dedica al corte de bobinas de acero, y la posterior integración de la fase de asignación de líneas de corte a la planificación de los patrones de corte.

El desarrollo de estos modelos ha motivado una transferencia científica al tejido industrial, al haber aportado soluciones innovadoras a complejos problemas de planificación. Además, en uno de los casos, se ha logrado la implantación de los modelos propuestos en el sistema de planificación de la empresa.

Chapter 1

Introduction

Contents

1.1 Motivation

Firms need to respond to emerging challenges in terms of its processes. To reach their business goals they need to efficiently use the resources they own. The increasingly complex processes make more difficult to optimise the resources manually and, therefore, it is necessary to use expert systems for this purpose.

Companies often rely on other experienced consulting firms to build these expert systems that will allow them to make more accurate decisions with fewer errors. In this context, the Spanish consulting firm *IDOM Consulting, Engineering, Architecture* (IDOM), leader in the industrial sector with presence in more than 125 countries and more than 3,000 employees, uses different technologies to address the problems of its clients. Among them there are complex simulation systems and some optimisation models.

The development of this PhD thesis is motivated by the need to provide IDOM with the necessary tools for the development of complex mathematical optimisation models and methods to assist the business decision-making process of their clients. The research has been developed under the framework of the industrial doctorate programme IND2018/TIC-9614 from the Government of the Region of Madrid, between the consulting firm IDOM and Rey Juan Carlos University.

1.2 Objectives

This PhD thesis has been developed under an industrial doctorate programme, and for that reason, the main objective is to develop decision support systems to help companies to tackle their complex business problems, and make optimal decisions that maximize their operational efficiency. In particular, mathematical optimisation is used as a tool for efficient decision making in the planning process of two well known international companies. This wide objective is achieved through the following particular objectives:

- To develop a strategical planning model to determine the appropriate management policy for a network of stores in the retail sector. The aim is to decide which type of management policy is the most suitable for each store and whether or not the store should continue to be part of the network.
- To develop an operational planning model for a steel manufacturing company to define a daily cutting plan. Its main process consists in cutting coils of steel lengthwise to obtain narrower coils known as strips. This cutting process is known as slitting. The aim is to provide the company with a cutting plan indicating which coils need to be cut and how to cut them to satisfy the demand of steel strips and minimise the leftovers generated in the process. The customers' restrictions and operational constraints need to be taken into account when defining the cutting patterns, making very complex to define the planning in a manual way.
- Based on the limitations observed in the operational model for the slitting of steel coils, an extension of the problem is studied to integrate the following phase of the production process: the allocation of coils to the slitting lines. The company owns several cutting lines with different characteristics and each of them imposes different conditions on the cutting patterns that can be obtained. Besides the goals considered in the previous problem the company would like to balance the workload among the different slitting lines.

1.3 Mathematical optimisation

Mathematical optimisation is a powerful discipline to optimise the resources and to help companies to make better and more informed decisions. An essential characteristic of a mathematical optimisation problem is that it involves a set of decisions that interact in complex ways, impacting various areas of the operations of companies and influencing other decisions.

Mathematical optimisation models are representations of real systems that allow us to deal with these complex problems. The decisions are represented by variables whose values we seek to determine, and the relationships between the elements of the system are represented by equations and inequalities (preferably linear). To find the best value of the decision variables an objective function is optimised.

Mathematical optimisation is considered to have its origins in 1947, when George Dantzig published the Simplex algorithm[1] (Dantzig, 1947), and has since become an essential tool in optimal resource planning and management. The Simplex algorithm allow us to solve problems where the decisions can be represented by continuous variables that appear linearly in the objective function and constraints of the model. The development of improved versions of the Simplex algorithm, including a polynomial-time algorithm proposed by Karmarkar (1984), and the current performance of computers make possible to solve virtually all continuous linear optimisation models.

However, the majority of business problems include some decisions that need to be modelled as integer variables (in particular, binary variables, which make possible to represent dichotomous decisions). Mathematical integer linear optimisation models are those in which some of the variables can only have integer values and appear in the objective and constraints linearly. The inclusion of integer variables introduces a high degree of complexity, making even small problems (hundreds of variables and/or constraints) difficult to solve in reasonable computational times.

The classical methods for solving this type of problems are based on a relaxation of the original problem. The two main algorithms are the Branch and Bound, introduced in Land and Doig (1960) and improved with the tree structure in Dakin (1965), and the Gomory Cutting planes, introduced in Gomory (1958) (for pure integer linear optimisation models) and Gomory (1960) (for mixed integer linear optimisation models). These algorithms were improved by an algorithm that integrates both procedures, the Branch and Cut algorithm, that was proved to be efficient for the Travelling Salesman Problem in Padberg and Rinaldi (1991). Nowadays, the Branch and Cut algorithm is the one implemented in the commercial optimisers for mixed integer linear optimisation models.

The use of integer variables allows us to make a more accurate representation of the real system, but even with the most efficient methods, it is not always possible to find a feasible solution or a good solution in reasonable computational time. In recent years, the development of approximate (non-exact) methods such as, metaheuristics or matheuristics, has increased and proved to be a useful tool to obtain solutions in reasonable computational times.

In this work, three mathematical integer optimisation models are presented to address

[1] Considered one of the most important algorithms developed in the XXth century(Cipra, 2000)

three complex business problems. In Chapter 2 the facilities delocation problem is introduced together with the mathematical non-linear integer optimisation model proposed, and its linear reformulation. In Chapter 3, the slitting problem in the steel industry and the mathematical linear integer optimisation model to solve it are presented. Chapter 4 extends the slitting problem described in the previous chapter with the integration of the slitting lines allocation. Finally, Chapter 5 summarizes the contributions produced in this thesis, and outlines some lines of future research. All models have been implemented using the algebraic modelling language AMPL (Fourer et al., 1990) and solved with the Gurobi optimiser (Gurobi Optimization, 2020).

1.4 Financial support

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- Rey Juan Carlos University, through its research assistance programme.
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- Government of Spain, with the grant RTI2018-094269-B-I00.
- Government of the Region of Madrid, via the industrial doctorate programme IND2018/TIC-9614.

Chapter 2

Facilities delocation in the retail sector.

Contents

2.1 Introduction

This chapter addresses the problem of facilities delocation in the retail sector by proposing a novel mixed 0-1 linear optimisation model. The aim of the problem is to decide whether to close existing stores or consider an alternative type of store management policy aimed at optimising the profit of the entire retail network. Each management policy has a different repercussion on the final profit of the stores due to the different margins obtained from the customers. Furthermore, closing stores can cause customers to leave the whole retail network according to their behavior.

Due to the commercial requirements concerning customer behavior, a set of non-linear constraints appears in the definition of the model. Classical Fortet inequalities are used in order to linearise the constraints and, therefore, obtain a mixed 0-1 linear optimisation model. As a result of the size of the network, border constraints have been imposed to obtain results in a reasonable computing time. The model implementation is done by introducing smart sets of indices to reduce the number of constraints and variables. The computational experiments present a real-world case study using data provided by the company and, a set of computational experiments using data randomly generated.

The findings of this chapter have been published in:

Sierra-Paradinas, M., Alonso-Ayuso, A., Martín-Campo, F.J., Rodríguez-Calo, F. & Lasso, E. (2020), 'Facilities Delocation in the Retail Sector: A Mixed 0-1 Nonlinear Optimization Model and Its Linear Reformulation', *Mathematics* **8**(11):1986. doi:10.3390/math8111986.

The chapter is organised as follows: Section 2.2 presents some literature review. Section 2.3 is intended to present a general description of the problem and the assumptions to be considered. Section 2.4 introduces the notation as well as the Mixed 0-1 Non-Linear optimisation model formulation. Section 2.5 presents the model reformulation to obtain a 0-1 Linear optimisation model. In Section 2.6, the main computational results are reported. Finally, Section 2.7 concludes and presents future lines of research.

2.2 Literature review

Facility location problems have been broadly studied in the literature. Their objective is to determine the best place to open new facilities within a region to satisfy the demand of the customers. The best location depends on the criteria considered from a wide range of options. The p-median whose first mathematical optimisation model was introduced in ReVelle and Swain (1970), the p-centre introduced in Hakimi (1964) and Hakimi (1965), or the capacitated facility location problems introduced by Balinski (1965) are some of the most well-known problems related to the field of location.

There are models available for locating public and retail facilities, emergency services and plants or warehouses, among others. These location problems can be solved by using (integer or continuous) linear optimisation, heuristic and meta-heuristic approaches. A comprehensive state of the art in this area can be found in Laporte et al. (2015) among many other interesting references.

The majority of the location problems involve the location or relocation of new facilities. The concept of delocation, defined as the operation cease of existing facilities (see Bhaumik (2010)), has been recently introduced. As shown in ReVelle et al. (2007), there have been several delocation problems in both the private and the public sectors. An example of facility closures in the transport sector is provided by Murray and Wu (2003) which detail various modelling approaches, that address a reduction of the current number of stops along a bus route in order to promote faster transit speeds and greater geographic coverage given a travel time budget.

Delocation has also been a subject of study for educational organisations. An example can be seen, in Bruno and Andersen (1982) where they present a model to determine school closures in a medium sized school district in California.

As regards the banking sector, the need to address the closure of facilities is shown in Morrison and O'Brien (2001), Wang et al. (2003), Monteiro and Fontes (2005), Ruiz-Hernández et al. (2015) and Ruiz-Hernández and Delgado-Gómez (2016) among others. A budget constraint location problem is presented in Wang et al. (2003) to locate and relocate bank-branches in a large-sized town. The authors consider both, opening new facilities while at the same time closing some of the existing ones. In Monteiro and Fontes (2005), a local search heuristic is proposed to address the problem of bank-branch restructuring. In Ruiz-Hernández et al. (2015), a model for re-sizing the bank network is presented with the aim of maintaining a constant service level, and deciding which branches should continue in business and which should be replaced. More recently, in Ruiz-Hernández and Delgado-Gómez (2016), a stochastic optimisation model has been introduced to restructure a network of capacitated bank-branches by considering uncertainty in the demand.

Store closures have also been a topic of concern in the retail sector. Evidence of this fact is shown in Shields and Kures (2007), which investigate the spatial and economic factors that influenced the decision of a major American retailer to close part of its network of stores. A further example on this interest is studied in ReVelle et al. (2007), which introduces two models to reduce the number of facilities in a given area with and without strong competition. In the first case the aim is to reduce the number of facilities to a fixed number, by minimising the impact on the loss of demand to the competitors. In the second case, the measure of the decline of the service is minimised. On the same line, in Bhaumik (2010), a model is presented to downsize an existing distribution network of a firm with known supplier locations. The firm seeks the closure of a fixed number of the supplier nodes. The model assumes that all demand nodes must be served by their respective supplier unless the existing supplier is removed. Recently, in Yavari and Mousavi-Saleh (2019), a problem of restructuring bi-level facilities was presented. The aim consisted of minimising costs by opening auxiliary facilities and closing or resizing the existing facilities maintaining an overall coverage.

The underlying problem presented in this chapter considers three different elements simultaneously: facilities delocation, modifications on management policies and network restructuring. As stated above, there are many references where some mathematical optimisation models have been proposed in order to deal with delocation problems. However, to our knowledge, there are no optimisation models that consider all these characteristics together. Then, we present a model that restructures the network of stores of a retail chain by delocating and changing management policy decisions in order to maximize the total profit of the network. As an initial approach, a Mixed 0-1 Non-Linear optimisation model is proposed. As a second step, the model is reformulated in order to linearise the non-linear expressions. As a result, a Mixed 0-1 Linear optimisation model is proposed. These models have been validated with the real-world data provided by a well-known international company.

2.3 Problem description

The delocation problem has been addressed in the literature for several reasons as discussed in Section 2.2. In our case, the manager of a retail chain seeks to optimise the management of the network of stores of the retail chain. The retail chain itself or an external dealer can operate these stores. In these case, the manager would like to know whether it is appropriate to either stop operating the stores or change the management policy.

The network consists of a set of stores all over a specific region. Currently, each store is managed by one of the different management policies considered in the network. Decisions about the change in the management policy are centralised and are mainly based on whether the store generates profits for the company or not. We distinguish two different types of stores in terms of the decisions that can be made about them: non-fixed and fixed stores. The impossibility of delocation or modification of the management policy characterise fixed stores. Then, decisions concerning delocation or modification of the management policy can be made only for non-fixed stores. It is the company, based on the contracts they have with the different stores, that decides which store is fixed or non-fixed.

The overall benefit is obtained from the purchase of goods made by customers in the stores. Each management policy has different repercussions on the final profit of the stores due to the profit margins imposed by the retail chain or external dealers. The customers of the company can be classified in two different classes: major customers, those who are large-scale consumers (usually professionals) with price fixing agreements, and the rest of the customers that usually have loyalty cards to obtain discounts or other advantages. Each customer consumes in a certain set of stores owing to the customer-store agreements. We can distinguish two types of customer behaviour with respect to each store regarding their commitment to the retail chain: customers with a tendency-to-abandon the network and customers with a tendency-to-stay in the network. On the one hand, a customer with a tendency-to-abandon a certain store will leave the network altogether if that store is delocated and, thus, will no longer consume in that retail chain. On the other hand, a customer with a tendency-to-stay in a certain store will continue consuming in the network in case the store is delocated. Notice that customers with loyalty cards may decide whether to continue consuming in the retail chain (in case one of their preferred stores is closed) whereas the major customers will remain in the network. The company know this aspect beforehand since it is aware of all of the costumer-store agreements. Therefore, a customer will only leave the network if a store where the customer has a tendency-to-abandon is delocated or if all stores where the customer consumes are delocated. Notice that this behaviour occurs particularity in this company chain since it depends on agreements and not on distances. The distances in this problem are not relevant, which is a feature that differs from the rest of the traditional literature. Furthermore, store capacities are not considered since the company is the one responsible for supplying the products to any given store within the network. Finally, the consumption in the delocated stores by those customers that do not leave the network is distributed among the remaining stores where they consume.

In order to maintain a certain service level, the retail chain has imposed a minimum number of stores that must stay open, but not which specific one of them. It is important to point out that the store delocation process has a fixed cost associated to it.

A more precise description of the issues involved, as well as the assumptions for addressing them, is presented below.

2.3.1 Types of stores and management policies

There are two different types of stores that are considered: fixed and non-fixed stores. Fixed stores must maintain their management policy and no decisions can be made over them. On the other hand, both decisions, on delocating or modifying management policy, can be issued on the non-fixed stores. Let us denote $\mathcal{J}$ as the set of stores in the network, where $\mathcal{J}^{\mathrm{F}} \subset \mathcal{J}$ is the subset of fixed stores and $\mathcal{J}^{\mathrm{NF}} \subset \mathcal{J}$ the subset of non-fixed stores.

For the stores in the latter set, a delocation cost given by c_j is considered and applied in case of delocation. Notice that $\mathcal{J} = \mathcal{J}^F \cup \mathcal{J}^{\mathrm{NF}}$.

As stated above, each store $j \in \mathcal{J}$ can be run by different management policies. Let us denote $\mathcal{K}$ as the set of management policies and $\mathcal{K}^j$ the specific set of management policies available for a certain store j.

For each store $j \in \mathcal{J}^{\mathrm{NF}}$, the decision made can be either to delocate the facility or to change the management policy within $\mathcal{K}^j$. In the case where the management policy changes from $k_1 \in \mathcal{K}^j$ to $k_2 \in \mathcal{K}^j$ there will be a variation in the volume of goods consumed, given by a percentage of the initial total consumption of goods which is given, e_{jk_2}. The extra volume will be charged at the profit margin r_{jk_2}.

In our case, we consider four different management policies depending on the administration and ownership of the stores. Type A refers to a policy whereby the company both, owns and operates the store. Type B means that the company owns the store but an external dealer operates it. Although one may think this is a franchise arrangement, it is not exactly the case, since a third party operates the store but not with the brand image and other common specifications within the franchise arrangements. Type C implies that an external dealer owns the store but the company operates it (which is usually named a lease agreement) and, finally, if the store is managed by Type D, an external dealer both, owns and operates the store, and the company simply supplies its products. Only the following management changes are allowed. If the store is managed as Type A, then it could be changed to Type B, meaning that an external dealer will operate the store. If the store is managed as Type C, then it could be changed to Type D. Types B and D management policies are not allowed to be modified although those stores can be delocated. Therefore, stores Type B or D must remain unchanged if they are not delocated.

2.3.2 Customers and their behaviour

The network is used by a set of customers $\mathcal{I}$ that can be classified in different ways depending on the types of stores where they consume and their behaviour given by the tendency to leave or not the network. This tendency mainly depends on the type of agreement between the company and the customer. For major customers, who have special agreements with the company, due to their large amount of consumption, the tendency to abandon is not common. However, when the agreement is in the form of a loyalty card, the customer may decide whether or not to continue consuming in the retail chain if one or some stores are closed. Moreover, a customer may have a subset of stores such that, if one of them is delocated, then, he/she leaves the network due to their dissatisfaction, for instance. This kind of customer is said to have a tendency-to-abandon the network in each store of this subset. However, if none of these stores is closed but some of the others in which the customers consume are delocated, they remain in the network and distribute their consumption in the delocated stores among those that remain open. The tendency to abandon or not is given and calculated by using machine learning techniques whatever the type of store (fixed or non fixed) where the customer consumes. Taking into account this tendency to abandon, customers will leave the network in the following situations when:

- At least one store in which they have a tendency to abandon is delocated.
- All stores in which they consume are delocated.

The behaviour of customers with respect to each store in which they consume, affects the total consumption in the whole network and, thus, the final profit of the retail chain. Notice that the information about the behaviour of customers is known in advance for all stores.

Regarding the types of stores where a customer consumes, we can distinguish between customers consuming in at least one non-fixed store and customers that only consume at fixed stores. The latter can be removed from the problem, since they will not change their pattern of consumption even though their incomes should be added to the total income of the company. We therefore assume that all customers in $\mathcal{I}$ consume in at least one non-fixed store. It is important to analyse the behaviour of these customers with respect to the non-fixed stores in which they consume. Then, the following subsets are defined:

- $\mathcal{I}^{\mathrm{L}}$, customers who can leave the network if some of the stores in which they consume are delocated. Three different kinds of customers can be considered here:
  - $\mathcal{I}^{\mathrm{A}}$, customers with a tendency-to-abandon the network in every non-fixed store where they consume. A customer in $\mathcal{I}^{\mathrm{A}}$ leaves the network if at least one non-fixed store in which he/she consumes is delocated.
  - $\mathcal{I}^{\mathrm{BA}}$, customers with a tendency-to-abandon the network in some, but not all, non-fixed stores where they consume. A customer in $\mathcal{I}^{\mathrm{BA}}$ leaves the network if at least one store in which he/she has a tendency-to-abandon is delocated. However, the client does not leave the network in case the stores where they do not have a tendency-to-abandon are delocated. In the latter case, their consumption in those delocated stores will be distributed among the non-delocated stores in which they still consume.
  - $\mathcal{I}^{\mathrm{NS}}$, customers consuming only in non-fixed stores and without a tendency-to-abandon in any of them. A customer in $\mathcal{I}^{\mathrm{NS}}$ leaves the network if *every* store in which the client consumes is delocated. However, if at least one store

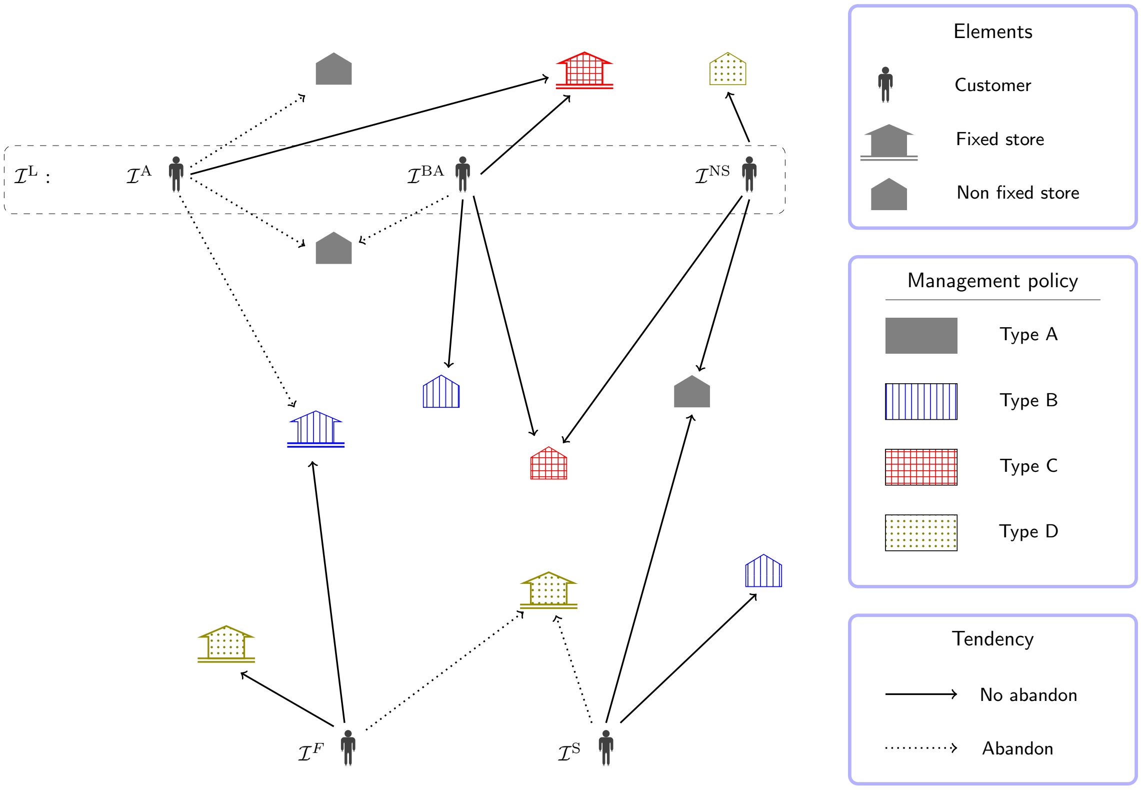


Figure 2.1: Classification of the customers.

in which they consume remains in the network, their consumption in those delocated stores will be distributed among the non-delocated stores in which he/she consumes.

- $\mathcal{I}^{\mathrm{S}}$, customers who will never leave the network whether or not any of the stores in which they consume are delocated. These customers consume in at least one fixed store and they do not have a tendency-to-abandon the network in any of the non-fixed stores where they consume. The customer's consumption in the delocated stores is distributed among the rest of stores where he/she consumes.

Figure 2.1 shows an example of these sets of customers. Fixed and non-fixed stores are represented by different shapes and each customer is linked by an arrow to the stores where he/she consumes; dotted arrows indicate that the customer has a tendency-to-abandon the network if the corresponding stores are delocated. Note that $\mathcal{I}^F$, the set of customers that only consume at fixed stores, will be removed from the problem since they have no impact on the optimal solution.

The amount of goods consumed by each customer in the stores is known in advance, as well as the profit margins that each customer provides to the chain; this profit dif-

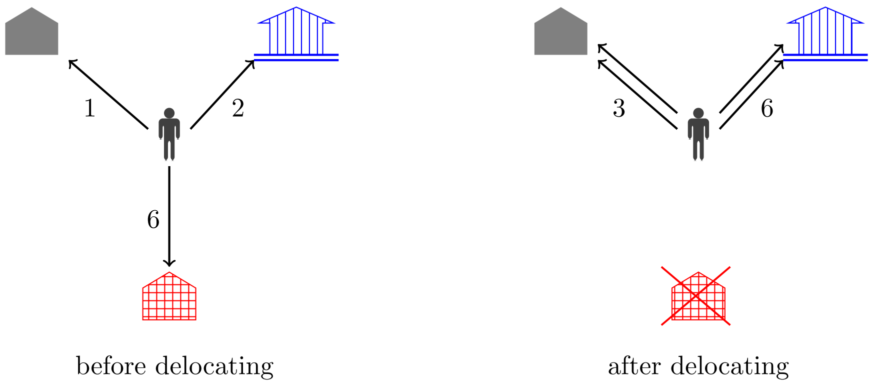


Figure 2.2: Proportional reassignment of goods.

fers for every customer, store and management policy. When a store is delocated, the customers can either abandon the whole network or keep consuming in the rest of the stores, depending on their behaviour. In the latter case, the goods they consumed in the delocating stores are distributed among the remaining stores in a proportional way, given by the amount of goods that each customer is consuming in the stores that remain open. Let us define the increase of goods by customer and store as y_{ij}. In order to illustrate the proportional reassignment of goods, Figure 2.2 shows one customer which consumes in three different stores, 1, 2 and 6 units, respectively. If the store in which the customer consumes 6 units is delocated, these units will be proportionally distributed in the two remaining stores, hence the final amounts of goods $3 = 1 + \frac{1 \cdot 6}{1+2}$ and $6 = 2 + \frac{2 \cdot 6}{1+2}$ units, respectively.

2.3.3 Problem hypotheses

In summary, the problem studied considers the following hypotheses:

- The network consists of a set of stores all over a specific region.
- At the present time, each store is managed by one of the four different management policies considered.
- The stores are divided into two groups regarding the decisions that can be made concerning them: non-fixed and fixed stores. Fixed stores are characterised by the impossibility of delocation or modification of management policy. On the other hand, both types of decisions can be made for non-fixed stores.
- The global profit is obtained from the consumption of goods by customers in the stores. Profit margins vary depending on the type of management policy.
- Each customer consumes in a certain set of stores due to customer-store agreements.

- We can distinguish two types of customer behaviour: customers with a tendency-to-abandon a certain store will leave the entire network if that store is delocated, customers with a tendency-to-stay in a certain store will remain consuming in the network in case the store is delocated. In the latter behaviour the consumption in the delocated stores is proportionally distributed among the remaining stores where the customer consumes.
- The tendency-to-abandon or to-stay is given as a parameter and calculated by using machine learning techniques for all stores (fixed or non fixed) where the customer consumes.
- This behaviour is particularly notable in this company chain since it depends on agreements and not on distances.
- Store capacities are not considered since the company is the one responsible of supplying the products to every store within the network.
- In order to maintain a service level, a minimum number of stores must remain open.
- The store delocation process has a fixed cost associated to it.

2.4 First approach: a Mixed 0-1 NonLinear optimisation model

A mathematical Mixed 0-1 NonLinear optimisation model is presented as follows, where all the elements are defined.

2.4.1 Notation

Sets

$\mathcal{J}$, set of stores, divided in two groups: $\mathcal{J}^{\mathrm{F}}$ and $\mathcal{J}^{\mathrm{NF}}$, set of fixed and non-fixed stores, respectively.

$\mathcal{I}$, set of customers. Note that only customers that consume in at least one non-fixed store are considered here, since, customers consuming only in fixed stores do not imply any change in the company's profit. Set $\mathcal{I}$ can be partitioned in the following subsets:

- $\mathcal{I}^{\mathrm{L}}$, customers who may leave the network if some of the stores in which they consume are delocated:
  - $\mathcal{I}^{\mathrm{A}}$, customers with a tendency to abandon the network in every non-fixed store where they consume.

  - $\mathcal{I}^{\mathrm{BA}}$, customers with a tendency to abandon the network in some, but not all, non-fixed stores where they consume.
  - $\mathcal{I}^{\mathrm{NS}}$, customers consuming only in non-fixed stores without a tendency to abandon any of them.
- $\mathcal{I}^{\mathrm{S}}$, customers consuming in at least one fixed store and without a tendency to abandon any non-fixed stores.

$\mathcal{J}_i$, set of stores where the customer i consumes in, $i \in \mathcal{I}$. $\mathcal{J}_i$ is divided in $\mathcal{J}_i^{\mathrm{F}}$ and $\mathcal{J}_i^{\mathrm{NF}}$, the sets of fixed and non-fixed stores, respectively.

$\mathcal{J}_i^{\mathrm{A}}$, set of stores where customer i has a tendency to abandon the network (i.e., customer i leaves the network if a store in $\mathcal{J}_i^{\mathrm{A}}$ is delocated), $i \in \mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}}$. Note: $\mathcal{J}_i^{\mathrm{A}} \subseteq \mathcal{J}_i^{\mathrm{NF}}$.

$\mathcal{K}$, set of management policies, such as company ownership, the company operates the store or there is an external dealer for instance. The set of possible management policies for store j will be denoted as $\mathcal{K}^j$, $j \in \mathcal{J}$.

Parameters

p, service level, given by the number of (fixed and non-fixed) stores that must remain open.

c_j, delocation costs of any store j, $j \in \mathcal{J}^{\mathrm{NF}}$.

e_{jk}, percentage of extra volume of goods consumed in store j with management policy k (if there is no profit, $e_{jk} = 0$), $j \in \mathcal{J}$, $k \in \mathcal{K}^j$.

r_{jk}, profit margin to be applied to the extra volume of goods in store j with management policy k, $j \in \mathcal{J}$, $k \in \mathcal{K}^j$.

g_{ij}, initial amount of goods consumed by customer i in store j, $i \in \mathcal{I}$, $j \in \mathcal{J}_i$.

m_{ijk}, unit profit obtained by customer i in store j with management policy k, $i \in \mathcal{I}$, $j \in \mathcal{J}_i$, $k \in \mathcal{K}^j$.

M_i, upper bound of the volume of goods consumed by customer i, $i \in \mathcal{I}$.

k_j^*, management policy of any fixed store $j \in \mathcal{J}^{\mathrm{F}}$. Note: $k_j^* \in \mathcal{K}^j$.

Decision variables

$\alpha_j = 1$, if the (non-fixed) store j is delocated and, 0 otherwise, $j \in \mathcal{J}^{\mathrm{NF}}$.

$\gamma_{jk} = 1$, if the (non-fixed) store j is managed with management policy k and, 0 otherwise, $j \in \mathcal{J}^{\mathrm{NF}}, k \in \mathcal{K}^j$.

$\beta_i = 1$, if customer i leaves the network and, 0 otherwise, $i \in \mathcal{I}^{\mathrm{L}}$.

x_i, total amount of goods consumed by customer i in all the delocated stores (must be transferred to the non-delocated stores), $i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}$.

y_{ij}, increase in the consumption of goods of customer i in store j if at least one of the stores where i consumes is delocated, $i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}$, $j \in \mathcal{J}_i$.

z_{ijk}, increase in the consumption of goods of customer i in store j if the store is managed by policy k, $i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}$, $j \in \mathcal{J}_i$, $k \in \mathcal{K}^j$.

2.4.2 Mathematical formulation

Objective function

The objective of the problem is to maximise the profit defined as the profit from the sales in the whole network minus the restructuring network costs:

$$\max z = PRS + PRO + PCM - CD \tag{2.1}$$

where PRS represents the Profit obtained from the Regular Sales, PRO represents the Profit obtained from the increase of sales in the stores that Remain Open, PCM represents the Profit obtained when there is a Change of Management policy and CD the Cost due to the Delocation of the stores.

$$\begin{aligned} PRS = & \sum_{i \in \mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}}} \Big(\sum_{j \in \mathcal{J}_i^{\mathrm{F}}} m_{ijk_j^*} g_{ij} + \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \gamma_{jk} \Big)(1-\beta_i) + \\ & \sum_{i \in \mathcal{I}^{\mathrm{NS}}} \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \gamma_{jk} (1-\beta_i) + \\ & \sum_{i \in \mathcal{I}^{\mathrm{S}}} \Big(\sum_{j \in \mathcal{J}_i^{\mathrm{F}}} m_{ijk_j^*} g_{ij} + \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \gamma_{jk} \Big) \end{aligned} \tag{2.1a}$$

$$PRO = \sum_{i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}} \sum_{j \in \mathcal{J}_i^{\mathrm{F}}} m_{ijk_j^*} y_{ij} + \sum_{i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}} \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} z_{ijk} \tag{2.1b}$$

$$PCM = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} r_{jk} e_{jk} g_{ij} \gamma_{jk} \tag{2.1c}$$

$$CD = \sum_{j \in \mathcal{J}^{\mathrm{NF}}} c_j \alpha_j \tag{2.1d}$$

Note that the expression (2.1a) has a quadratic term in the first and second terms. In Section 2.5 an equivalent formulation with linear expressions is presented.

Constraints

1. Type of management: Constraints (2.2) state that a non-fixed store must be either managed with only one policy or be delocated.

$$\sum_{k\in\mathcal{K}^j} \gamma_{jk} + \alpha_j = 1 \quad \forall j \in \mathcal{J}^{\mathrm{NF}} \tag{2.2}$$

2. Service level: Constraint (2.3) ensures a proper service level by maintaining a minimum of p stores open.

$$|\mathcal{J}| - \sum_{j\in\mathcal{J}^{\mathrm{NF}}} \alpha_j \geq p \tag{2.3}$$

3. Customer's behaviour: Constraints (2.4) force a customer to leave the network if a store in which he/she has a tendency to abandon is delocated; on the other hand, constraints (2.5) state that a customer in $\mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}}$ does not leave the network if none of the stores in which he/she has a tendency to abandon are delocated, that is, if he/she leaves the network, at least one store in which he/she has a tendency to abandon must have been delocated. For customers without a tendency to abandon the network that only consume at non-fixed stores, constraints (2.6) ensure those customers remain in the network if at least one of their stores remains open, while constraints (2.7) force them to leave the network if all stores where they consume are delocated, that is, if the customer does not abandon the network, then, at least one of the stores in which he/she consumes remains open.

$$\alpha_j \leq \beta_i \quad \forall i \in \mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}}, j \in \mathcal{J}_i^{\mathrm{A}} \tag{2.4}$$

$$\beta_i \leq \sum_{j\in\mathcal{J}_i^{\mathrm{A}}} \alpha_j \quad \forall i \in \mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}} \tag{2.5}$$

$$\beta_i \leq \alpha_j \quad \forall i \in \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i \tag{2.6}$$

$$\sum_{j\in\mathcal{J}_i} \alpha_j \leq |\mathcal{J}_i| - 1 + \beta_i \quad \forall i \in \mathcal{I}^{\mathrm{NS}} \tag{2.7}$$

4. Consumption in the network: Constraints (2.8) compute, for each customer, the amount of goods that must be transferred from the delocated stores in which he/she consumes to the stores that remain open. Constraints (2.9)–(2.12) compute the increase of consumption of goods in some stores for those customers in $\mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}$, thas is, customers that could stay in the network although some of the stores where they consume are delocated. For each customer, goods originally consumed in the delocated stores must be transferred to the non-delocated stores in which the customer

continues consuming. The increase in store j will be zero if customer i leaves the network ($\beta_i = 1$) or if store j is delocated ($\alpha_j = 1$); otherwise, the total amount of goods consumed in all delocated stores where customer i consume,

$$\sum_{j \in J_i^{NF}} g_{ij}\alpha_j,$$

is distributed among the non-delocated stores where the customer consumes in proportion to the quantity that the customer originally consumed in those stores, that is.

$$\frac{g_{ij}}{\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'}(1-\alpha_{j'})}.$$

Notice that the denominator in this weighting factor could be 0 for customers that only consume in non-fixed stores if all these stores are delocated; in such a case, constraints (2.9) cannot force y_{ij} to be equal to 0, but constraints (2.13)-(2.14), together with (2.2), will do so.

$$\sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij}\alpha_j = x_i \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}} \tag{2.8}$$

$$\Big(\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'}(1-\alpha_{j'})\Big) y_{ij} = g_{ij}(1-\beta_i)(1-\alpha_j)x_i \quad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.9}$$

$$\Big(\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'}(1-\alpha_{j'})\Big) y_{ij} = g_{ij}(1-\alpha_j)x_i \quad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.10}$$

$$\Big(\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'}(1-\alpha_{j'})\Big) y_{ij} = g_{ij}(1-\beta_i)x_i \quad \forall i \in \mathcal{I}^{\mathrm{BA}}, j \in \mathcal{J}_i^{\mathrm{F}} \tag{2.11}$$

$$\Big(\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'}(1-\alpha_{j'})\Big) y_{ij} = g_{ij}x_i \quad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{F}} \tag{2.12}$$

5. Profit in the network: Constraints (2.13)-(2.14) relate variables γ_{jk}, y_{ij} and z_{ijk} to include the increase of goods with the appropriate profit margin depending on the final management policy. These constraints are only defined for non-fixed stores, as

they are the only ones whose management policy can be modified.

$$y_{ij} = \sum_{k \in \mathcal{K}^j} z_{ijk} \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.13}$$

$$z_{ijk} \leq M_i \gamma_{jk} \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.14}$$

6. Variables' domain: Constraints (2.15)-(2.20) define the domain for the variables in the model.

$$\alpha_j \in \{0,1\} \quad \forall j \in \mathcal{J}^{\mathrm{NF}} \tag{2.15}$$

$$\gamma_{jk} \in \{0,1\} \quad \forall j \in \mathcal{J}^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.16}$$

$$\beta_i \in \{0,1\} \quad \forall i \in \mathcal{I}^{\mathrm{L}} \tag{2.17}$$

$$x_i \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}} \tag{2.18}$$

$$y_{ij} \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i \tag{2.19}$$

$$z_{ijk} \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, k \in \mathcal{K}^j \tag{2.20}$$

2.5 Model reformulation: A Mixed 0-1 Linear optimisation model

The above described model contains non-linear expressions. The non-linearities appear in the model through the term (2.1a) in the objective function and the constraints (2.9)–(2.12). The algorithms developed for these types of models do not provide an optimal solution in a reasonable computational time. Hence we will work with a new Mixed 0-1 Linear optimisation model by substituting the non-linear equations in the model with an equivalent linear representation.

2.5.1 Linearisation of the nonlinear equations

Objective function

In order to linearise the quadratic term defined by the product of two 0-1 variables in term (2.1a) of the objective function, new auxiliary 0-1 variables σ_{ijk} are defined representing the product $(1-\beta_i)\gamma_{jk}$, for $i \in \mathcal{I}^{\mathrm{L}}$, $j \in \mathcal{J}_i^{\mathrm{NF}}$ and $k \in \mathcal{K}^j$, such that, $\sigma_{ijk} = 1$ if customer i does not leave the network and store j uses management policy k and 0 otherwise.

Variables σ_{ijk} are computed from the variables β_i and γ_{jk} through the following constraints:

$$\begin{aligned} \sigma_{ijk} &\leq \gamma_{jk} && \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \\ \sigma_{ijk} &\leq 1 - \beta_i && \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \\ \sigma_{ijk} &\geq \gamma_{jk} - \beta_i && \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \end{aligned}$$

Constraints related to the increase of goods consumption

Further down we present an equivalent linear formulation to constraints (2.9)–(2.12) in order to obtain a completely linear model, reducing complexity although the dimensions of the model increase.

Constraints (2.9)–(2.12) contain a quadratic term in the left-hand-side (the product of a continuous variable y_{ij} and, a 0-1 variable, α_j). An equivalent linear representation of this product can be obtained by using Fortet inequalities. The Fortet inequalities are presented in Fortet (1960) and Hammer and Rudeau (1968). They deal with the linearisation of the product involving a binary, say b, and a nonnegative continuous variable, say c. In order to linearise the product $c \cdot b$, a new nonnegative continuous variable k is introduced, where $k = c \cdot b$ as well as the following set of constraints:

$$k \leq c, \qquad k \leq Mb, \qquad c - k \leq M(1 - b),$$

where M is a big enough parameter that should be adjusted to strengthen the linear relaxation of the problem. From this system, if $b = 0$ then $k = 0$, and if $b = 1$ then, necessarily $k = c$, which models the product $c \cdot b$.

Applying Fortet inequalities scheme, product $(1 - \alpha_{j'})y_{ij}$ can be replaced with the auxiliary continuous variable $u^1_{ijj'} = (1 - \alpha_{j'})y_{ij}$, together with the following set of constraints:

$$\begin{aligned} u^1_{ijj'} &\leq y_{ij} && \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \\ u^1_{ijj'} &\leq M_{ij}(1 - \alpha_{j'}) && \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \\ y_{ij} - u^1_{ijj'} &\leq M_{ij}\alpha_{j'} && \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \end{aligned}$$

where M_{ij} can be set as

$$M_{ij} = \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}} : j' \neq j} g_{ij'}.$$

With this reformulation, the left-hand-side is linearised.

The right-hand-side of the constraints also contains non linear terms that are linearised by using Fortet inequalities once more:

- Constraints (2.9), defined for $i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}$ and $j \in \mathcal{J}_i^{\mathrm{NF}}$:

  Constraints (2.9) contain a cubic term, $(1-\alpha_j)(1-\beta_i)x_i$. A new binary auxiliary variable δ_{ij} is introduced to represent the product $(1-\alpha_j)(1-\beta_i)$, such that, if either one of α_j or β_i is equal to 1, then automatically $\delta_{ij} = 0$. Then, $\delta_{ij} = 1$ if customer i does not leave the network and store j is not delocated and 0 otherwise. Variables δ_{ij} are computed by using variables α_j and β_i through the following constraints:

$$\begin{aligned} \delta_{ij} + \alpha_j &\leq 1 && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ \delta_{ij} + \beta_i &\leq 1 && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ \delta_{ij} + \alpha_j + \beta_i &\geq 1 && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \end{aligned}$$

  Replacing this new variable in $x_i(1-\alpha_j)(1-\beta_i)$, the result is $x_i\delta_{ij}$. The latter expression is a product of a continuous variable, x_i, and a $0-1$ variable δ_{ij}, that can also be linearised using the Fortet inequalities scheme. Therefore, a new continuous variable has to be introduced, $u_{ij}^2 = x_i\delta_{ij}$, together with the following set of additional constraints:

$$\begin{aligned} u_{ij}^2 &\leq x_i && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ u_{ij}^2 &\leq M_{ij}\delta_{ij} && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ x_i - u_{ij}^2 &\leq M_{ij}(1-\delta_{ij}) && \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \end{aligned}$$

  where M_{ij} can be set as

$$M_{ij} = \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}} : j' \neq j} g_{ij'}.$$

- Constraints (2.10), defined for $i \in \mathcal{I}^{\mathrm{S}}$ and $j \in \mathcal{J}_i^{\mathrm{NF}}$:

  These constraints contain a quadratic term, $(1-\alpha_j)x_i$. By using the Fortet inequalities scheme, this product can be replaced by a new continuous variable, $u_{ij}^3 = x_i(1-\alpha_j)$, and the following set of additional constraints:

$$\begin{aligned} u_{ij}^3 &\leq x_i && \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ u_{ij}^3 &\leq M_{ij}(1-\alpha_j) && \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \\ x_i - u_{ij}^3 &\leq M_{ij}\alpha_j && \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \end{aligned}$$

where M_{ij} can be set as

$$M_{ij} = \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}} : j' \neq j} g_{ij'}.$$

- Constraints (2.11), defined for $i \in \mathcal{I}^{\mathrm{BA}}$ and $j \in \mathcal{J}_i^{\mathrm{F}}$:

  These constraints contain a quadratic term, $(1-\beta_i)x_i$. Yet again, the Fortet inequalities scheme can be used to linearise this quadratic term. Therefore, a new continuous variable has to be introduced, $u_i^4 = x_i(1-\beta_i)$, and the following set of additional constraints:

$$\begin{aligned} u_i^4 &\leq x_i & \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \\ u_i^4 &\leq M_{ij}(1-\beta_i) & \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \\ x_i - u_i^4 &\leq M_{ij}\beta_i & \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \end{aligned}$$

  where M_{ij} can be set as

$$M_{ij} = \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}} : j' \neq j} g_{ij'}.$$

- Constraints (2.12), defined for $i \in \mathcal{I}^{\mathrm{S}}$ and $j \in \mathcal{J}_i^{\mathrm{F}}$:

  Once the left hand side of the constraints has been linearised, these constrains become linear.

2.5.2 Mathematical formulation

Taking into account all the linearisation process performed above, model (2.1)–(2.20) can be replaced by an equivalent linear reformulation substituting the non-linear constraints (2.1a) and (2.9)–(2.12) by their corresponding sets of constraints and auxiliary variables.

Then, the notation for the Mixed 0-1 linear optimisation reformulation has the following different elements with respect to the non-linear model:

Additional auxiliary binary variables

$\delta_{ij} = 1$ if customer $i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}$ leaves the network or store $j \in \mathcal{J}_i^{\mathrm{NF}}$ is delocated and 0 otherwise.

$\sigma_{ijk} = 1$ if customer $i \in \mathcal{I}^{\mathrm{L}}$ leaves the network or store $j \in \mathcal{J}_i^{\mathrm{NF}}$ is operated with management policy $k \in \mathcal{K}^j$ and 0 otherwise.

Additional auxiliary continuous variables

$u^1_{ijj'}$, nonnegative continuous variables for each customer $i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}$ and stores $j \in \mathcal{J}_i$, $j' \in \mathcal{J}_i^{\mathrm{NF}}$.

u^2_{ij}, nonnegative continuous variables for each customer $i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}$ and store $j \in \mathcal{J}_i^{\mathrm{NF}}$.

u^3_{ij}, nonnegative continuous variables for each customer $i \in \mathcal{I}^{\mathrm{S}}$ and store $j \in \mathcal{J}_i^{\mathrm{NF}}$.

u^4_{i}, nonnegative continuous variables for each customer $i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset$.

New linear constraints

- Equation (2.1a) must be substituted by its equivalent set of linear constraints:

$$\begin{aligned} PRS = &\sum_{i \in \mathcal{I}^{\mathrm{A}} \cup \mathcal{I}^{\mathrm{BA}}} \Big(\sum_{j \in \mathcal{J}_i^{\mathrm{F}}} m_{ijk_j^*} g_{ij} (1 - \beta_i) + \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \sigma_{ijk} \Big) + \\ &\sum_{i \in \mathcal{I}^{\mathrm{NS}}} \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \sigma_{ijk} + \\ &\sum_{i \in \mathcal{I}^{\mathrm{S}}} \Big(\sum_{j \in \mathcal{J}_i^{\mathrm{F}}} m_{ijk_j^*} g_{ij} + \sum_{j \in \mathcal{J}_i^{\mathrm{NF}}} \sum_{k \in \mathcal{K}^j} m_{ijk} g_{ij} \gamma_{jk} \Big) \end{aligned} \tag{2.21}$$

$$\sigma_{ijk} \leq \gamma_{jk} \qquad \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.22}$$

$$\sigma_{ijk} \leq 1 - \beta_i \qquad \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.23}$$

$$\sigma_{ijk} \geq \gamma_{jk} - \beta_i \qquad \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.24}$$

- Constraints (2.9)–(2.12) must be substituted by their equivalent set of linear constraints:

$$u^1_{ijj'} \leq y_{ij} \qquad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.25}$$

$$u^1_{ijj'} \leq M_{ij}(1 - \alpha_{j'}) \qquad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.26}$$

$$y_{ij} - u^1_{ijj'} \leq M_{ij} \alpha_{j'} \qquad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.27}$$

$$\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} y_{ij} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'} u^1_{ijj'} = g_{ij} u^2_{ij} \qquad \forall i \in \mathcal{I}^{\mathrm{BA}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.28}$$

$$\sum_{j' \in \mathcal{J}_i} g_{ij'} u^1_{ijj'} = g_{ij} u^2_{ij} \qquad \forall i \in \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i \tag{2.29}$$

$$\delta_{ij} \leq 1 - \alpha_j \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.30}$$

$$\delta_{ij} \leq 1 - \beta_i \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.31}$$

$$\delta_{ij} \geq 1 - \alpha_j - \beta_i \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.32}$$

$$u^2_{ij} \leq x_i \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.33}$$

$$u_{ij}^2 \leq M_{ij}\delta_{ij} \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.34}$$

$$x_i - u_{ij}^2 \leq M_{ij}(1-\delta_{ij}) \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.35}$$

$$\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} y_{ij} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'} u_{ijj'}^1 = g_{ij} u_{ij}^3 \qquad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.36}$$

$$u_{ij}^3 \leq x_i \qquad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.37}$$

$$u_{ij}^3 \leq M_{ij}(1-\alpha_j) \qquad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.38}$$

$$x_i - u_{ij}^3 \leq M_{ij}\alpha_j \qquad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.39}$$

$$\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} y_{ij} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'} u_{ijj'}^1 = g_{ij} u_i^4 \qquad \forall i \in \mathcal{I}^{\mathrm{BA}}, j \in \mathcal{J}_i^{\mathrm{F}} \tag{2.40}$$

$$u_i^4 \leq x_i \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \tag{2.41}$$

$$u_i^4 \leq M_{ij}(1-\beta_i) \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \tag{2.42}$$

$$x_i - u_i^4 \leq M_{ij}\beta_i \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \tag{2.43}$$

$$\sum_{j' \in \mathcal{J}_i^{\mathrm{F}}} g_{ij'} y_{ij} + \sum_{j' \in \mathcal{J}_i^{\mathrm{NF}}} g_{ij'} u_{ijj'}^1 = g_{ij} x_i \qquad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{F}} \tag{2.44}$$

- The model can be tightened by adding new constraints that allow certain feasible solutions to be cutoff of its linear relaxation without eliminating any other feasible solution from the original model:

$$\delta_{ij} = \sum_{k \in \mathcal{K}^j} \sigma_{ijk} \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.45}$$

$$\sigma_{ijk} \leq \delta_{ij} \qquad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.46}$$

- Variables' domain:

$$\sigma_{ijk} \in \{0,1\} \quad \forall i \in \mathcal{I}^{\mathrm{L}}, j \in \mathcal{J}_i^{\mathrm{NF}}, k \in \mathcal{K}^j \tag{2.47}$$

$$\delta_{ij} \in \{0,1\} \quad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.48}$$

$$u_{ijj'}^1 \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I} \setminus \mathcal{I}^{\mathrm{A}}, j \in \mathcal{J}_i, j' \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.49}$$

$$u_{ij}^2 \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I}^{\mathrm{BA}} \cup \mathcal{I}^{\mathrm{NS}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.50}$$

$$u_{ij}^3 \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I}^{\mathrm{S}}, j \in \mathcal{J}_i^{\mathrm{NF}} \tag{2.51}$$

$$u_i^4 \in \mathbb{R}_0^+ \quad \forall i \in \mathcal{I}^{\mathrm{BA}} : \mathcal{J}_i^{\mathrm{F}} \neq \emptyset \tag{2.52}$$

2.6 Computational experience

The model described above has been tested by using real-world data taken from a case involving an international company interested in knowing the best management policy

Category of customers	$\mathcal{I}^{\mathrm{A}}$	$\mathcal{I}^{\mathrm{BA}}$	$\mathcal{I}^{\mathrm{NS}}$	$\mathcal{I}^{\mathrm{S}}$
Distribution	63.0	12.5	19.6	4.9

Table 2.1: Distribution (%) of the customers into the different categories.

for its network of stores. The goal of the company is to choose the most appropriate management policy for each store so as to maximise the total profit of the company. The possibility of delocating the stores was considered in conjunction with four different types of management policy.

2.6.1 Case study analysis

The network consists of 20 stores (6 fixed and 14 non fixed). In the initial configuration of the network, the stores belong to any of types A (owned and operated by the company), C (owned by an external dealer but operated by the company) or D (owned and operated by an external dealer). The model can make decisions on the following management policy changes: if the store is managed as Type A, then it could be replaced with Type B; if the store is managed as Type C, then it could be changed to Type D and, finally, the stores under management type D must remain under this management policy or otherwise they are delocated. In our experiments, there is no limit in the number of stores that can be delocated.

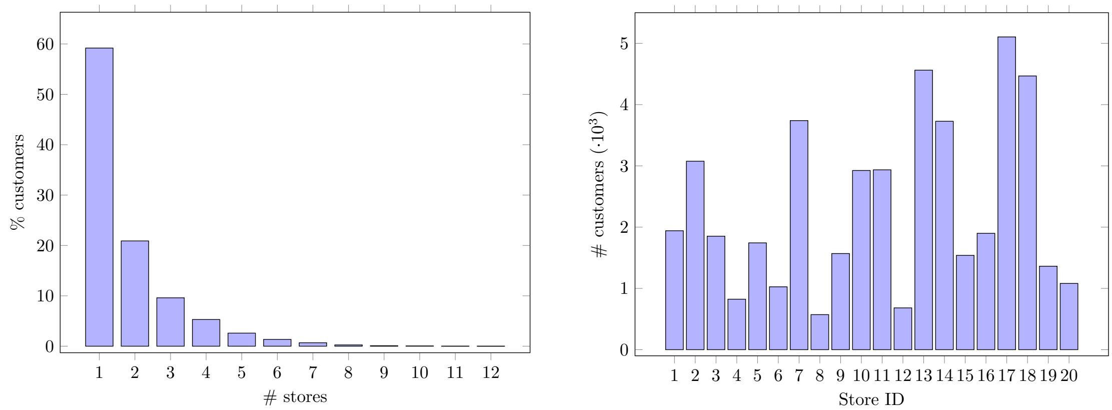
The network serves a number of between 15000 and 20000 customers, each of them consuming in one or more stores which are distributed in a region of approximately 1200 km^2. On average, each customer consumes in 2.1 different stores. Fig. 2.3a shows the percentage of customers in the network consuming in one store, two stores, etc. Note that almost 60% of the customers visit only one store. There are some customers who visit more than 6 stores, although they can be considered outliers. Fig. 2.3b shows the percentage of customers that are served by each store. There are five stores attending a high number of customers (more than 3000), but the other 15 have a number of customers, approximately between 1000 and 3000. Table 2.1 reports the distribution of the customers in the different categories.

In order to guarantee the confidentiality of the data, the initial profit given by the term

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i}\sum_{k\in\mathcal{K}_j} m_{ijk}g_{ij},$$

and the initial amount of goods sold in the stores given by

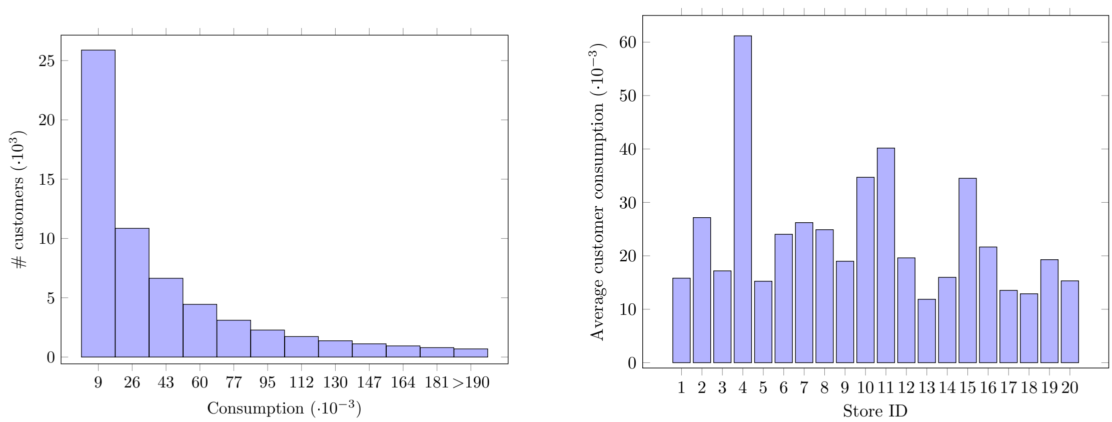
$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}_i} g_{ij}$$



(a) Distribution of the number of stores visited per customer

(b) Number of customers consuming in each store

Figure 2.3: Distribution of customers in the network.



(a) Total consumption per customer

(b) Average customer consumption per store

Figure 2.4: Distribution of consumption.

are set to 1000. Using this number as a reference, the consumption of goods per customer varies depending on the stores, with an average consumption of 0.05 units. Fig. 2.4 shows the distribution of the consumption per store and customer. Fig. 2.4a shows the distribution of consumption of each customer in the stores. It clearly shows an asymmetric distribution, where most of the customers consume very little while there are few customers with a much higher level of consumption. Focusing on the stores, for each of the 20 stores we have calculated the average consumption that each customer consumes in the store; this amount is represented in Fig. 2.4b. Note that stores #4, #11, #12, and #15 have customers with higher levels of consumption (specially store #4), while there are no significant differences among the rest of the stores.

Result	**Value**
Final profit (Obj. Func.)	1262.1
Initial profit	1000.0
Amount of lost sales (%)	28.3
Churn rate (% customers)	19.9
Number of delocated stores	3

Table 2.2: Results of the case study.

The commercial margins are different depending on the customer, the store and the type of management policy, and in some cases they are even found to be negative. Finally, the percentage of customers who have a tendency to leave the network in at least one store is about 75%. This is used to classify the customers in different sets, taking into account their tendency to leave the network and the stores where they consume.

The model has been implemented in the algebraic modelling language AMPL (Fourer et al., 1990) and solved using the Gurobi v.7.0.0 optimiser (Gurobi Optimization, 2020) in a computer with an Intel Core i7-7700HQ, 2.80GHz, 16GB RAM, Xubuntu 16.04 SO. The main characteristics of the proposed solution are reported in Table 2.2.

This solution, which is obtained in 310 secs., implies the delocation of 3 stores with a 26.21% increase in the total profit while the 17 remaining stores sell 28.3% less. Furthermore, there is a 19.9% decrease in the number of customers (churn rate).

Table 2.3 reports more detailed information about the network of stores before and after optimisation: whether the store is fixed or not; management policy; amount of product sold and profit of each store before and after the optimisation process; and the churn rate (percentage of customers of each store leaving the network).

There is evidence that the solution proposed here involves changes in six of the stores. The three stores owned and managed by the company (type A) are put into a dealer operating arrangement (type B), and the three stores of type D (an external dealer both owns and operates the store) are delocated, while the other 14 stores remain in the same situation. Note that the model does not consider the customers re-allocation, that is, if a store is delocated, then its customers either leave the network or continue to purchase in the remaining stores where they were already consuming, however, they do not start to purchase in new stores. In our case, the delocation of three stores implies that between 84% to 91.8% of their customers leave the network, but there are some other customers that continue purchasing in the other stores where they were already consuming.

The distribution of profit that the company obtains at each store before and after the optimisation is shown in Figure 2.5.

Store		Management policy		Consumption		Profit		Churn
ID	Fixed	Initial	Final	Initial	Final	Initial	Final	Rate (%)
1	No	TypeD	TypeD	39.8	36.3	12.2	11.4	11.4
2	No	TypeD	TypeD	108.1	89.9	-55.5	-45.0	15.3
3	No	TypeD	TypeD	41.1	36.6	-23.4	-20.3	15.1
4	No	TypeD	Delocated	65.3	0.0	-56.8	0.0	85.6
5	Yes	TypeD	TypeD	25.3	21.1	-21.2	-16.1	18.4
6	No	TypeD	TypeD	31.9	27.9	-1.4	-1.5	18.4
7	Yes	TypeD	TypeD	59.4	49.9	-36.3	-30.0	9.8
8	No	TypeD	TypeD	18.4	16.6	2.9	3.5	15.9
9	No	TypeD	TypeD	38.6	32.2	-24.0	-19.1	18.9
10	No	TypeA	TypeB	131.5	120.4	394.6	460.0	9.7
11	Yes	TypeD	TypeD	39.5	30.7	2.0	2.7	8.8
12	Yes	TypeC	TypeC	8.8	8.0	31.7	29.5	15.5
13	Yes	TypeC	TypeC	24.3	21.0	103.5	87.7	5.7
14	Yes	TypeC	TypeC	26.3	23.7	100.5	89.3	5.9
15	No	TypeD	Delocated	68.8	0.0	-50.8	0.0	84.0
16	No	TypeD	Delocated	53.3	0.0	-59.4	0.0	91.8
17	No	TypeA	TypeB	89.5	80.7	297.9	308.8	10.6
18	No	TypeA	TypeB	74.6	69.3	246.6	264.5	11.6
19	No	TypeA	TypeB	34.0	31.6	119.6	120.7	11.6
20	No	TypeD	TypeD	21.4	20.8	17.3	16.0	23.1
Network				1000.0	716.7	1000.0	1262.1	19.9

Table 2.3: Characteristics of the stores before and after optimisation.

When comparing the profits before and after the optimisation process, we observe that several stores initially had losses. This is due to the fact that some of their customers had a negative commercial margin. After optimisation, three of the stores with higher losses are delocated (stores #4, #15 and #16) and overall losses are reduced by 60%. However, there are some stores that also have large losses, store #2 for instance, that continue in the network. The explanation behind this is that even though some of their customers do not provide profits in these stores, the profit they provide in other stores where they also consume offset the losses. Therefore, although the model focuses on individual stores, it proposes a reorganisation based on the network as a whole and not only on the stores with losses.

2.6.2 Extended experiments analysis

Since the company is interested in solving the problem in the future, for a larger network covering all the stores, different instances where solved by increasing the number of facilities. Table 2.4 reports the results obtained for the different instances. For each instance, the following data are reported: the number of stores and their initial distribution among the different types (A, B, C and D, fixed, non fixed); the range of the number of customers (in thousands), the number of constraints and variables in the optimisation

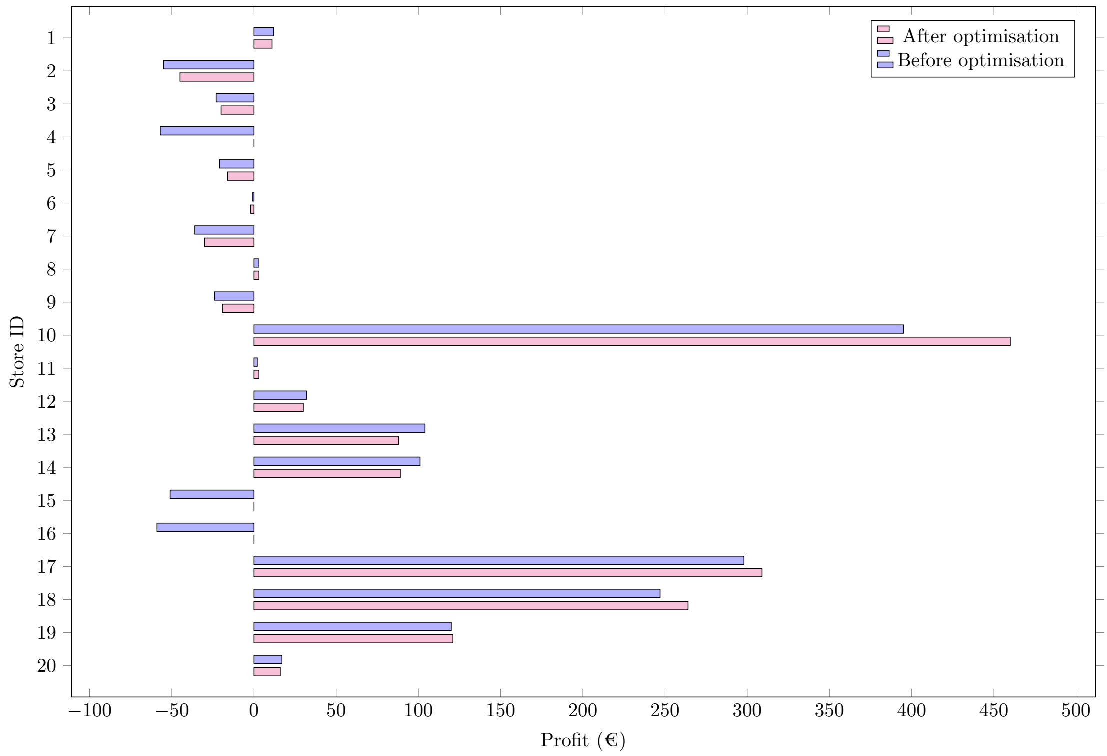


Figure 2.5: Distribution of sales profit in the stores.

model); z_{IP}, the value of the objective function for the optimal solution (as was previously presented, the initial profit for each instance has been set to 1000); the value of the objective function for the linear relaxation of the problem (z_{LP}), the relative gap between z_{LP} and z_{IP} in % (GAP), computed as $100\frac{Z_{LP}-Z_{IP}}{Z_{IP}}$ and the total computing time in seconds. Furthermore, the variation (in %) on profit and consumption of the solution is reported as well as the percentage of customers that leave the network (churn rate) and the number of stores that are delocated (all of them initially were type D).

The solution obtained includes the following changes: all A-stores are changed to B-stores, B and C-stores remain under the same management policy and D-stores remain under the same management policy or they are delocated (see the last column in Table 2.4).

The computational time needed to obtain the optimal solution increases as the number of stores increases. The largest instance solved in a reasonable computing time (less than 24 hours, fixed as stopping criterion) has 35 stores. Hence, the model faces a problem when considering larger areas of the network. It should be noted that the higher the dimensions of the problem, the greater the gap between the optimal solution for the continuous linear relaxation and the optimal solution of the problem (see GAP in Table 2.4). This illustrates the complexity involved in solving large instances. Therefore, other alternative lines of

	# Stores	#Custm. (in 1000)	#cons.	#vars.	Z_{IP}	Z_{LP}	GAP (%)	time (s)	Profit Var. (%)	Consump. Var. (%)	Churn Rate (%)	#Deloc. Stores
10	(1,0,0,9)	10-15	227750	95774	2858.3	4144.5	45.00	44.31	185.83	-65.42	10.18	5
12	(1,0,1,10)	15-20	273971	114724	2272.7	3526.1	55.15	220.45	127.27	-55.48	8.34	5
14	(1,0,3,10)	20-25	379001	163313	1239.6	1686.4	36.04	867.73	23.96	-48.78	5.75	5
16	(1,9,3,12)	20-25	429712	183070	1408.3	2023.9	43.72	692.23	40.83	-56.40	6.46	7
18	(3,0,3,12)	20-30	589940	251692	1224.6	1863.4	52.17	3687.44	22.46	-22.48	1.61	3
20	(4,0,3,13)	20-30	638280	271088	1199.9	1863.2	55.28	4714.70	19.99	-21.89	1.48	3
22	(5,0,3,14)	20-30	710482	304345	1233.5	2038.6	65.27	7664.40	23.35	-20.28	1.29	3
24	(6,0,3,15)	20-30	804612	337805	1259.8	2036.3	61.64	7951.16	25.98	-6.55	0.23	1
26	(6,7,4,16)	20-30	864425	360838	1263.0	2177.2	72.38	10279.33	26.30	-15.06	0.70	2
28	(7,1,4,16)	20-30	942775	392050	1265.1	2158.3	70.61	12173.89	26.51	0.00	0.00	0
30	(8,2,4,16)	30-35	1079980	445472	1246.7	2154.8	72.84	13246.83	24.67	0.00	0.00	0
35	(11,3,4,17)	30-35	1342280	547812	1224.9	2247.4	83.48	25757.67	22.49	0.00	0.00	0
40	(12,4,5,19)	30-35	1416861	653290	*	2453.8	*	16693.94				

(*) Out of memory

Table 2.4: Performance of the model for different instances.

research must be investigated in order to deal with problems of this kind.

2.7 Conclusions and future research

A model to address the problem of redesigning the network of stores of a retail chain has been presented in this chapter. The model aims to decide whether the stores should be removed from the network, or whether the type of management policy applied to some of these stores should be modified. Under the proposed model, simultaneous consideration is given to: the delocation of facilities, changes in management policies, and the restructuring of the network. In addition, customer behaviour is also considered. This behaviour is based on the type of customers: major customers and the remaining customers who usually have a loyalty card. These customers can decide whether to continue consuming in the retail chain on the basis of their behaviour when some of the stores are closed. Moreover, it is possible to adapt this model to different business sectors by considering a network with retail facilities and agreements with their customers.

The model has been tested using a network with real data. In this particular instance the total profit increased by more than 20% and the percentage of non-fixed delocated stores was around 40%. At the same time, it tends to delocate the stores with negative profits as long as it does not cause too many customers to leave. Moreover, it also allows a change in the management policy whenever it brings higher profits to the company.

Our approach has also been tested in different instances, by increasing the number of stores considered. The model achieved results considering up to 35 stores in a reasonable computational time for these types of decisions (less than 24h).

From a business point of view, the application of this model will provide a better management of the network and therefore increase the company's profits. It will also provide a better knowledge of the network of stores facilitating the decision making process.

In order to be able to solve larger instances, a future line of research would be to implement a metaheuristic approach as for example evolutionary algorithms. In this case, we would consider each store as an individual and the objective function $\mathcal{F}$ would act as the fitness function.

From a business perspective it would be interesting to perform a sensitivity analysis to obtain a clearer understanding of the effect of each of the model's parameters on the final configuration of the network.

Chapter 3

The slitting problem in the steel industry

Contents

3.1 Introduction

The previous chapter introduced a strategic planning problem and a mathematical optimisation model to facilitate the decision making process and provide a better knowledge of the network of stores of a retail chain. In this chapter, a mathematical methodology for operational planning is presented. Within this context, the chapter proposes a mathematical optimisation model to solve the slitting problem in *Cortichapa*, a Spanish steel manufacturing company which is part of *Comercial de Laminados* group.

The slitting problem occurs when large width steel coils are slit into narrower coils, known as strips, to meet the requirements of the customers. A major challenge is defining a cutting plan to fulfil all these requirements, as well as ongoing operational constraints and customer demands. As part of the cutting plan, the coils to be used and their cutting patterns are decided. The company looks for a reduction of the leftovers generated in the entire process, while maximising the overall accuracy of the orders. These leftovers may be used in the future as part of new orders provided they are able to respond to specific requirements, or otherwise they are discarded and considered as scrap.

This chapter introduces a mixed integer linear optimisation model to respond to the specific slitting problem of *Cortichapa*. The model is validated with real data and it outperforms the results obtained by the company in different ways: by adjusting the orders that are to be served, by reducing the amount of scrap and by using the retails for future orders. Furthermore, the model is solved in only a few minutes, while the company needs several hours to prepare the scheduling in the current planning process.

The findings of this chapter have been published in:

Sierra-Paradinas, M., Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (2021), 'An exact model for a slitting problem in the steel industry', *European Journal of Operational Research* **295**(1), 336-347. doi:10.1016/j.ejor.2021.02.048.

The remaining part of this chapter is organised as follows. Section 3.2 presents some literature review related to the slitting problem in the steel industry and in the field of Operations Research. Section 3.3 introduces and details the problem under study. Section 3.4 presents the mixed integer linear optimisation model proposed to solve the problem. In Section 3.5, an extensive computational experience, based on a real-world situation, is introduced. Finally, Section 3.6 provides the conclusions and future research.

3.2 Literature review

3.2.1 The slitting problem in the steel industry

Cutting processes are an essential part of the operations carried out in steel manufacturing companies and, therefore, need to be efficiently planned. On a worldwide basis, the steel industry plays an important role. In 2020, steel mills produced 1,878 million tonnes of crude steel, 7.4% of which was produced within the European Union and 56.7% within China. In 2020, the largest producer in the world was China Baowu Group with an annual tonnage of 115 million tonnes, followed by ArcelorMittal with an annual tonnage of 78 million tonnes (World Steel Association, 2021).

The steel industry needs to respond to emerging challenges in terms of its processes, with optimisation methods being considered as an essential tool for the continuous improvement of the production processes and how they are managed (Mukherjee and Ray, 2006). Applications of optimisation methods in the steel industry can be seen in Haessler (1978), Ferreira et al. (1990), Vasko et al. (1992), Valério de Carvalho and Rodrigues (1995), Dutta and Fourer (2001) and Santos et al. (2018). In particular, the slitting problem has been frequently investigated, such as in Coffield and Crisp (1976), Haessler (1978), Sarker (1988), Sweeney and Haessler (1990), Ferreira et al. (1990), Vasko et al. (1992) and Valério de Carvalho and Rodrigues (1995).

3.2.2 The slitting problem in OR

In the field of Operations Research, the problem of laying out cutting patterns has been tackled through the family of Cutting Stock Problems (CSPs). The CSP, as stated in Ben Amor and Valério de Carvalho (2005), consists in determining the most effective way to cut a set of large objects into smaller ones. The CSP has been broadly studied in the literature and has been also known as Trim Loss Problems (TLP) (see Dyckhoff et al. (1985)). Kantorovich (1960) and Eisemann (1957) are responsible for the fundamental works of the CSP. Kantorovich's work, first published in 1939, introduces the first mathematical formulation for the *one-dimensional* TLP. Eisemann's work defines the TLP for rolls of material and proposes a linear programming model to solve this problem. A few years later, Gilmore and Gomory (1961) introduced a method based on duality with the aim of determining all possible cutting patterns, overcoming the difficulty of dealing with a large number of variables. This method used to solve the original CSP is considered as the classical approach. Consequently, these authors extended and adapted the method to the specific trim problem (Gilmore and Gomory, 1963) and to the *two-dimensional* Cutting Stock Problem (2D-CSP) (Gilmore and Gomory, 1965). Later, in Valério de Carvalho (1998) a new formulation is introduced for the *one-dimensional* Cutting Stock Problem

(1D-CSP) and an exact solution is found using column generation and Branch-and-Bound techniques. On the other hand, there have been a number of alternative approaches to solve the different variants of the problem (see Hinxman (1980) and Delorme et al. (2016)). Also an extensive revision of linear programming models for bin packing and cutting stock problems can be found in Valério de Carvalho (2002).

An initial, simplified version of the problem presented in this chapter can be viewed more as a generalisation of the classical 1D-CSP. The coils held in stock have a specific width and external diameter, which are not necessarily the same for all of them, in other words, the stock is heterogeneous. These coils are then cut to produce strips to match the widths requested by the customers, taking into account that the number of knives used in the cutting process is limited. In the same way as in the 1D-CSP, the model should include constraints to ensure that the maximum number of slits and that the total width of the coil are not exceeded. Considering the cutting and packing typology described in Dyckhoff (1990), this first simplified version of the problem can be classified as *1/V/D/M*, where *1* refers to a *one-dimensional* problem, *V* indicates that all of the items requested should be allocated to a set of large objects, *D* indicates that the large objects available may be of different sizes, while *M* means that there are many items and different sizes.

However, in the case of our problem, the orders are not based on the number of strips required, but rather by the total weight to be served for a specific strip width. Since not all coils share the same diameter, the weight of a strip will depend on the selected coil from which it is obtained. Therefore, it is impossible to know in advance the number of strips needed to meet the demand. It should be noted that if all coils share the same diameter, the transformation between the total weight and number of strips could be performed and we would be facing a 1D-CSP. CSPs with two relevant dimensions, where one is fixed (the width of the strips) and the other is variable (the weight of the strips) are named *one-and-a-half-dimensional* CSP (1.5D-CSP) (Hinxman, 1980). This type of problem also differs from the 2D-CSP, where rectangles of a fixed length and width are cut from the rectangular stock. Examples of 2D-CSP can be found in Beasley (1985) and Hifi and M'Hallah (2006).

Several authors have considered different versions of 1.5D-CSP. Haessler (1978) dealt with the 1.5D-CSP in the metal industry. They assumed that each selected coil should be completely processed and proposed a heuristic procedure that would sequentially satisfy the requirements of each order, while controlling both trim losses and slitter changes. Saraç and Özdemir (2003) proposed a genetic algorithm to solve a multi-objective mathematical model for the 1.5D assortment problem. Gasimov et al. (2007) presented a 1.5D cutting stock and assortment problem which involved determining the number of different widths of the rolls in stock and the cutting patterns used. They propose a new multi-objective mixed integer linear programming model and an equivalent nonlinear version. A detailed survey of 1.5D-CSP's from 1965 to 1990 can be found in Sweeney and Paternoster (1992).

The problem addressed in this chapter also includes additional differences relative to the 1D-CSP. These include certain requirements, which should be fulfilled by the products that are ordered, such as the maximum diameter allowed for the strips being supplied. In many instances, the maximum diameters allowed for the orders are smaller than the diameter of the coils in stock. When this occurs, slitting the coils is not sufficient to meet these requirements so additional crosscuts are necessary to reduce the diameter. These crosscuts are guillotine cuts that cross the entire width of the coil from one edge to the other. This reduces the diameter of the coil and the strips obtained from it, to either by half or by a third and so forth, depending on the number of crosscuts performed. Although the problem considers crosscuts, it cannot be regarded as a 2D-CSP, since the length of the strips is not pre-set and it implies a continuous decision variable in the mathematical model.

Another important characteristic of our problem that differentiates it from the classic CSP, is the assortment of large objects. The coils held in stock are strongly heterogeneous. According to the extended typology of Wäscher et al. (2007), our problem can be classified as a residual cutting stock problem (RSCP). Gradišar et al. (1999) argue that a traditional pattern-oriented approach is possible only when the stock is of the same size or of several standard sizes, thus inappropriate for this type of problem. There is a need to use item-oriented approaches, which are characterised by treating each item to be cut, individually. Therefore, we propose an item-oriented solution based on a mixed integer programming model. In this regard, other item-oriented approaches have been studied in the literature. For example, Gradišar et al. (1999) proposes a Sequential Heuristic Procedure to solve the problem of reducing trim losses in one-dimensional stock cutting, when all stock lengths are different.

As a consequence of the manner in which the coils are processed and cut, there is a wide variety of sizes held in stock. When the coils are cut into the exact number of strips required, this may result in a large number of strips that are not assigned to any specific order (these are the leftovers of the cutting process). As stated in Tomat and Gradišar (2017) the existence of leftovers is common during the cutting process. If they are greater than a certain threshold, they are considered as usable leftovers (UL) and are returned to the stock to be used for future orders. This problem is known as the Cutting Stock Problem with Usable Leftovers (CSPUL) (Cherri et al., 2009). In this respect, several papers have previously dealt with UL. In Coelho et al. (2017) the possibility of generating standard pieces from the leftovers during the cutting process, is considered to reduce waste material. In Abuabara and Morabito (2008) two mixed integer programming formulations are presented to deal with a one-dimensional cutting stock problem that arises in the manufacturing of agricultural light aircraft. Both models minimise the total trim loss considering the possibility of generating leftovers for future reuse. As in the CSP, the CSPUL can be classified as 1D, 1.5D and 2D. Cherri et al. (2012) consider the 1D-CSPUL where the UL are kept for future use, prioritising the use of leftovers compared to the standard pieces held in stock. A survey on the 1D-CSPUL can be seen in Cherri et al. (2014). Examples of the 2D-CSPUL are found in Andrade et al. (2014) and Birgin et al. (2019). Therefore, we consider the leftovers of the process to be usable if specific conditions are met and are preferred to as opposed to obtaining scrap.

The underlying problem studied in this chapter can be classified as a 1.5 dimensional residual cutting stock with usable leftovers (1.5D-RCSPUL), where crosscuts are permitted to reduce the diameter of the resulting strips and knives constraints are considered. The stock consists of coils of different sizes containing leftovers from previous cutting processes. We have developed a mixed integer linear optimisation model for this problem taking into account two main goals, which are related to the leftovers generated in the process, and the precision carried out when serving the customer's orders. Insofar as we are aware, no such problem has been previously discussed. Furthermore, our methodology has been validated by the company that proposed the problem, and it has significantly improved its current planning operation.

3.2.3 Summary of references

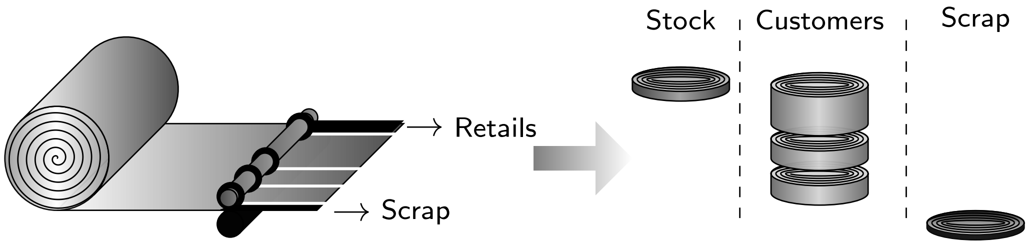
Table 3.1 presents a summary of the main characteristics and solution methods studied in the references that have been cited. In the table, the type of industry is specified only where the papers mention it, the rest could be applied to a wide range of them. The check mark "✓" refers to a defined problem characteristic. It should be noted that the problem characteristic is undefined for the cells marked with "-". Finally, in the Modelling approach and Solution approach columns, MILP/MINLP represent mixed-integer linear/nonlinear programming; ILP represents integer linear programming; LP represents linear programming; DP represents dynamic programming and "Survey" indicates that it is a survey type research.

3.3 Problem description

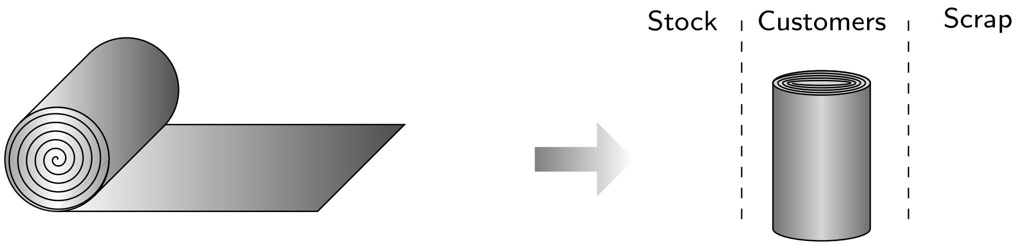
The production planning of steel strips is mainly based on the customer demand. Customers place orders for the strips, by specifying a certain width and a total weight. A combination of knives is set in the slitting machine (one knife more than the number of strips obtained). The strips are slit from the coils in stock which, besides new coils, also include leftovers from older cutting processes. Once a coil is selected to be processed, it

				Characteristics				
Literature	Dimension	Industry	Slitting	Two-stage	Usable leftovers	Residual stock	Modeling approach	Solution approach
Eisemann (1957)	1D	-	-	-	-	-	LP	General LP solver
Kantorovich (1960)	1D	-	-	-	-	-	LP	Method of resolving multipliers
Gilmore and Gomory (1961)	1D	-	-	-	-	-	ILP	Column generation
Gilmore and Gomory (1963)	1D	Paper	-	-	-	-	ILP	Column generation
Gilmore and Gomory (1965)	2D	-	-	-	-	-	ILP	Column generation
Coffield and Crisp (1976)	1D	-	✓	-	-	-	LP	LP solver (MPS/360)
Haessler (1978)	1.5D	Metal	✓	-	-	-	MILP	Heuristic (SGP)
Hinxman (1980)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Beasley (1985)	2D	-	-	-	-	-	DP	Dynamic-progr. and heuristic
Dyckhoff et al. (1985)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Sarker (1988)	1D	-	✓	-	-	-	DP	Dynamic progr.
Dyckhoff (1990)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Ferreira et al. (1990)	1.5D	Steel	✓	✓	-	-	MINLP	Heuristic
Sweeney and Haessler (1990)	1D	Paper	✓	-	-	-	ILP	Two-stage heuristic
Sweeney and Paternoster (1992)	1D,1.5D,2D	-	-	-	-	-	Survey	Survey
Vasko et al. (1992)	1.5D	Steel	✓	-	-	-	MILP	Heuristic (MINSET)
Valério de Carvalho and Rodrigues (1995)	1.5D	Steel	✓	✓	-	-	LP	Column generation
Valério de Carvalho (1998)	1D	-	-	-	-	-	ILP	Column generation and B&B
Gradišar et al. (1999)	1D	-	-	-	-	✓	MILP	Sequential Heuristic (SHP)
Dutta and Fourer (2001)	-	Steel	-	-	-	-	Survey	Survey
Valério de Carvalho (2002)	1D	-	-	-	-	-	LP	Survey
Saraç and Özdemir (2003)	1.5D	-	-	-	-	-	MILP	Implicit enum. / genetic heuristic
Hifi and M'Hallah (2006)	2D	-	-	-	-	-	-	Heuristic (SGA)
Mukherjee and Ray (2006)	-	Metal	-	-	-	-	Survey	Survey
Wäscher et al. (2007)	1D,2D	-	-	-	-	✓	Survey	Survey
Gasimov et al. (2007)	1.5D	Corrugated box	✓	-	-	-	MILP, MINLP	Conic scalarization
Abuabara and Morabito (2008)	1D	Metal	-	-	✓	-	MILP	General LP solver
Cherri et al. (2009)	1D	-	-	-	✓	✓	-	Residual heuristics
Cherri et al. (2012)	1D	-	-	-	✓	✓	ILP	Heuristic (RGR)
Andrade et al. (2014)	2D	-	-	-	✓	✓	MILP	MIP solver (CPLEX 12.1.0)
Cherri et al. (2014)	1D	-	-	-	✓	-	Survey	Survey
Delorme et al. (2016)	1D	-	-	-	-	-	Survey	Survey
Coelho et al. (2017)	1D	-	-	-	✓	✓	ILP	Two residual heuristics
Tomat and Gradišar (2017)	1D	-	-	-	✓	✓	MILP	Heuristic search process
Santos et al. (2018)	1D	Steel	-	-	-	-	MILP	Two heuristics
Birgin et al. (2019)	2D	-	-	✓	✓	✓	MILP	MIP solver (CPLEX 12.8.0)
This research	1.5D	Steel	✓	-	✓	✓	MILP	MIP solver (GUROBI 9.0.2)

Table 3.1: Literature review.



(a) Slit coil producing different strips



(b) No slits performed

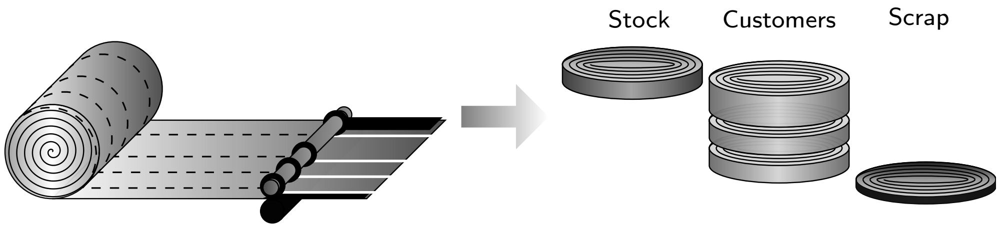
Figure 3.1: Different possibilities to allocate orders to coils.

is unwound in the slitting machine, which is equipped with a variable and limited number of knives to perform the corresponding cuts. While the slitting process is carried out, the coil is rewound, and a set of strips is obtained.

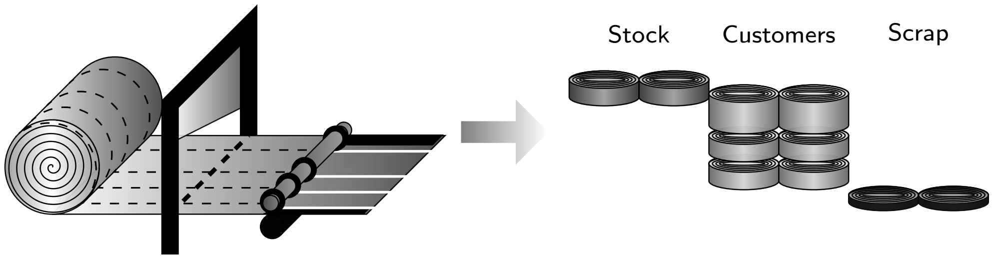
Figure 3.1a represents an unwound coil, where four knives have been set into the slitting machine to obtain three different strips, which is represented by the grey area. Knives at both extremes are necessary in order to keep the coil uniform, therefore, a minimum edge trim will always be required. The material on the outer sides of the extreme knives, which is illustrated by the black area in the figure, represents the leftovers of the process. These leftovers are usable and kept in stock when their width and weight are greater than a certain threshold. Whenever this occurs they are referred to as retails, otherwise, the leftovers are considered as scrap. When the width of the order perfectly matches the width of the coil, the coil is served as a complete strip so edge trimming is not required, since no slits are performed, as shown in Figure 3.1b.

3.3.1 Restrictions of the customers

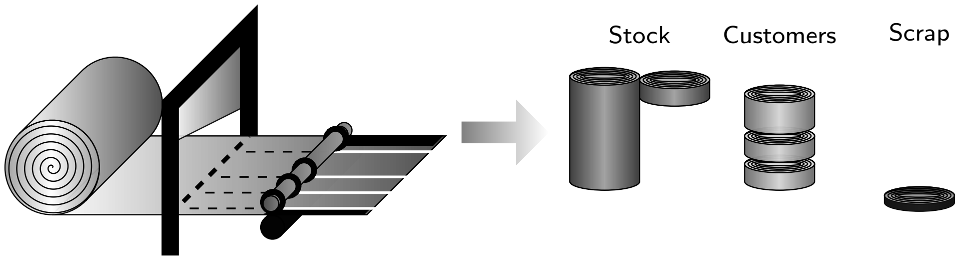
In the case of this company, there are particular restrictions imposed by the customers that affect the way their orders need to be served. These restrictions include the maximum weight of the strip and the maximum external diameter allowed. It should be noted that given some properties such as width, thickness, density and internal diameter of the strip, the weight of a strip can be obtained from its external diameter. Hence, both requirements can be given in terms of a maximum allowed weight for the strips where the most restrictive



(a) Strips obtained without crosscuts



(b) Strips obtained when a crosscut is made and the slitting process continues



(c) Strips obtained when a crosscut is made and the slitting process is stopped

Figure 3.2: Example of strips obtained.

one is considered. For each order, the maximum allowed weight for the strips, as well as the size of the coil, would define the number of strips served.

In order to reduce the diameter, and consequently the weight of the strips to meet the customers' requirements, it is possible to make one or more guillotine crosscuts with a shear blade. These cuts affect all the strips assigned to the coil and must ensure that each resulting strip has a certain weight which lies within the limits set by the customer. These limits may vary from customer to customer. For this reason, it is worth noting that the knives cannot be redirected after a crosscut remaining the cutting pattern the same. Figure 3.2 presents an example of how a crosscut reduces the external diameter or weight of the finished strips. In Figure 3.2a the external diameter of the finished strips is greater than those in Figure 3.2b where a crosscut has been performed.

Each time a crosscut is carried out, a decision must be made on whether to continue cutting or stop the cutting process. In the latter case, the remaining part of the coil is rewound and kept in stock for future cutting processes. It should be noted that this partial rewound process produces coils in stock with a variety of external diameters (see Figure 3.2c).

3.3.2 Compatibility of stock and orders

An additional feature that complicates the problem lies in the quality requirements requested by the customer, regarding the product being served. The company handles more than 20 parameters that define the technical characteristics of each coil, the type of material, its thickness, etc. For each of these parameters, the customer requests specific values and, in some cases, small tolerances are allowed. For example, the type of material has to match both the order and stock. However, if the order requires a thickness of 2 mm with a tolerance of 0.1 mm, the thickness of the stock could range between 1.9 mm and 2.1 mm. These requirements should be considered in order to determine the set of compatible coils for each order. Nevertheless, since the tolerances are so small, the costs involved for the company are fairly insignificant.

These compatibility requirements create a problem that is more difficult to solve, as the same coil can be used to serve orders with different characteristics. As a result, the problem cannot be easily separated into several sub-problems. An illustrative example is shown in Figure 3.3, where any R type coil is compatible with order 1, but not with order 2. However, the M-coil is compatible with both orders 1 and 2. This situation can arise, for example, when customers allow their orders to be served with better quality products than requested, or with a certain thickness tolerance. It should be noted that this characteristic introduces a new difficulty: an order can be served with coils of different densities or weights per linear metre. Therefore, converting weight to length on a general basis for all coils is not possible and has to be carried out for each coil individually.

3.3.3 Cutting patterns

A number of cutting scenarios may arise, resulting in various types of cutting patterns, which are shown in Figure 4. With regard to slitting the coil, two situations may occur: (A) the strip width matches the coil width and, (B) the strip width is narrower than the coil width. In case A, the coil is not slit while in case B, the coil is slit and at least two knives are needed for the slitting machine to perform the cuts. In both cases, A and B, the coil can be fully used lengthwise (cases A1, A2, B1 and B2 in Figure 3.4) or partially used (cases A3 and B3 in Figure 3.4). In cases A3 and B3, the remaining part without slits is

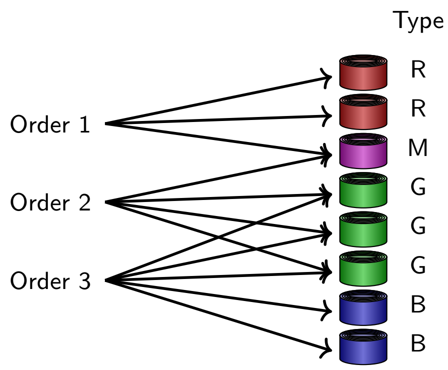


Figure 3.3: Illustrative example of compatibility between orders and stock.

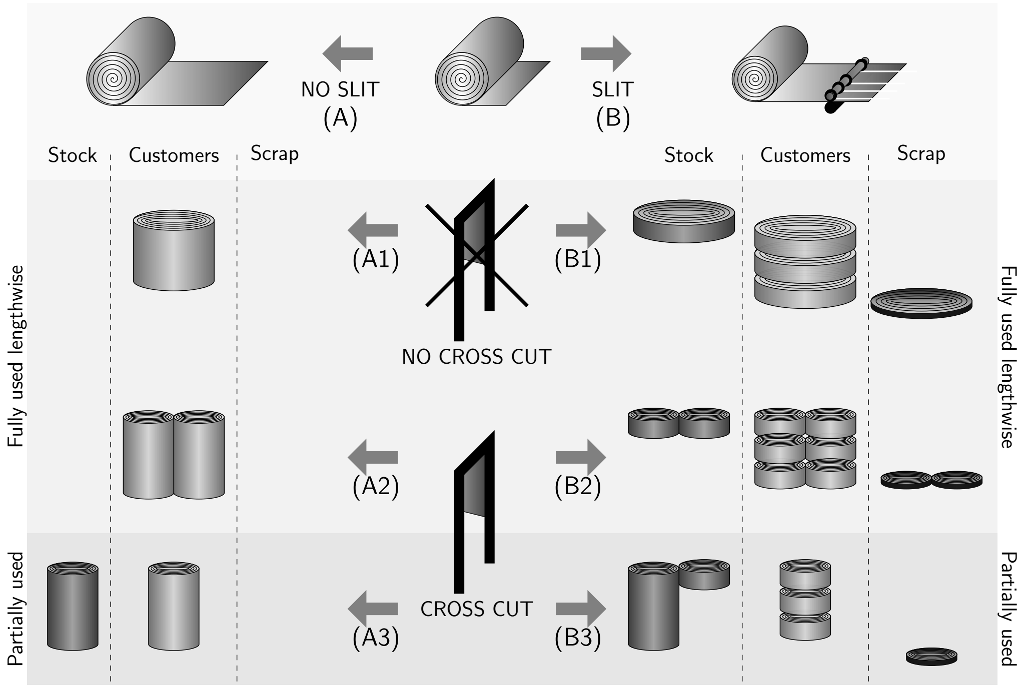


Figure 3.4: Different cutting patterns considered.

rewound and restocked. One or more guillotine crosscuts may be performed in order to meet the maximum strip weight requirements (cases A2, A3, B2 and B3).

3.3.4 Goals

The company pursues the following goals, namely:

- Maximise the utilisation of each coil.

  The company currently uses about 50-60% of the weight of the coils to meet demand. The leftovers of the cutting process are 1) discarded if their weight or width are not

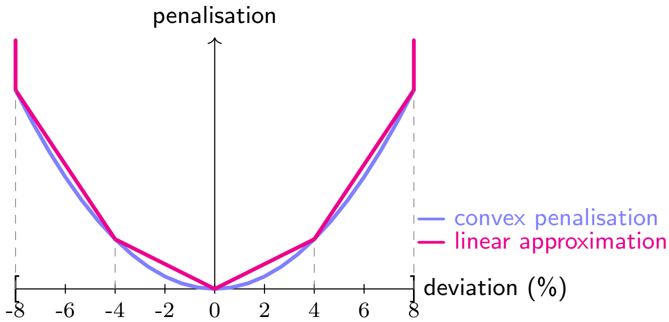


Figure 3.5: Penalisation of deviation from actual weight for each order.

large enough; 2) stocked for future use; or 3) used to prepare strips for expected future orders. Based on past record data, the company can forecast future demand and then seek to make efficient use of the unused pieces by anticipating future demand. These three situations imply economic costs to the company that include the generation of scrap, inventory costs and the risk of producing non-demanded strips that would cause an increase in the stock.

In order to improve the current operation process, the model includes a penalisation for the weight of the leftovers. This penalisation is greater for scrap than for retails. It should be noted that the third option, that of anticipating possible future demand, is only carried out by the company as a last option to reduce scrap, and it is not contemplated in the model.

- Adjust the weight served to the real demand.

Owing to the way the company operates, it is very difficult to serve the exact amount of product that each customer has requested. Currently, the company works with a $\pm 20\%$ tolerance; this seems reasonable for the company since most of the customers are regular ones and, therefore, part of the demand can be transferred to other days, usually at a cost in the form of a discount. To reduce this deviation, the objective function includes a penalisation. A non-linear (convex) function is used which results in larger deviations being more penalised than smaller ones. This convex function can be approximated by a piece-wise linear function without the need of binary variables. More specifically, we have considered two intervals for the approximation: up to a certain level, deviations are acceptable as it is rather difficult to meet the exact weight ordered for all of the customers, but for deviations over the maximum *desired* deviation, a greater penalisation is imposed. Either way, a maximum *allowed* deviation is established. Figure 3.5 shows an example, where a deviation up to 4% of the weight ordered is less penalised than a deviation from 4% to 8%. The latter is the maximum deviation allowed.

3.3.5 Problem hypotheses

In summary, the problem under study considers the following hypotheses:

- Customers place their orders by specifying a certain width and total weight. Neither the number of strips nor their weight are predefined.
- All strips supplied should meet the customer's requirements in terms of a maximum permitted weight.
- The existing stock consists of coils with different sizes, and includes the retails from former cutting processes.
- Compatibility requirements imply that different types of coils are considered for a given order.
- Both slits and guillotine crosscuts are considered.
- After a crosscut, the reconfiguration of knives is not allowed. Either the rest of the coil is cut with the same configuration of the knives or it is rewound and kept in stock.
- A minimum edge trim is required when slitting the coil.
- Leftovers are divided into retails (usable leftovers) and scrap.
- Customer's orders have to be fulfilled. A maximum allowed deviation on the weight served with respect to the requested weight is considered.

3.4 Mixed Integer Linear optimisation model

Below, a Mixed Integer Linear optimisation model to solve the problem is presented[1].

3.4.1 Notation

- $\mathcal{O}$, set of customer's orders. The following information is given for each order $o \in \mathcal{O}$:

  w_o, width (m) of strips.

  b_o, required weight (kg).

  $\overline{b}_o$, maximum weight (kg) allowed for each single strip of the order.

  $\overline{u}_o$**,** $\overline{u}_o^{\mathbf{d}}$**,** maximum absolute deviation on the required weight (kg) that is permitted and desired, respectively.

[1]For the sake of clarity, the notation presented in this chapter differs slightly from the one introduced in Sierra-Paradinas et al. (2021).

- $\mathcal{C}$, set of coils in stock. For each coil $c \in \mathcal{C}$ the following data are known:

  W_c, width (m).

  B_c, weight (kg).

  L_c, length (m).

  $\underline{L}_c, \overline{L}_c$, for partially used coils, minimum and maximum length (m) used from the coil, respectively. $\overline{L}_c$ is such that it guarantees that the rest of the coil can be rewound and kept in stock.

  D_c, weight (kg) per m^2. It depends on the density and thickness of the coil.

  k_c, maximum number of knives.

  $\mathcal{J}_c = \{1, \ldots, k_c - 1\}$, index set for the slits performed.

  $\mathcal{O}_c$, set of orders that are compatible with the coil.

- Parameters for operation settings:

  r, minimum edge trim required for quality purposes.

  $\underline{a}, \underline{b}$, minimum width and weight required for a trim waste to be reusable, respectively.

- Objective function:

  q, q^{d} penalisation for the allowed and desired deviation of the weight served with respect to the required weight, respectively. Note: $q > q^{\mathrm{d}}$.

Decision variables

$\beta_c = 1$, if coil c is used, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_c^{\mathrm{T}} = 1$, if coil c is fully used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_c^{\mathrm{P}} = 1$, if coil c is partially used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\alpha_{c0} = 1$, if coil c is used without slitting, 0 otherwise, $c \in \mathcal{C}$.

$\alpha_{cj} = 1$, if the j-th slit is performed in coil c, 0 otherwise, $c \in \mathcal{C}, j \in \mathcal{J}_c : j > 0$.

$\mu_{ocj} = 1$, if order o is assigned to the j-th slit of coil c, 0 otherwise, $c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c$.

$\theta_c = 1$ if trim waste of coil c is reusable, 0 otherwise, $c \in \mathcal{C}$.

δ_c, number of guillotine crosscuts made in the used part of coil c, $c \in \mathcal{C}$. It should be noted that this variable does not count the last crosscut made if the coil is partially used, in other words, it only counts the crosscuts made to assure that the weights

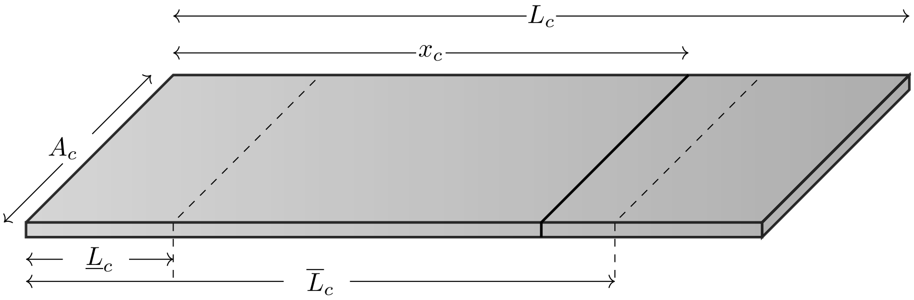


Figure 3.6: Parameters referred to a given coil.

	Fully used lengthwise		Partially used
	$\gamma_c^{\mathrm{T}} = 1, x_c = L_c$		$\gamma_c^{\mathrm{P}} = 1, \underline{L}_c \leq x_c \leq \overline{L}_c$
	(1) No cross-cut	(2) Cross-cut	(3) Cross-cut
(A) No slit	$\alpha_{c1} = 0,\ \delta_c = 0$	$\alpha_{c1} = 0,\ \delta_c > 0$	$\alpha_{c1} = 0,\ \delta_c \geq 0$
(B) Slit	$\alpha_{c1} = 1,\ \delta_c = 0$	$\alpha_{c1} = 1,\ \delta_c > 0$	$\alpha_{c1} = 1,\ \delta_c \geq 0$

Table 3.2: Relationship between variables and the different cutting patterns for a coil c.

of the strips are less than the maximum allowed. Observe Figure 3.2b where $\beta = 1$ and Figure 3.2c where $\beta = 0$.

x_c, used length of coil c, $c \in \mathcal{C}$.

v_{ocj}, length of the strip of order o obtained from the j-th slit made in coil c, $c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c$.

u_o^+, u_o^-, excess and lack of weight served for order o, $o \in \mathcal{O}$.

$u_o^{\mathrm{d}+}, u_o^{\mathrm{d}-}$, excess and lack of weight served for order o within the maximum desired deviation, $o \in \mathcal{O}$.

y_c, width of the leftovers of coil $c \in \mathcal{C}$ which is broken down in the sum of y_c^{r}, for retails and y_c^{s}, for scrap, $c \in \mathcal{C}$.

$z_c^{\mathrm{r}}, z_c^{\mathrm{s}}$, weight of the retails and scrap of coil c, $c \in \mathcal{C}$.

Figure 3.6 shows some elements of the notation on an unwound coil and Table 3.2 indicates how the values of the variables determine the different cutting patterns described in Figure 3.4.

3.4.2 Mathematical formulation

Objective function

The objective function is a weighted sum of three elements:

$$\min \ \omega_1 f_1 + \omega_2 f_2 + \omega_3 f_3, \tag{3.1}$$

where ω_1, ω_2 and ω_3 represent the weights assigned to each component:

f_1: Usable leftovers or retails. There are two kinds of usable leftovers: the rewound coils obtained from the partially used coils and retails that appear when a coil is not fully used widthwise.

$$f_1 = \sum_{c \in \mathcal{C}} (B_c \beta_c - D_c W_c x_c) + z_c^{\mathrm{r}} \tag{3.2}$$

f_2: Scrap. Leftovers whose weight and/or width are below a certain threshold.

$$f_2 = \sum_{c \in \mathcal{C}} z_c^{\mathrm{s}} \tag{3.3}$$

f_3: Difference between the weight served and the weight that is actually ordered by each customer. Two different penalisations are considered: q^{d} for deviation within the desirable limits and q for the excess deviation over these desirable limits.

$$f_3 = \sum_{o \in \mathcal{O}} \Big(q(u_o^+ + u_o^-) + (q^{\mathrm{d}} - q)(u_o^{\mathrm{d}+} + u_o^{\mathrm{d}-}) \Big) \tag{3.4}$$

Constraints

1. Cutting patterns: Constraints (3.5) state, for each used coil, whether it is served with or without slitting. Constraints (3.6) force not to make a slit if the previous slit has not been performed, in other words, introduce an order in the slits. Constraints (3.7) state, for used coils, whether they are completely or partially slit. Constraints (3.8) guaranty that no guillotine crosscuts are performed on unused coils.

$$\alpha_{c0} + \alpha_{c1} = \beta_c \quad \forall c \in \mathcal{C} \tag{3.5}$$

$$\alpha_{cj} \leq \alpha_{cj-1} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c : j > 1 \tag{3.6}$$

$$\gamma_c^{\mathrm{T}} + \gamma_c^{\mathrm{P}} = \beta_c \quad \forall c \in \mathcal{C} \tag{3.7}$$

$$\beta_c \leq G_c \beta_c \quad \forall c \in \mathcal{C}, \tag{3.8}$$

Where G_c is an upper bound of the number of guillotine crosscuts in coil c.

2. Assignment of orders to strips: Constraints (3.9) assign exactly one order to each strip (it should be noted that an order can be assigned to one or more strips). Constraints (3.10) ensure that the width of each coil is not exceeded. Constraints (3.11) ensure a minimum edge trim in slit coils.

$$\sum_{o \in \mathcal{O}_c} \mu_{ocj} = \alpha_{cj} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c \tag{3.9}$$

$$\sum_{j \in \mathcal{J}_c} \sum_{o \in \mathcal{O}_c} a_o \mu_{ocj} + y_c = W_c \beta_c \quad \forall c \in \mathcal{C} \tag{3.10}$$

$$2r\alpha_{c1} \leq y_c \leq W_c \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{3.11}$$

3. Bounds on the used length of the coils: Constraints (3.12) force the used length of the coils to be equal to the total length of the coil when it is completely slit and between the given bounds when it is partially slit.

$$L_c \gamma_c^{\mathrm{T}} + \underline{L}_c \gamma_c^{\mathrm{P}} \leq x_c \leq L_c \gamma_c^{\mathrm{T}} + \overline{L}_c \gamma_c^{\mathrm{P}} \quad \forall c \in \mathcal{C} \tag{3.12}$$

4. Length of the strips ordered: Constraints (3.13) allow to assign a length only to the strips assigned to the orders. Constraints (3.14) guaranty the length of all the strips in a coil to be equal to the used length of the coil.

$$\underline{L}_c \mu_{ocj} \leq v_{ocj} \leq L_c \mu_{ocj} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{3.13}$$

$$0 \leq x_c - v_{ocj} \leq L_c(1 - \mu_{ocj}) \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{3.14}$$

5. Guillotine crosscuts: Constraints (3.15) compute the number of guillotine crosscuts needed in coil c to keep the weight of the strips lower than the maximum allowed for each order assigned to this coil.

$$(D_c w_o) v_{ocj} \leq \overline{b}_o (\beta_c + 1) \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{3.15}$$

6. Demand: Constraints (3.16) –(3.17) compute the lack or excess weight served to each order.

$$\sum_{c \in \mathcal{C}: o \in \mathcal{O}_c} \sum_{j \in \mathcal{J}_c} (D_c w_o) v_{ocj} - u_o^+ + u_o^- = b_o \quad \forall o \in \mathcal{O} \tag{3.16}$$

$$u_o^{\mathrm{d}+} \leq u_o^+, \; u_o^{\mathrm{d}-} \leq u_o^- \quad \forall o \in \mathcal{O} \tag{3.17}$$

7. Leftovers: When leftovers are rewound for further use, they are considered as usable.

Alternatively, the material is discarded and considered as scrap. When leftovers are usable, there is always a weight that represents the minimum edge trim required which is considered as scrap. Thus, the width of the leftovers can be divided into usable (retails) and non-usable (scrap) parts (3.18). In an analogous manner, the weight of the leftovers can also be divided into usable and non-usable parts (3.19). Constraints (3.20)-(3.23) assure that if leftovers are usable, their width and weight should be at least the minimum established. Failure to comply with at least one of these conditions will result in the leftovers as being considered scrap. Finally, constraints (3.24) state that leftovers only appear when the coil is slit.

$$y_c = y_c^{\mathrm{r}} + y_c^{\mathrm{s}} \quad \forall c \in \mathcal{C} \tag{3.18}$$

$$(D_c W_c) x_c - \sum_{o \in \mathcal{O}_c} \sum_{j \in \mathcal{J}_c} (D_c w_o) v_{ocj} = z_c^{\mathrm{r}} + z_c^{\mathrm{s}} \quad \forall c \in \mathcal{C} \tag{3.19}$$

$$\underline{a}\theta_c \leq y_c^{\mathrm{r}} \leq W_c \theta_c \quad \forall c \in \mathcal{C} \tag{3.20}$$

$$\underline{b}\theta_c \leq z_c^{\mathrm{r}} \leq B_c \theta_c \quad \forall c \in \mathcal{C} \tag{3.21}$$

$$r\theta_c \leq y_c^{\mathrm{s}} \leq r\theta_c + \overline{s}_c(1-\theta_c) \quad \forall c \in \mathcal{C} \tag{3.22}$$

$$(D_c r \underline{L}_c)\theta_c \leq z_c^{\mathrm{s}} \leq (D_c r \overline{L}_c)\theta_c + B_c(1-\theta_c) \quad \forall c \in \mathcal{C} \tag{3.23}$$

$$\theta_c \leq \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{3.24}$$

Constant $\overline{s}_c$ in constraints (3.22) are an upper bound of the width of the leftovers of coil c to be considered as scrap. It can be computed as the maximum between $\underline{a}$ and the width corresponding to the leftovers of weight $\underline{b}$:

$$\overline{s}_c = \max\left\{\underline{a}, \frac{\underline{b}}{D_c \underline{L}_c}\right\}$$

8. Variables' domain:

$$\beta_c, \gamma_c^{\mathrm{T}}, \gamma_c^{\mathrm{P}}, \theta_c \in \{0,1\} \quad \forall c \in \mathcal{C} \tag{3.25}$$

$$x_c, y_c, y_c^{\mathrm{r}}, y_c^{\mathrm{s}}, z_c^{\mathrm{r}}, z_c^{\mathrm{s}} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C} \tag{3.26}$$

$$\delta_c \in \mathbb{Z}_0^+ \quad \forall c \in \mathcal{C} \tag{3.27}$$

$$\alpha_{cj} \in \{0,1\} \quad \forall c \in \mathcal{C}, j \in \mathcal{J}_c \tag{3.28}$$

$$u_o^+, u_o^- \in [0, \overline{u}_o] \quad \forall o \in \mathcal{O} \tag{3.29}$$

$$u_o^{\mathrm{d}+}, u_o^{\mathrm{d}-} \in [0, \overline{u}_o^{\mathrm{d}}] \quad \forall o \in \mathcal{O} \tag{3.30}$$

$$\mu_{ocj} \in \{0,1\} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{3.31}$$

$$v_{ocj} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, j \in \mathcal{J}_c \tag{3.32}$$

3.5 Computational experience

This section presents the computational results of the experiments that have been carried out to validate the model proposed above: a comparison with the current operation implemented by the company, a sensitivity analysis varying the weights of the objective function and the limits of the desired deviation, and an extended experiment to asses the limits of the model.

3.5.1 Data description

For the experiments, we have used real data provided by Cortichapa, a Spanish steel manufacturing company interested in improving the planning of steel strip production. The company holds a permanent stock level of about 40,000 tonnes, consisting of approximately 3,000 coils of different sizes and types of products (including retails from previous cutting processes). Table 3.3 reports the range of values for the main characteristics of the coils in stock: their width, thickness, weight and external diameter. The thickness of the coil is directly related to the maximum number of knives that can be configured in the slitting machine (varying from 4 to 18 knives). Furthermore, the company uses around twenty additional parameters to define the compatibility between the orders and coils, such as type of material, quality, mechanical and physical characteristics of the coil.

It should be noted that the unused and processed coils have different characteristics, thus making the problem more difficult to deal with. Fig. 3.7 shows the distribution of widths for both types of coils. It can be observed that the processed coils are much narrower than the unused ones, but in both cases, there is a great diversity of widths in the stock. The company currently designs the patterns manually and, given the difficulty of the problem, it tends to select the unused coils, which are wider and easier to obtain feasible cutting patterns from. These operations gradually lead to an increase in the number of processed coils in the stock.

Concerning the demand, Table 3.4 reports the range of values for the main characteristics of the orders: the width and thickness of the strips, the total amount of weight ordered, and the maximum weight allowed for each strip. The widths of the strips are extremely diverse depending on the order, even for orders from the same customer. For illustrative purposes, Figure 3.8 shows the distribution of the widths of the strips for four different customers, demonstrating the diversity of the demand, even for the same customer. Such diversity leads to the need to lay out completely different cutting patterns, depending on the demand at the time of planning.

Some customers allow a certain degree of tolerance in the thickness; this tolerance can range from 0.05 mm to 0.21 mm. The coils held in stock have an external diameter that

	Min.	Max.
Width (mm)	19	2,000
Thickness (mm)	0.32	5.0
Weight (kg)	40	27,530
External diameter (mm)	600	1,980

Table 3.3: Main characteristics of coils in stock.

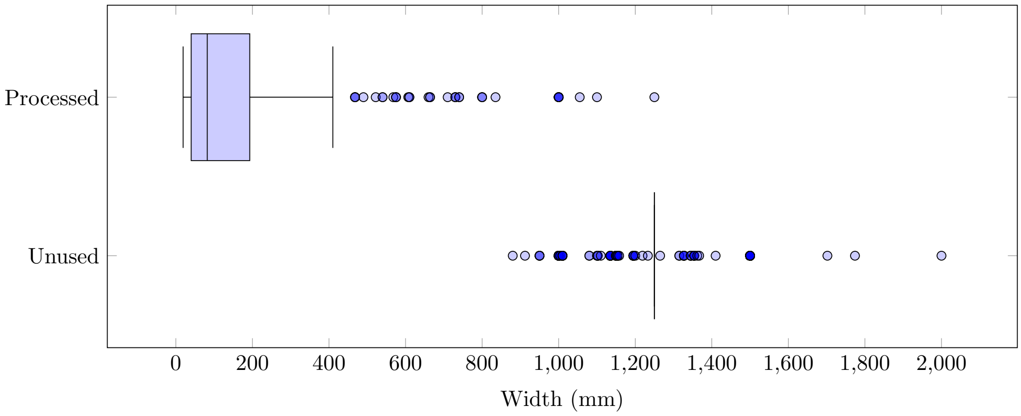


Figure 3.7: Coils width distribution.

	Min.	Max.
Width (mm)	19	1500
Thickness (mm)	0.32	5.0
Total weight (kg)	80	25,000
Max. strips' weight (kg)	120	24,000

Table 3.4: Main characteristics of orders.

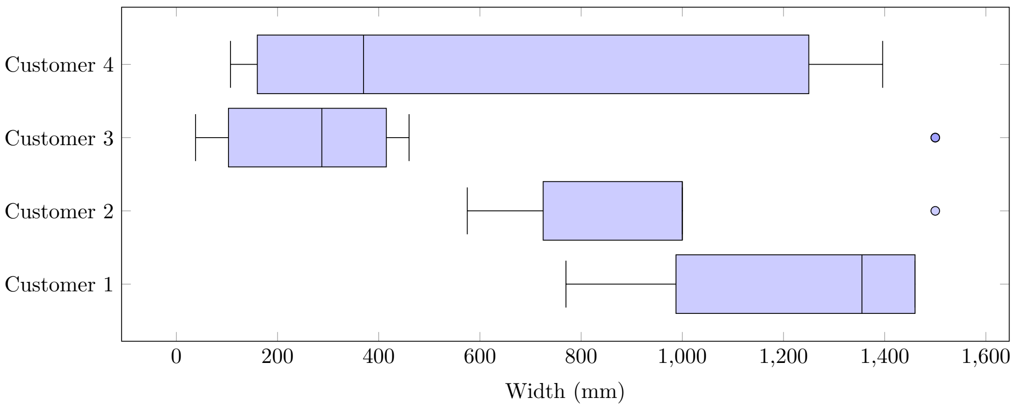


Figure 3.8: Orders' width distribution by customer.

Instance	# orders	Total required weight (t)	# compatible coils	Weight of compatible coils (t)
I01	15	115	380	5,192
I02	7	104	76	1,085
I03	14	83	222	2,972
I04	9	90	175	2,486
I05	14	112	193	2,74
I06	11	97	271	4,024
I07	8	74	65	954
I08	7	92	207	2,027
I09	7	123	154	2,442
I10	13	123	180	1,941
I11	10	61	68	999

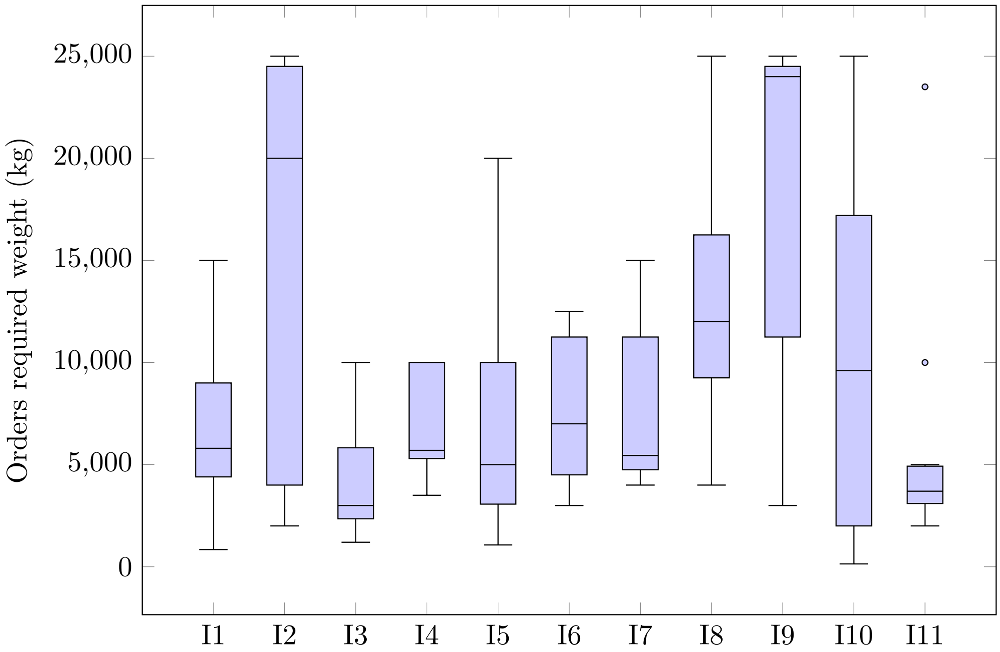
Table 3.5: Main characteristics of instances.

may reach 1980 mm, providing that the maximum diameter established by the customer may be under that value, it would be necessary to include crosscuts to ensure that the strips served do not exceed this limit. Another aspect that may imply crosscuts is the maximum weight per strip that varies from 120 to 24000 kg. Finally, the widths of the coils vary from 19 mm to 2000 mm, while customers demand strips that range from 19 mm to 1500 mm.

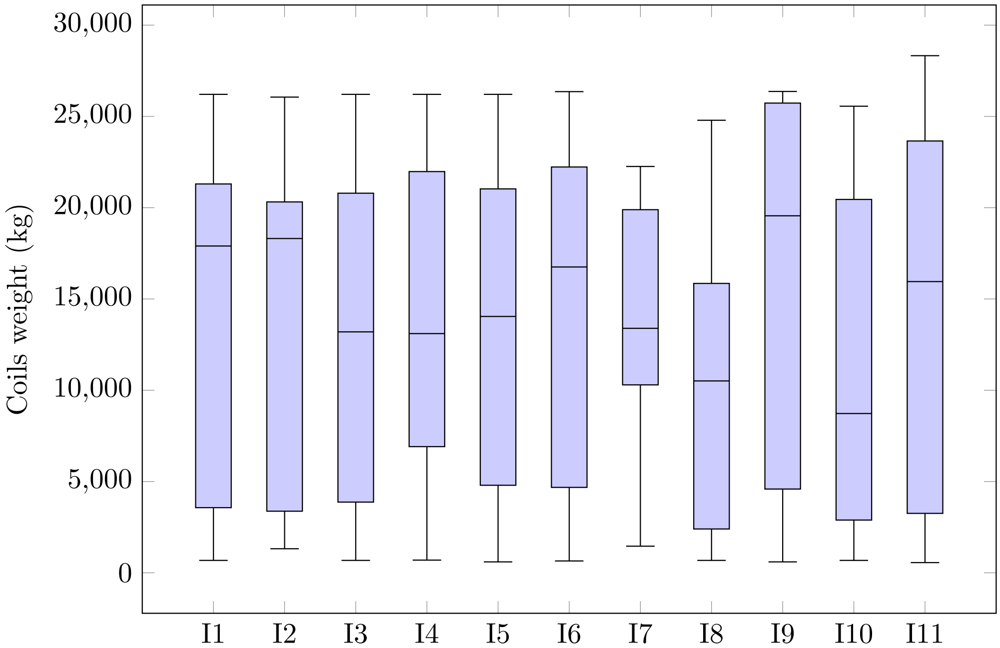
3.5.2 Current operation

For 11 working days in 2019, we were provided with the solution applied by the company which was compared with the one provided by the model. The actual planning of the cutting process takes several hours and very often, there are situations where the company cannot find a way to complete and fulfil all its orders in one day. In other words, the weight served in one day is not within the permitted deviations from the total weight required. Consequently, those orders remain open for the next day. In addition, as mentioned before, the anticipation of future demand is practised by the company (a posteriori) by planning make-to-stock orders to complete the coils used. In order to validate the model, and at the company's request, the records only include confirmed orders that have been completely fulfilled, thereby allowing there to be a comparison between both solutions.

Table 3.5 reports the main characteristics of the instances tested: number of orders, total weight required in tonnes, number of compatible coils in stock and their total weight in tonnes. Additionally, for each instance, Figure 3.9 shows the distribution of the weights of the orders (3.9a) and the distribution of the weights of the compatible coils in the stock (3.9b). It should be noted that instances $I03$ and $I11$ have mainly small orders and little variability, while the variability in the weight of the order is greater for instances $I02$, $I09$



(a) Distribution of weight required



(b) Distribution of coil's weight

Figure 3.9: Distribution of order and compatible coils sizes for the current operation instances.

and $I10$. Regarding the weight of the coil, the instances are more homogeneous.

Table 3.6 reports the main performance indicators of the solution currently implemented in the company. In each instance, the number of coils used in the solution and their total weight in kg are indicated, together with the coils served, the retail weights and the weights of the scrap. In addition, the percentage that each weight represents over the total weight used is also indicated. Leftovers are considered retail when their widths and weights are greater than 100 mm and 500 kg respectively. The *Slit* and *Cross* columns provide the number of slits performed on average per coil, and the total number of guillotine crosscuts made in the solution, respectively. The *Rew.* column provides the number of coils that are rewound. Deviations up to ±20% from the required weight are allowed and, in order to measure how the solution behaves in this matter, we define the order accuracy as the ratio between the required weight and the served weight. In this way, the order accuracy values over 1 indicate that the served weight is more than the required weight. Finally, the last three columns of Table 3.6 report the minimum, average and maximum values of the order accuracy obtained using the current solution established by the company. As an illustrative example of this indicator, in instance 1, at least 84% (resp. 117% maximum) of the required weight was served for all of the orders, the average being 101%.

3.5.3 Experiments

The model has been implemented using the algebraic modeling language AMPL (Fourer et al., 1990) and solved using the MIP Gurobi v.9.0.2 optimiser (Gurobi Optimization, 2020) in a virtual machine managed by OpenStack with 4 CPUs, 8GB RAM, Ubuntu 18.04 OS. The default Gurobi settings are used, except for the time limit, which is set to 10 minutes, unless otherwise stated.

Different experiments have been performed varying the weights of the objective function: minimizing both retail sales and scrap, as well as the deviation from the weight required for each order.

Regarding the first goal, a higher penalisation has been assigned to the scrap than to the retails, while for the second goal; the maximum deviation allowed has been set equal to ±20%, which is the one currently used by the company. In addition to this value, different limits have been tested for the desired deviation: ±20%, ±10% and ±5%. In order to achieve solutions that are adjusted as close as possible to the required limits, deviations above the required limit have been further penalised. In particular, the values used for q and q^{d} are 10 and 1, respectively.

Furthermore, three different weight combinations $(\omega_1, \omega_2, \omega_3)$ have been tested and

	Used coils		Served		Retail		Scrap		Cuts		Rew.	Order accuracy		
Ins.	**#**	**Weight**	**Weight**	**%**	**Weight**	**%**	**Weight**	**%**	**Slit**	**Cross**	**#**	**Min**	**Mean**	**Max**
I01	14	251340	113680	45.23	136438	54.28	1222	0.49	6	4	3	0.84	1.01	1.17
I02	8	126780	104032	82.06	22086	17.42	662	0.52	7	2	2	0.99	1.01	1.04
I03	12	217140	86416	39.80	129177	59.49	1547	0.71	4	2	0	0.98	1.04	1.18
I04	12	207130	88942	42.94	117168	56.57	1020	0.49	4	1	0	0.89	1.01	1.11
I05	14	186293	110688	59.42	73127	39.25	2478	1.33	6	8	2	0.88	1.00	1.08
I06	15	251175	98899	39.37	150746	60.02	1530	0.61	4	3	3	0.92	1.04	1.13
I07	7	147920	73395	49.62	73577	49.74	948	0.64	9	0	0	0.93	1.02	1.14
I08	17	174800	90505	51.78	83050	47.51	1245	0.71	2	0	0	0.83	1.01	1.12
I09	9	186490	114793	61.55	70562	37.84	1135	0.61	7	0	2	0.88	0.95	1.00
I10	18	227102	120210	52.93	104852	46.17	2040	0.90	4	2	7	0.81	0.98	1.06
I11	11	125652	62994	50.13	61362	48.84	1296	1.03	5	0	1	0.95	1.03	1.20
Avg.	12.4	191075	96778	50.65	92922	48.63	1375	0.72	5.3	2.0	1.8	0.90	1.01	1.11

Table 3.6: Real operation performance indicators.

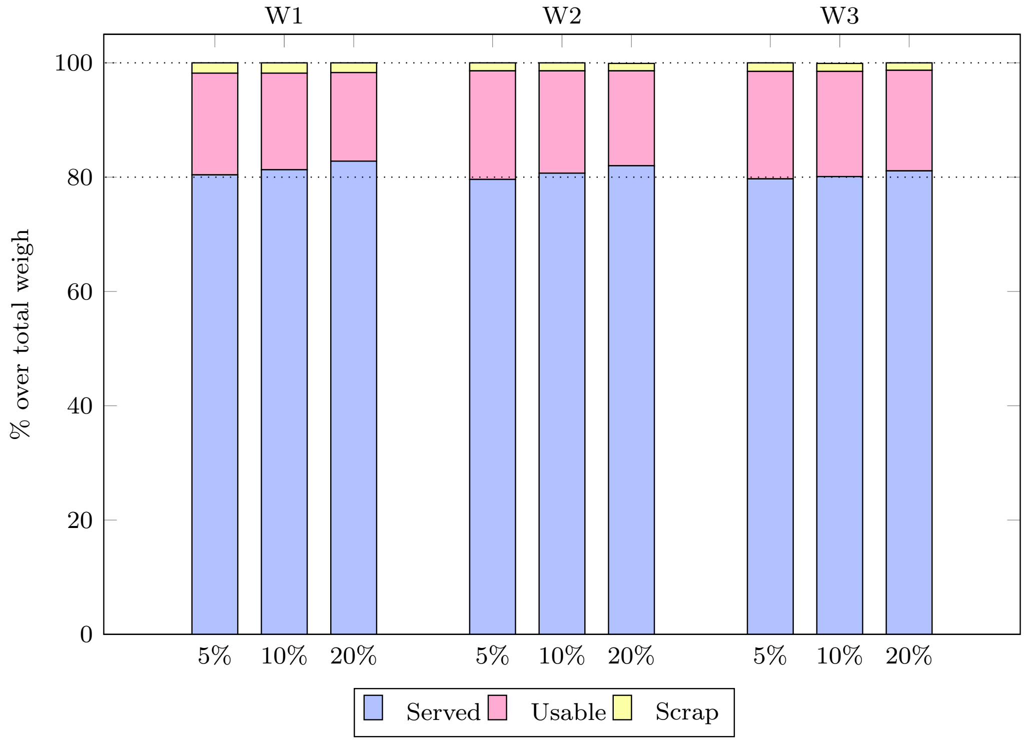


Figure 3.10: Utilisation of coils for the different combinations of weights and desired deviations.

are used to penalise retails, scrap and deviation from the weight ordered, respectively, $W1 = (1, 3, 2)$, $W2 = (1, 4, 2)$ and $W3 = (1, 4, 3)$.

Each of the combinations has been tested in the set of instances presented above. Figure 3.10 shows the performance of the model: the percentage of the weight served to customers over the weight of the used coils, the percentage of the weight of the retails held in stock and the percentage of scrap that is discarded. These data correspond to the aggregation of all the orders included in all instances.

Moreover, it can be observed that in almost all cases, more than 80% of the total weight is served to customers, and the higher the maximum desired deviation, the more efficient the coil usage, although the differences are not really significant. The performance of the model for both, $W2$ and $W3$ weights, is very similar. It is worth noting that the penalisation set $W1$ provides an increase of 1% in the weight served together with an increase of 1% in scrap.

The distribution of the order accuracy is represented in Figure 3.11. Each box-plot shows the distribution of the accuracy obtained in all orders of the instances. As would be expected, one can observe that in the three penalisation sets used, the variability of the deviations is reduced as is the maximum desired deviation. Although the differences are

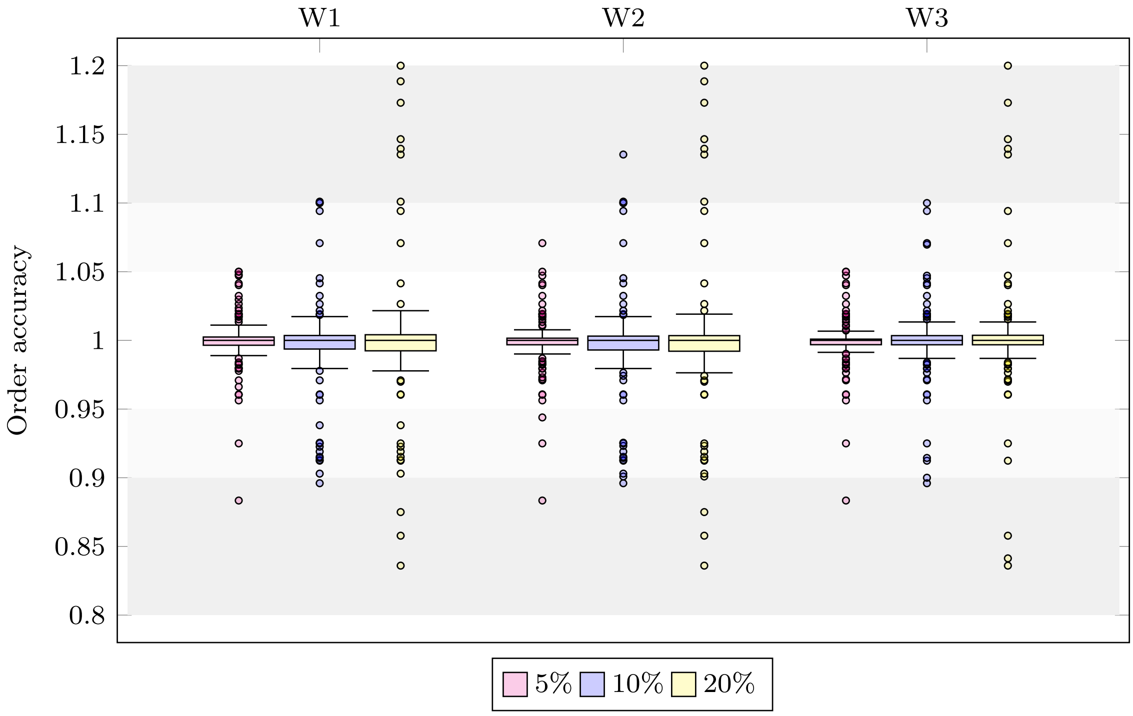


Figure 3.11: Order accuracy variability for the different combinations of weights and desired deviations.

not very relevant, we have decided to discard the penalisation set $W1$ since it increases the scrap, which is contrary to one of the goals of the company. It is worth pointing out that in all cases, very few orders are outside the desired limits, although it can be observed that the penalisation set $W3$ is slightly better adjusted than $W2$.

Therefore, taking into account the information in Figures 3.10 and 3.11 and although the differences are not very distinguishable, we will use penalisation set $W3$ and a maximum desired deviation of $\pm 5\%$ for the remaining analysis.

Table 3.7 reports a number of statistics on the computational performance of the model. The following information is presented for each instance: the number of constraints, the variables, the integer and binary variables, the value of the objective function within the best feasible solution provided, and the optimality gap (in %) for this solution. It is possible to observe that in eight of the instances, the model provides an optimal solution, and in only three instances, I03, I07 and I08, the optimality gap is greater than 9%. It is worth noting that, as we will observe below, in instance I08, there is an optimality gap greater than 23%, 98% of the weight of the used coils is served to the customers and the order accuracy varies between 0.99 and 1.01. Therefore, it appears that the optimality gap is quite large because of the quality of the lower bound, while the proposed solution is

Ins.	cons.	vars.	int.vars.	0-1 vars.	time (s)	Z_{IP}	gap
I01	57590	30077	380	17510	264.39	34501.3	0.00
I02	8233	4378	76	2560	1.51	17484.1	0.00
I03	39154	20404	222	11388	600.19	27811.7	10.24
I04	28045	14650	175	8242	275.53	15310.5	0.00
I05	23984	12644	193	7489	10.28	39692.5	0.00
I06	39503	20669	271	11918	125.63	40634.8	0.00
I07	12215	6383	65	3482	600.21	20654.3	9.28
I08	40573	21058	207	11688	600.23	7457.3	23.63
I09	23733	12402	154	7114	126.60	20167.0	0.00
I10	20090	10659	180	6208	3.20	80280.3	0.00
I11	8760	4632	68	2736	1.74	68563.4	0.00

Table 3.7: Computational statistics: Model dimensions and solution.

close to the optimal one. The model was run for two hours for each instance, and obtained a gap of 0% for instances I03 and I07 with the same objective value, whereas the gap of instance I08 was reduced to 7% and the best known solution improved 5%.

3.5.4 Interpretation of the obtained results

Table 3.8 reports a number of statistics on the quality of the solution after solving the proposed mathematical model in the same terms as in Table 4. It also indicates the number and weight of coils used, the coils served, the retails and the scrap, including the percentage that each of these weights represents over the total weight used, the number of slits and crosscuts performed and, finally, the minimum, the average and the maximum order accuracy achieved.

Figures 3.12 and 3.13 provide a graphical comparison between the performance of the solution proposed by the mathematical model and the solution which is currently implemented in the company.

Figure 3.12 shows the utilisation of the coils for both solutions. On average, under the solution that is currently applied, 52.3% of the total weight is used to serve the orders, 47% of it is retails and the remaining 0.7% is considered as scrap. On the other hand, under the solution proposed by the model, 79.7% of the total weight is served, 18.8% is stocked as retails and the remaining 1.5% is considered as scrap. Although the amount of scrap increases from 0.7% to 1.5%, there is a more efficient use of the coils, since the model manages to reduce the weight of the retails to one third. This implies a direct reduction in the management costs of these retails, and, consequently, a reduction in the management costs in the warehouse. In general terms, the mathematical model aims at increasing the use of the coils by using the available stock, while at the same time, reducing it since the

	Used coils		Served		Retail		Scrap		Cuts		Rew.	Order accuracy		
Ins.	**#**	**Weight**	**Weight**	**%**	**Weight**	**%**	**Weight**	**%**	**Slit**	**Cross**	**#**	**Min**	**Mean**	**Max**
I01	18	140732	115587	82.13	22562	16.03	2583	1.84	8	1	2	0.99	1.00	1.02
I02	10	118173	103445	87.54	14143	11.97	585	0.49	6	3	1	0.99	1.00	1.00
I03	10	102744	83330	81.10	17119	16.66	2295	2.23	6	4	4	0.98	1.00	1.05
I04	11	101970	89874	88.14	11329	11.11	767	0.75	9	0	2	0.98	1.00	1.03
I05	20	141963	111580	78.60	28280	19.92	2103	1.48	6	7	3	0.92	0.99	1.04
I06	17	130054	97289	74.81	29586	22.75	3178	2.44	4	3	5	1.00	1.00	1.02
I07	7	91032	73982	81.27	16645	18.28	406	0.45	8	1	2	0.96	1.00	1.05
I08	13	93922	91794	97.73	578	0.62	1550	1.65	9	0	0	0.99	1.00	1.00
I09	13	138591	123634	89.21	13159	9.49	1798	1.30	6	1	1	1.00	1.00	1.01
I10	19	194924	123098	63.15	69631	35.72	2195	1.13	5	1	3	0.96	1.00	1.02
I11	16	113373	60308	53.19	50016	44.12	3049	2.69	5	5	6	0.88	0.99	1.05
Avg.	14.0	124316	97629	78.53	24823	19.97	1864	1.50	6.0	2.4	2.6	0.97	1.00	1.03

Table 3.8: Model performance indicators.

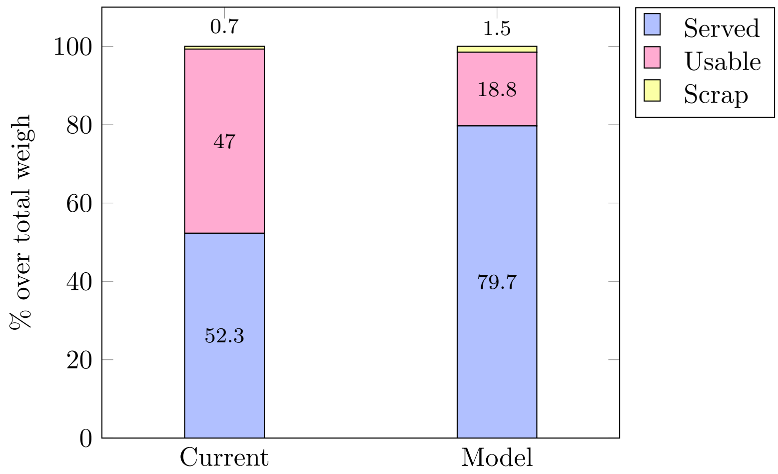


Figure 3.12: Utilisation of coils in the solution proposed by the model and the solution currently implemented.

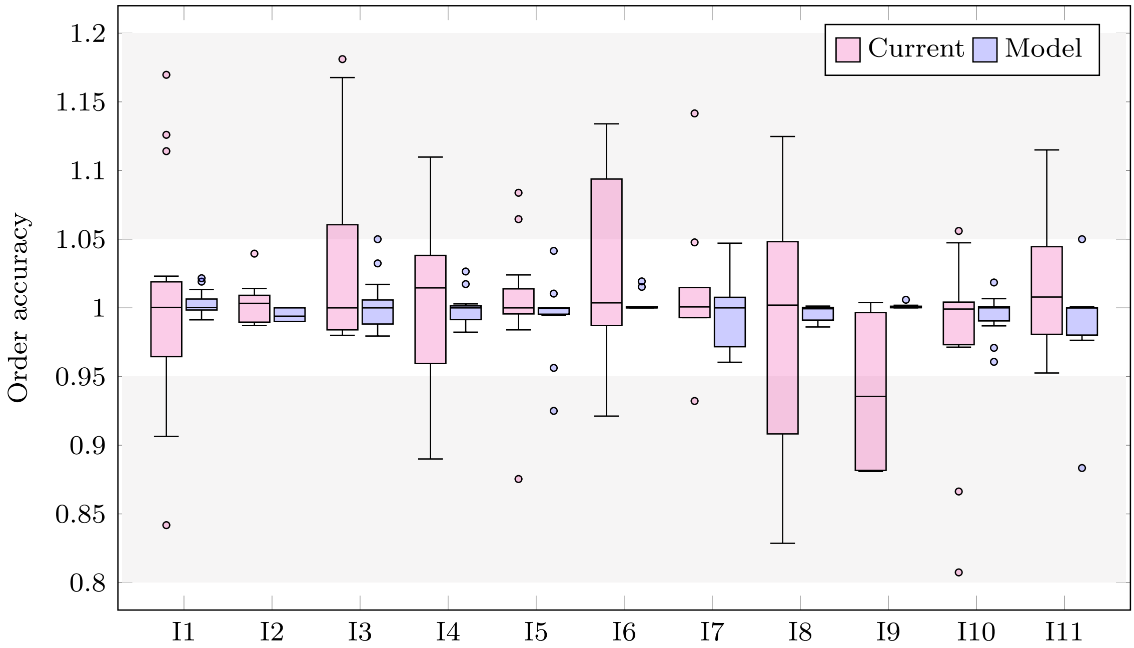


Figure 3.13: Order accuracy variability in the solution proposed by the model and the solution currently implemented.

amount of retails is also reduced.

Figure 3.13 shows the distribution of the order accuracy for both solutions. The horizontal band with a white background corresponds to the maximum desired deviation of 5%. It is evident how clearly the model provides solutions as most of the orders are within these limits (except for instances I05 and I11 which have an order accuracy below 95%). In addition, it can be observed that in the boxplot boxes (representing the distance

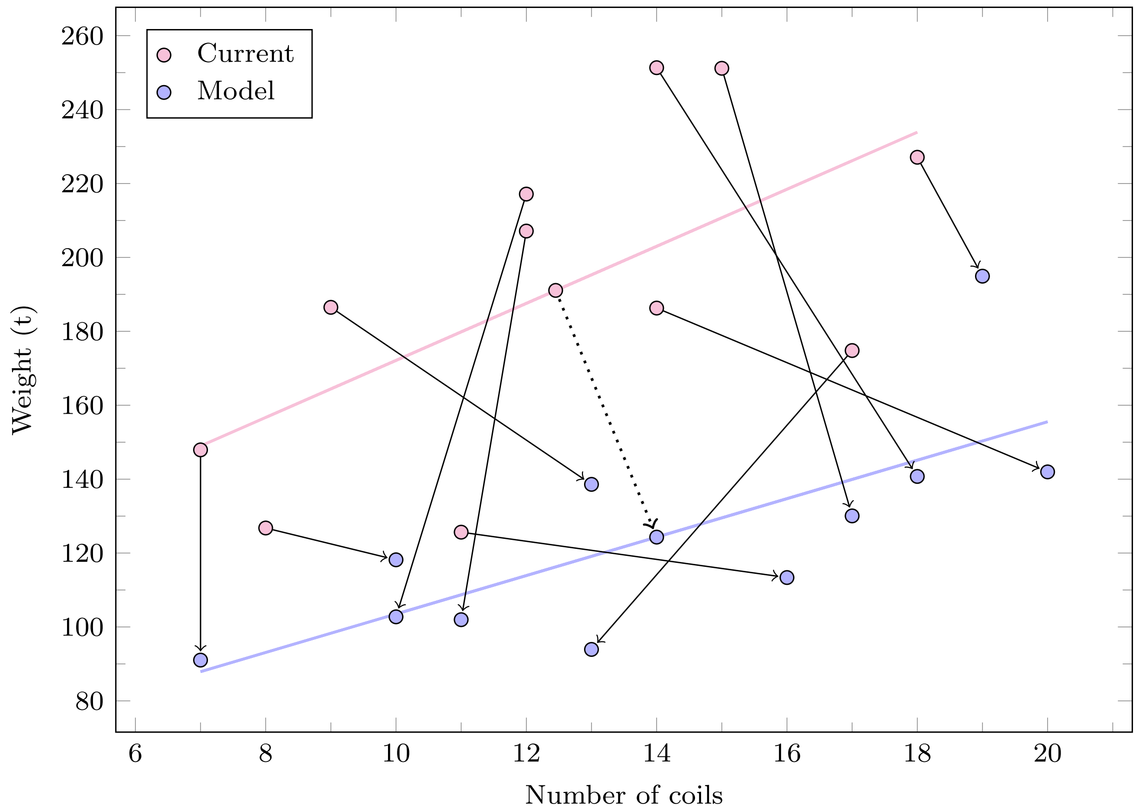


Figure 3.14: Number and weight of the used coils in the current and model solutions.

between the first and third quartiles), the variability of the model solution is far lower since the boxes are much narrower and the whiskers considerably shorter. In other words, using the mathematical model, is possible to provide solutions that are better suited to the weight required by the customers with a consequent saving in material.

Finally, Figure 3.14 shows a scatter plot indicating the number of used coils along with their weight, for both, the currently used and the model solutions. Each instance is represented by two points connected by an arrow. One point for the current solution and the other for the model solution. A trend line is also shown, representing the relationship between the number of coils and their weight. It can be observed that in all cases, the weight of the used coils is lower in the solution proposed by the model (downward arrows) and there is also a clear tendency to use more coils in this solution (rightward arrows): there are only three cases where the current solution uses more coils. In addition, the dotted arrow shows the average values in terms of used weight and number of coils for the current and model solutions. On average, the number of coils increases while the weight used decreases. Therefore, it can be determined that the model proposes solutions where the coils are smaller. These small coils often correspond to retails from previous days which are more difficult to allocate, since the company tends to use large coils on a daily basis, which provides more flexibility in the planning. It is worth noting that this strategy will imply an increase in the stock, contrary to the solutions provided by the model which optimises the management of the available stock more efficiently.

3.5.5 Extended computational experiment

In order to assess the limits of the model, larger instances have been generated and solved by increasing the number of orders and coils in stock. Instances with 15, 30, 60, 90 and 120 orders are created and for each orders' size two different sets of stock are generated resulting in a total of ten different scenarios. For each scenario, 5 different instances are created by randomly selecting orders from a list of orders of the company and coils from the available stock, taking into account stock-orders compatibility. Therefore, the computational experiment includes 50 instances. The computational time limit for solving each instance has been set to 20 minutes.

Table 3.9 reports the average values obtained for each scenario. The following data are reported: the number of orders and coils, the dimensions of the model (number of constraints, variables, integer and binary variables), the computational time in seconds (time), the value of the objective function for the best known feasible solution (Z_{IP}) and the optimality gap given in % (gap). Last columns present the proportion of the weight of the coils used to serve orders, retails and scrap (minimum, mean and maximum values of the instances solved for each scenario are reported).

Regarding the performance of the model, it may be observed that as the size of the instances increases so does the optimality gap. Furthermore, in scenarios 9 and 10 only 4 and 3 out of 5 instances, respectively, were solved within 20 minutes. These results suggest that for larger problems the model will need more time to obtain a feasible solution and other strategies would have to be investigated to obtain good feasible solutions in reasonable time, such as heuristic or metaheuristic approaches.

However, if we observe the quality of the solution, we may highlight that the utilisation of the coils is similar in all scenarios, even for those with a large optimality gap. On average, 76.6% of the coils is used to serve orders, 22.3% is intended for retails and only 1.1% is considered as scrap. This distribution is similar to the one obtained in the previous instances (see Table 3.8).

3.6 Conclusions and future research

A mixed integer linear optimisation model has been presented to address the specific cutting stock problem in a Spanish steel manufacturing company. The model has been validated with real data provided by the company, and has succeeded in surpassing its

Scenarios			Model dimensions				Performance			Served (%)			Retail (%)			Scrap (%)		
n	$\lvert\mathcal{O}\rvert$	$\lvert\mathcal{C}\rvert$	**cons.**	**vars.**	**int.vars.**	**0-1 vars.**	**time**	Z_{IP}	**gap**	**min**	**mean**	**max**	**min**	**mean**	**max**	**min**	**mean**	**max**
1	15	250	36732	19241	250	11025	641.1	66880	1.4	65.5	73.6	81.2	17.4	25.3	33.4	0.7	1.1	1.4
2	15	500	65137	34198	500	19958	561.1	43894	0.6	78.8	80.6	83.3	15.0	18.1	20.1	0.8	1.3	2.3
3	30	500	74906	39224	500	22338	968.9	145884	2.1	72.4	75.7	81.0	17.5	23.2	26.9	0.8	1.1	1.5
4	30	750	109974	57589	750	32655	985.5	118786	5.6	66.3	73.5	84.4	14.6	25.5	32.7	0.8	1.0	1.2
5	60	750	128897	67334	750	37320	1200.0[c]	266420	6.6	71.0	73.6	76.5	22.6	25.4	28.0	0.9	1.0	1.1
6	60	1000	167540	87518	1000	48553	1200.0[c]	239925	7.6	70.4	76.4	81.5	17.1	22.4	28.4	0.7	1.2	1.4
7	90	1000	205745	107011	1000	58059	1200.0[c]	323209	15.6	75.8	78.4	80.9	18.1	20.4	23.0	1.0	1.2	1.5
8	90	1500	311683	161911	1500	87799	1200.0[c]	296803	17.3	76.6	78.9	80.8	18.4	20.1	22.4	0.8	1.0	1.3
9[a]	120	1500	371608	192450	1500	102540	1200.0[c]	457106	28.5	76.5	78.4	79.5	19.2	20.5	22.6	0.9	1.1	1.3
10[b]	120	2000	464941	240958	2000	129066	1200.0[c]	512079	42.1	72.2	76.9	80.1	18.7	21.7	26.1	1.2	1.4	1.8
Avg.											76.6			22.3			1.1	

[a] 4 out of 5 instances were solved within 20 minutes.
[b] 3 out of 5 instances were solved within 20 minutes.
[c] No instance was solved to optimality within 20 minutes.

Table 3.9: Computational statistics: Average values for each scenario.

current performance. One of the main benefits of this approach is the reduction of the time needed to design the cutting plans.

Mathematical optimisation is able to provide solutions which are difficult to analyse manually. Moreover, mathematical optimisation avoids mistakes caused by human errors, such as scheduling wrong quantities, which imply higher costs for the company. In technical terms, the model provides us with solutions that increase the size of each used coil that is effectively sent to the customers, and decrease the retails accordingly. This fact represents an improvement in the stock management such as saving time in locating the coils, as well as reducing the management costs of the raw materials. Besides these issues, the model is able to respond more efficiently to customers' orders by delivering the orders with a weight that is much closer to the one ordered.

As it has been studied in the extended computational experience, as the number of orders and available stock increase, so does the difficulty in solving the problem. A heuristic approach is currently being investigated to deal with larger instances. Due to the characteristics of the problem, a matheuristic strategy that decomposes the orders according to a rule based in the compatibility matrix between orders and stock could also be investigated in order to deal with larger instances.

As far as the future investigation is concerned, the company is interested in carrying out a planning process over several days simultaneously and identify the more appropriate slitting line among the ones they own. Therefore, in our particular case, this new model needs to include more orders with their corresponding deadlines, and an allocation of the workload over different slitting lines, indicating which coils should be cut in each line to maximise the productivity of the cutting process. This new model will be far more complex and it will require more computational effort to solve full-size instances.

The new problem that arises when introducing the slitting lines allocation, and the mathematical optimisation model developed to solve it are described in the upcoming chapter.

Chapter 4

The slitting problem in the steel industry with slitting lines allocation

Contents

4.1 Introduction

This chapter introduces an extension of the mathematical optimisation model presented in Chapter 3 in order to include the slitting lines allocation. After deciding which coils to cut and how to cut them to satisfy the demand of the steel strips, one key element in the planning of the slitting process is deciding the slitting line where each coil will be slit. The coils have to be processed on one slitting line from a heterogeneous set of lines, and therefore the cutting pattern also needs to meet the ongoing operational constraints imposed by the cutting lines.

The technical differences among the lines limit the allocation of the coils-pattern-slitting line, producing a high impact on the company performance. Therefore, it is necessary to integrate the slitting line allocation into the coil selection and pattern definition. The integrated model is validated with real data provided by the same Spanish steel manufacturing company, and outperforms the results obtained when this integration is not considered, thereby improving the company's performance, without reducing the quality of the solutions. Additionally, three different objectives are considered: the reduction of the leftovers generated in the process, the maximisation of the overall accuracy of the orders and the minimisation of the makespan, by performing a multi-criteria analysis.

In addition to the slitting lines allocation, the mathematical optimisation model presented in this chapter introduces some other differences with respect to the model presented in the previous chapter to better fit the problem of the company:

- We consider the restrictions imposed by the slitting lines in the definition of the cutting patterns. In particular, the maximum number of knives allowed now will also depend on the slitting line, and the crosscuts could also be performed due the slitting lines limitations.
- Regarding the customer's orders, we will have some orders that do not need to be completed within the cutting plan, allowing the model to decide whether to plan those orders or not. These orders can be seen as a way of anticipating future demand, being only served if the leftovers are reduced.
- In the previous model, edge trimming was considered in only one edge of the coil, and the other edge was considered as the leftovers (at least edge trim wider). In the new formulation, we consider that in case the coil is slit the coil is trimmed at both edges. It might be one strip dedicated to leftovers if it is not assigned to orders widthwise.
- To decide whether the leftovers are reusable or not we will consider the weight of each resulting strip obtained after crosscuts are performed. In the previuous model

we would consider the corresponding weight of the total length used.

The main results of this chapter have been submitted to:

> **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (submitted March 2022), 'An exact model for the 1.5-dimensional cutting stock problem in the steel industry with heterogeneous parallel slitting lines allocation', *European Journal of Operational Research*

The remaining part of this chapter is organised as follows. Section 4.2 presents some literature review relevant to this field. Section 4.3 introduces a detailed problem description. Section 4.4 presents the mixed integer linear optimisation model proposed to solve the problem. In Section 4.5 an extensive computational experiment is carried out, based on a real-world situation, including a multi-criteria analysis. Finally, Section 4.6 provides the conclusions obtained and outlines several future research lines.

4.2 Literature review

A detailed literature review of the general CSP introduced in this work has been presented in the previous chapter in Section 3.2, and therefore this section is devoted to introduce some state of the art in the combination of CSP's with other other close problems.

In recent years, owing to the development in optimisation software and computers, it is common for CSP to be combined with other related problems. For instance, once the cutting patterns are defined, the original products have to be cut into cutting lines. The sequence in which patterns are assigned in the cutting lines has a great impact on the performance, since changing the pattern requires the machine setup to be adjusted, with a consequent loss of time. In Giannelos and Georgiadis (2001) a Mixed Integer Programming (MIP) model is presented to address the scheduling of cutting operations on multiple parallel slitting machines. In Meng et al. (2019) a Genetic Algorithm is proposed to solve the scheduling of cutting-stock processes on multiple identical parallel machines. In these works, the set of possible patterns is determined a priori, reducing the cutting problem to the selection of the pattern applied to each element of the stock. This strategy of using a predefined set of patterns is very common in the literature, since it reduces the complexity of the problem and allows calculating the pattern change time of the cutting lines, which is a key element in sequencing.

However, regarding our problem, there is a wide variety of coils in stock (including leftovers from previous slitting processes) and a wide range of order widths that vary over time. Consequently, this makes the predefined pattern approach inappropriate. Additionally, the company has different types of slitting lines, as a result of its product needs or because they have been acquired over time. Each line has different characteristics, such as the maximum number of knives and minimum distance between them, or the limits in the width of the material to be cut, among others. Due to these differences, it may happen that some patterns cannot be cut by any available slitting line, or that some lines end up overloaded. In order to avoid these situations so as to obtain valid cutting patterns, one alternative is to integrate the slitting line allocation and the cutting patterns layout, in such a way that the pattern for a coil is laid out taking into account the restrictions of the slitting line, on which the coil will be cut. We address a variant of the CSP that combines the layout of the cutting patterns and the assignment of the slitting line, where each coil will be slit with its corresponding speed. To the best of our knowledge, this integration has not been studied previously in the literature.

The most common objectives in CSP are the reduction of leftovers, the reduction of the amount of product used, and when sequencing is included, the reduction of machine usage time. It is common to combine more than one of these objectives. Campello et al. (2019) introduce a multi-objective model for the lot-sizing and cutting stock problem where different goals are studied: production cost, object storage and machine setup for lot sizing, and waste of material and holding costs for the resulting pieces. Song et al. (2006) present a multi-objective formulation to deal with two goals: trim loss and production time. Braga et al. (2016), combine production time and waste generated in the same objective function to deal with scheduling alongside the cutting plan. In our problem, we deal with three objectives: the minimisation of leftovers generated in the process, the maximisation of productivity of the slitting lines, and the minimisation of the deviation between the served and ordered weight.

4.3 Problem description

As stated before, planning the slitting process refers to the process of selecting a subset of compatible coils, laying out the cutting patterns for these coils, and assigning the slitting line (and the speed) where each coil will be cut. The planning of the slitting process is done on a make-to-order basis. At the beginning of the planning horizon, the company has a list of orders. Each order corresponds to an amount (in kg) of material (with specific characteristics such as thickness and other quality parameters) that can be served in one or more strips of a given width. Additionally, each customer establishes upper and lower limits on the weight and external diameter of each strip. Due to the complexity of the process, it is rather difficult to serve the exact weight demanded by all the customers and, as a consequence, deviations between the weight ordered and the total weight served, are acceptable to a certain extent (±20% tolerance). For better planning, some orders are required to be completed in the current cutting plan, while others are not. The latter can be partially completed, and they remain open until they are completed in the following cutting plans.

At a first step, compatible coils, with the material characteristics of the orders, are selected from the available stock. This stock is formed by unused coils and processed coils (reusable leftovers from previous cutting processes) of different widths and external diameters. Customers allow for certain tolerances on the thickness and quality parameters, which makes it possible to serve an order with coils of different characteristics, and to use the same coil to serve orders with different characteristics. Although these tolerances provide an advantage by giving more flexibility, they increase the level of difficulty, making the problem non-separable by either quality characteristics or thickness.

The cutting patterns must not only satisfy the restrictions imposed by the customers on the final strips, but also the restrictions imposed by the slitting lines. Usually, a company has more than one slitting line with different characteristics. Therefore, the allocation of the coil to a slitting line depends on the coil characteristics (width, thickness, weight, and external diameter) lying between the minimum and maximum values allowed by the slitting line, as well as the cutting pattern laid out for the coil (number of knives, minimum cutting width, etc.). Furthermore, these restrictions are different for each cutting speed of the slitting line.

4.3.1 Slitting lines

Once the patterns for all selected coils have been laid out, the slitting process is carried out on the slitting lines and begins with the loading of the steel coil onto the uncoiler. The uncoiler is one of three major parts of the slitting line along with the slitter and the recoiler. After the coil is uncoiled and flattened, it is moved through the slitter, which consists of two parallel arbours mounted with rotary cutting knives. These knives penetrate the coil as it moves, performing the cuts and separating the strips from one another. The recoiler collects the slit strips and rolls them up. Finally, the finished strips are pushed off of the recoiler. The slitting line has a shear blade to perform guillotine crosscuts used to reduce the diameter of the final strips. We will say a coil is cut in $i + 1$ passes if i crosscuts are performed. If no crosscuts are performed ($i = 0$) the coil is cut in a single pass. A diagram of a slitting line is shown in Fig. 4.1.

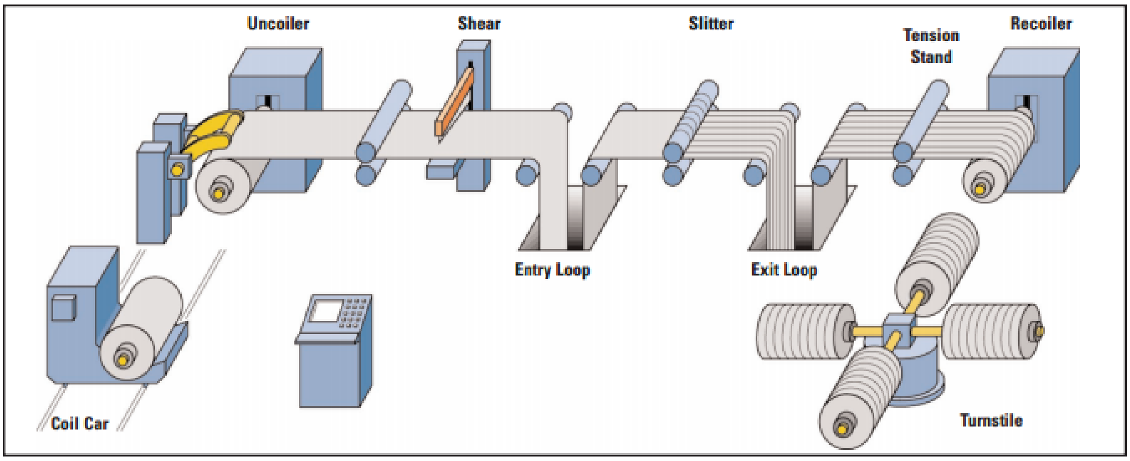


Figure 4.1: Diagram of a slitting line. Image extracted from Coowor.com (2022).

4.3.2 Operational restrictions

The following restrictions have to be taken into account when laying out the cutting patterns:

- Customer's restrictions: In addition to the width of the strips and total weight to be served, customers set maximum weight and maximum external diameter limitations for the strips. This restriction directly affects the number of crosscuts required. Note that given the width, thickness, density and internal diameter, the weight can be obtained from the external diameter and vice versa.

- Slitting line restrictions:

  - Maximum number of knives, which depends on the thickness and hardness of the coil, and the cutting speed of the slitting line, making the maximum number of strips on a coil dependent on the assigned slitting line.

  - Minimum cutting width between two knives, which prevents some orders from being allocated to coils slit on certain machines. As with the maximum number of knives, the minimum cutting width will vary depending on the speed of the slitting line, and the thickness and hardness of the coil. Note that this restriction applies also to the strip of leftovers.

  - Edge trim: To maintain uniformity in the cutting process, a minimum edge trimming is required when slitting the coil. These trims require one knife on each side of the coil so that n knives yield $n-1$ strips. Edge trimming is not required on coils served as strips without slitting.

  - Maximum weight of strips. This restriction, combined with that imposed by

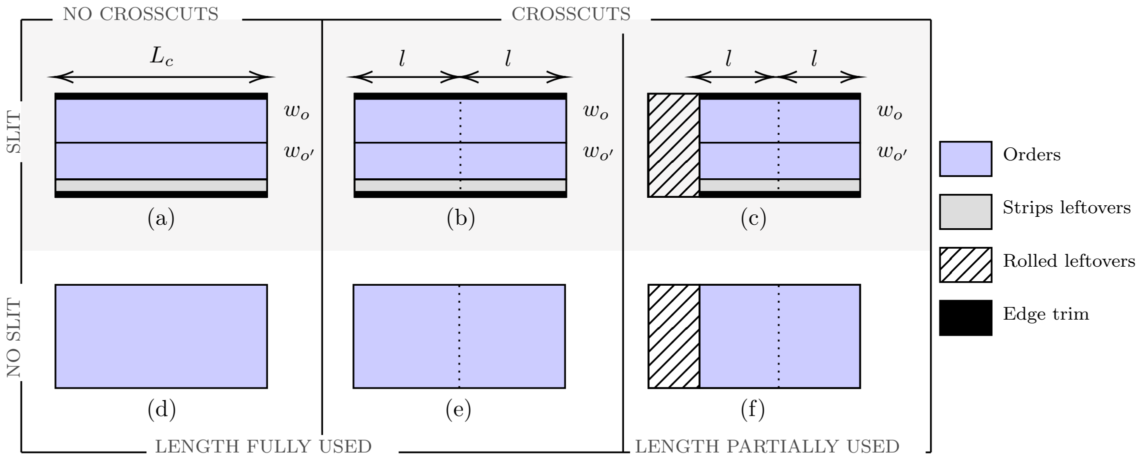


Figure 4.2: Typologies of cutting patterns.

customers, makes it necessary to make crosscuts to reduce the weight of the set of final strips. Note that it is imposed on the sum of strips obtained in each pass and not on every single strip.

– Lower and upper limits on the external diameter of the set of strips. The upper limit can force crosscuts.

Note that these restrictions can be different for each slitting line.

4.3.3 Cutting patterns

Despite the new restrictions imposed by the slitting lines, the cutting patterns that can be obtained are the same as the ones introduced in the previous chapter. The six types of cutting patterns are shown in Fig. 4.2 and respond to three key decisions: whether or not the coil is cut, whether or not crosscuts are made, and whether the length of the coil is partially or fully used. Note that in these cases the edge trim is represented in both extremes of the coil. The simplest cutting pattern is represented in Fig. 4.2(a), where different orders of strip widths are assigned to the coil widthwise. If the sum of the widths of the strips assigned to the coil is lower than the width of the coil, the last strip will be considered as a leftover.

The cutting patterns may also include guillotine crosscuts, but in this case, they may be due to the slitting line restrictions. Note that a guillotine crosscut imposes the same length (l) for all strips in the coil (for every pass), even if they belong to different orders or leftovers, and that the configuration of knives remains the same after the crosscut (see Fig. 4.2(b)). Therefore, it must be ensured that all strips have a specific weight and diameter within the limits set by the customer and the slitting line.

Each time a crosscut is performed, it is possible to continue slitting the coil or stop cutting, rewind the remainder of the coil and stock it for future use. In Fig. 4.2(c) two crosscuts have been performed: one to reduce the diameter of the resulting strips and another to rewind the rest of the coil. The slitting pattern is maintained after the first crosscut. Processed coils cannot be rewound after a crosscut and have to be used completely lengthwise.

Occasionally, a coil in stock may meet all the requirements of an order: its width is exactly as ordered and its weight and external diameter are within the limits specified by the customer. In this case, it is not necessary to use the slitting line and the coil is automatically served to the customer (see Fig. 4.2(d)). If the coil exceeds the maximum weight or external diameter limits, crosscuts can be made; in this case, edge trimming is not required (see Fig. 4.2(e,f)).

4.3.4 Goals

In addition to the goals described in the previous chapter: a reduction in leftovers and customer satisfaction, the company pursues to maximise the productivity of the process.

The most important of these goals is the reduction of leftovers. As stated before, a leftover is defined as the part of the coil that is not used to serve orders; they appear lengthwise if the coil is rewound after a crosscut (rolled leftovers, see Fig. 4.2(c,f)) or widthwise if it is not possible to complete the width of the coil with orders (see Fig. 4.2(a,b,c)). Leftovers with a weight and width above a given minimum are considered to be *retails*, otherwise, they are considered as *scrap*. The retails are stored for future use and, therefore, are preferred to scrap. Rolled leftovers are permitted only if they can be reused and are therefore included with the retails. Edge trim is considered as scrap.

The second goal considered by the company is related to productivity: The less time-consuming a cutting plan takes, the better it is. The three relevant elements for reducing the processing time are: 1) the number of crosscuts; 2) the load balance between the different slitting lines, since if all the lines have a similar workload, the makespan is reduced; and 3) the speed at which each coil is cut.

The last goal remains to be, as in the previous chapter, to maximise the satisfaction of their customers. As in Chapter 3, this is measured by the order accuracy, which is an indicator of the deviation between the weight ordered and the weight served. The better the accuracy of the orders, the higher the customer satisfaction.

4.4 Mixed Integer Linear optimisation model

In this section, we present a Mixed Integer Linear optimisation model to solve the previously described problem. For the sake of clarity, the complete notation, as well as the model equations, are introduced in detail.

4.4.1 Notation

- $\mathcal{O}$, set of customers' orders. The following information is given for each order $o \in \mathcal{O}$:

  w_o, width (m) of strips.

  b_o, required weight (kg).

  $\overline{b}_o$, maximum weight (kg) allowed for each single strip in the order.

  $\overline{u}_o$, maximum deviation allowed (kg) for the ordered weight.

- $\mathcal{M}$, set of slitting lines. The following information is given for each slitting line $m \in \mathcal{M}$:

  $\mathcal{S}_m$, set of different available speeds.

  $\mathcal{C}_m$, set of compatible coils.

  $\overline{b}_m$, maximum output weight (kg) for the set of strips obtained in each pass.

- $\mathcal{C}$, set of coils in stock. For each coil $c \in \mathcal{C}$, the following information is known:

  $\mathcal{O}_c$, set of compatible orders.

  $\mathcal{M}_c$, set of compatible slitting lines.

  W_c, B_c, L_c, width (m), weight (kg) and length (m), respectively.

  $\underline{L}_c, \overline{L}_c$, minimum and maximum length (m) used if the coil is partially used. $\underline{L}_c$ is such that it ensures that the external diameter of the resulting strips does not exceed the limits of the slitting lines. The maximum length is such that it ensures that the remainder of the coil can be rewound and kept in stock to be reused.

  D_c, weight (kg) per m^2. It depends on the density and thickness of the coil.

  $\hat{p}_{cm}$, maximum number of passes that can be performed if it is cut on the slitting line m (it is obtained from the minimum external diameter allowed for the resulting strips).

  I_c, maximum number of passes that can be performed, $I_c = \max_{m \in \mathcal{M}_c} \{\hat{p}_{cm}\}$.

$\hat{k}_{cms}$, maximum number of knives allowed if it is cut on the slitting line m at speed s.

J_c, maximum number of strips that can be obtained, $J_c = \max_{m\in \mathcal{M}_c, s\in \mathcal{S}_m} \{\hat{k}_{cms} - 1\}$.

$\hat{w}_{cms}$, minimum width of strips (m) if it is cut on the slitting line m at speed s.

$\underline{w}_c$, minimum width of strips (m), $\underline{w}_c = \min_{m\in \mathcal{M}_c, s\in \mathcal{S}_m} \{\hat{w}_{cms}\}$.

$\underline{l}_{cm}, \bar{l}_{cm}$, minimum and maximum strip lengths (m) if it is cut on slitting line m (obtained from the external diameter limits allowed on each slitting line).

$\hat{q}_{cm}$, maximum process time (min) on slitting line m (including the time to perform possible crosscuts).

$\hat{t}_{cms}$, cutting time (min) on slitting line m at speed s.

- $\mathcal{O}^*$, set of orders that need to be completed within the cutting plan, $\mathcal{O}^* \subset \mathcal{O}$.
- $\mathcal{C}^*$, set of processed coils in stock, $\mathcal{C}^* \subset \mathcal{C}$.
- Parameters for operation settings:

$\hat{W}$, largest minimum cutting width (m) among all slitting lines.

$\bar{t}$, time (min) used to perform a crosscut.

r, minimum edge trim (m) required to maintain the uniformity of the cut.

$\underline{a}, \underline{b}$, minimum width (m) and weight (kg) of strip leftovers to be considered retails, respectively.

Decision variables

$\alpha_{cj} = 1$, if the j-th strip is obtained from coil c, 0 otherwise, $c \in \mathcal{C}, 1 \leq j \leq J_c$.

$\beta_c^{\mathbf{E}} = 1$, if edge trimming is needed in coil c, 0 otherwise, $c \in \mathcal{C}$.

$\beta_c^{\mathbf{T}} = 1$, if there is no need for edge trimming in coil c, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_c^{\mathbf{T}} = 1$, if coil c is fully used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\gamma_c^{\mathbf{P}} = 1$, if coil c is partially used lengthwise, 0 otherwise, $c \in \mathcal{C}$.

$\delta_{ci} = 1$, if strips from coil c are served in i passes, 0 otherwise, $c \in \mathcal{C}, 1 \leq i \leq I_c$.

$\lambda_c = 1$, if coil c needs to go through the slitting lines, 0 otherwise, $c \in \mathcal{C}$.

$\mu_{ocj} = 1$, if order o is assigned to the j-th strip of coil c, 0 otherwise, $o \in \mathcal{O}_c, c \in \mathcal{C}, 1 \leq j \leq J_c$.

$\varphi_{oc} = 1$, if order o is assigned to coil c, 0 otherwise, $o \in \mathcal{O}_c, c \in \mathcal{C}$.

$\tau_c = 1$, if coil c has leftovers widthwise, 0 otherwise, $c \in \mathcal{C}$.

$\theta_c = 1$, if the strip of leftovers of coil c is for retails, 0 otherwise, $c \in \mathcal{C}$.

$\rho_{cms} = 1$, if coil c is cut on slitting line m at speed s, 0 otherwise, $c \in \mathcal{C}, m \in \mathcal{M}_c, s \in \mathcal{S}_m$.

x_c, used length of coil c, $c \in \mathcal{C}$.

$\hat{x}_{ci}$, length of strips of coil c if it is cut in i passes, $c \in \mathcal{C}, 1 \leq i \leq I_c$.

v_{ocj}, length of the j-th strip of order o obtained from coil c, $o \in \mathcal{O}_c, c \in \mathcal{C}, 1 \leq j \leq J_c$.

u_{ocji}, weight of each piece of the j-th strip of order o obtained from coil c if it is cut in i passes, $o \in \mathcal{O}_c, c \in \mathcal{C}, 1 \leq j \leq J_c, 1 \leq i \leq I_c$.

u_o^+, u_o^-, excess and lack, respectively, of weight served for order o, $o \in \mathcal{O}$.

y_c, width of the strip of leftovers of coil $c \in \mathcal{C}$, which is broken down in the sum of y_c^{r}, for retails and y_c^{s}, for scrap, $c \in \mathcal{C}$.

z_{ci}, weight of each piece of the leftover strip of coil c if it is cut in i passes, which is broken down in the sum of z_{ci}^{r}, for retails and, z_{ci}^{s}, for scrap, $c \in \mathcal{C}, 1 \leq i \leq I_c$.

e_c, weight of edge trim of coil c, $c \in \mathcal{C}$.

t_{cm}, processing time of coil c on slitting line m (considering both, the time used to perform crosscuts and the time to cut the coil), $c \in \mathcal{C}, m \in \mathcal{M}_c$.

C_{max}, makespan of all slitting lines.

4.4.2 Mathematical formulation

Objective function

The objective function is a weighted sum of three elements that represent the different goals considered by the company:

$$\min \ \omega_1 f_1 + \omega_2 f_2 + \omega_3 f_3, \tag{4.1}$$

where ω_1, ω_2 and ω_3 are decision maker's weights assigned to each component:

f_1: Reduction in leftovers. It considers the strips of leftovers (retails and scrap), and the rolled leftovers. Each term includes a weight (s_1, s_2 and s_3) to penalise more the scrap and rolled leftovers.

$$f_1 = \Big(s_1 \sum_{c \in \mathcal{C}} \sum_{1 \leq i \leq I_c} i z_{ci}^{\mathrm{r}} + s_2 \sum_{c \in \mathcal{C}} \sum_{1 \leq i \leq I_c} i z_{ci}^{\mathrm{s}} + s_3 \sum_{c \in \mathcal{C}} (B_c \alpha_{c1} - D_c W_c x_c)\Big) \tag{4.2}$$

f_2: Productivity maximisation. It consists in the minimisation of a combination of the sum of the processing time of coils and the makespan. The makespan is prioritised through the extra weights s_4 and s_5.

$$f_2 = \Big(s_4 \sum_{m\in\mathcal{M}} \sum_{c\in\mathcal{C}_m} t_{cm} + s_5 C_{max}\Big) \tag{4.3}$$

f_3: Customer satisfaction. Measured by the deviation of the served weight from the ordered weight. Note that the lack of weight is only penalised for orders that have to be completed within the cutting plan.

$$f_3 = \Big(\sum_{o\in\mathcal{O}} u_o^+ + \sum_{o\in\mathcal{O}^*} u_o^-\Big) \tag{4.4}$$

Constraints

1. Cutting patterns: Constraints (4.5) state, for used coils ($\alpha_{c1} = 1$), whether they are served with or without edge trimming, while constraints (4.6) state whether they are completely or partially used lengthwise. Constraints (4.7) guarantee that processed coils cannot be rewound. Constraints (4.8) force not to assign a strip if the previous strip has not been assigned, inducing an order in the strips. Constraints (4.9) force edge trimming in coils with at least two strips. Constraints (4.10) force the choice of the number of pieces into which the strips will be divided and, therefore, define the number of passes. Note that all strips in the same coil will be divided into the same number of pieces. Constraints (4.11)–(4.13) force a coil to be sent to a slitting line either if it is slit, partially used lengthwise or if a crosscut has been performed, while constraints (4.14) guarantee that only the coils in one of the previous situations are sent to a slitting line. Constraints (4.15) state that a coil is cut only if it is used.

$$\beta_c^{\mathrm{T}} + \beta_c^{\mathrm{E}} = \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{4.5}$$

$$\gamma_c^{\mathrm{T}} + \gamma_c^{\mathrm{P}} = \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{4.6}$$

$$\gamma_c^{\mathrm{P}} = 0 \quad \forall c \in \mathcal{C}^* \tag{4.7}$$

$$\alpha_{cj} \le \alpha_{cj-1} \quad \forall c \in \mathcal{C}, 2 \le j \le J_c \tag{4.8}$$

$$\alpha_{c2} \le \beta_c^{\mathrm{E}} \quad \forall c \in \mathcal{C} \tag{4.9}$$

$$\sum_{1\le i\le I_c} \delta_{ci} = \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{4.10}$$

$$\beta_c^{\mathrm{E}} \le \lambda_c \quad \forall c \in \mathcal{C} \tag{4.11}$$

$$\gamma_c^{\mathrm{P}} \le \lambda_c \quad \forall c \in \mathcal{C} \tag{4.12}$$

$$\sum_{2\leq i\leq I_c} \delta_{ci} \leq \lambda_c \quad \forall c \in \mathcal{C} \tag{4.13}$$

$$\lambda_c \leq \beta_c^{\mathrm{E}} + \gamma_c^{\mathrm{P}} + \sum_{2\leq i\leq I_c} \delta_{ci} \quad \forall c \in \mathcal{C} \tag{4.14}$$

$$\lambda_c \leq \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{4.15}$$

2. Assignment of the strips: Constraints (4.16) assign the second strip to leftovers or orders, while constraints (4.17) guarantee that coils from which only one strip can be obtained, do not have leftovers widthwise. The assignment of the leftover strip to the second strip is a technical procedure to reduce the set of feasible solutions: the strip which is considered as leftover is irrelevant in terms of feasibility or optimality. Constraints (4.18) guarantee that every other strip is assigned to an order (it should be noted that an order can be assigned to one or more strips). Constraints (4.19) and (4.20) guarantee that an order is assigned to a coil if the order has been assigned to at least one strip of the coil. Constraints (4.21) ensure that the width of each coil is not exceeded. This means that if the coil is used, the sum of the width of the strips assigned to orders, the width of the leftover strip, and the width of the edge trim, is exactly the width of the coil. Constraints (4.22) ensure that the width of the strip assigned to leftovers is zero for the coils without a strip of leftovers.

$$\sum_{o\in\mathcal{O}_c} \mu_{oc2} + \tau_c = \alpha_{c2} \quad \forall c \in \mathcal{C} \tag{4.16}$$

$$\tau_c = 0 \quad \forall c \in \mathcal{C} : J_c = 1 \tag{4.17}$$

$$\sum_{o\in\mathcal{O}_c} \mu_{ocj} = \alpha_{cj} \quad \forall c \in \mathcal{C}, 1 \leq j \leq J_c : j \neq 2 \tag{4.18}$$

$$\mu_{ocj} \leq \varphi_{oc} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.19}$$

$$\varphi_{oc} \leq \sum_{1\leq j\leq J_c} \mu_{ocj} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c \tag{4.20}$$

$$\sum_{1\leq j\leq J_c} \sum_{o\in\mathcal{O}_c} a_o \mu_{ocj} + y_c + 2r\beta_c^{\mathrm{E}} = W_c \alpha_{c1} \quad \forall c \in \mathcal{C} \tag{4.21}$$

$$y_c \leq (W_c - 2r - \underline{w}_c)\tau_c \quad \forall c \in \mathcal{C} \tag{4.22}$$

3. Bounds on the used length of the coils: Constraints (4.23) force the used length of the coils to be equal to the total length of the coil when it is completely used lengthwise, and between given bounds when it is partially used lengthwise.

$$L_c\gamma_c^{\mathrm{T}} + \underline{L}_c\gamma_c^{\mathrm{P}} \leq x_c \leq L_c\gamma_c^{\mathrm{T}} + \overline{L}_c\gamma_c^{\mathrm{P}} \quad \forall c \in \mathcal{C} \tag{4.23}$$

4. Length of the strips: Constraints (4.24) guarantee that the sum of the lengths of

all the strips in a coil is equal to the used length of the coil. Constraints (4.25) ensure that only the assigned strips have a non-negative length. Constraints (4.26) and (4.27) calculate the length of final strips (pieces obtained from one strip after crosscuts).

$$0 \leq x_c - v_{ocj} \leq L_c(1 - \mu_{ocj}) \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.24}$$

$$\underline{L}_c \mu_{ocj} \leq v_{ocj} \leq L_c \mu_{ocj} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.25}$$

$$\sum_{1 \leq i \leq I_c} i\hat{x}_{ci} = x_c \quad \forall c \in \mathcal{C} \tag{4.26}$$

$$i\hat{x}_{ci} \leq L_c \delta_{ci} \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.27}$$

5. Weight of the strips: Constraints (4.28) and (4.29) calculate the weight of the final strips after a crosscut has been performed, and force the weight of these strips to be under the maximum allowed for each order. Constraints (4.30)–(4.31) calculate the weight of the leftover strips. Constraints (4.32)–(4.34) calculate the weight of the edge trim. Note that we are applying the Fortet inequalities (see Fortet (1960); Hammer and Rudeau (1968)) to obtain a linear equivalence of the term $2rD_c x_c \beta_c^E$ which corresponds to the weight of edge trim.

$$\sum_{1 \leq i \leq I_c} iu_{ocji} = D_c a_o v_{ocj} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.28}$$

$$u_{ocji} \leq \bar{b}_o \delta_{ci} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c, 1 \leq i \leq I_c \tag{4.29}$$

$$\sum_{1 \leq i \leq I_c} \sum_{o \in \mathcal{O}_c} \sum_{1 \leq j \leq \mathcal{J}_c} iu_{ocji} + \sum_{1 \leq i \leq I_c} iz_{ci} + e_c = A_c D_c x_c \quad \forall c \in \mathcal{C} \tag{4.30}$$

$$iz_{ci} \leq B_c \delta_{ci} \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.31}$$

$$e_c \leq 2rD_c x_c \quad \forall c \in \mathcal{C} \tag{4.32}$$

$$e_c \leq 2rD_c L_c \beta_c^E \quad \forall c \in \mathcal{C} \tag{4.33}$$

$$2rD_c x_c - e_c \leq 2rD_c L_c (1 - \beta_c^E) \quad \forall c \in \mathcal{C} \tag{4.34}$$

6. Slitting lines: Constraints (4.35) assign one slitting line to the coils to be cut. Constraints (4.36) limit the number of knives. Constraints (4.37) and (4.38) guarantee the compatibility between a slitting line and the width of the strips in the coil. Constraints (4.39) and (4.40) guarantee that the weight of the set of strips obtained is between the limits specified by the slitting line chosen. Constraints (4.41) and (4.42)

ensure that the length of the strips is between the limits specified by the slitting line.

$$\sum_{m \in \mathcal{M}_c} \sum_{s \in \mathcal{S}_m} \rho_{cms} = \lambda_c \quad \forall c \in \mathcal{C} \tag{4.35}$$

$$\alpha_{c\hat{k}_{cms}} + \rho_{cms} \leq 1 \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c, s \in \mathcal{S}_m \tag{4.36}$$

$$y_c \geq \sum_{m \in \mathcal{M}_c} \sum_{s \in \mathcal{S}_m} \hat{w}_{cms}\rho_{cms} - \hat{W}(1 - \tau_c) \quad \forall c \in \mathcal{C} \tag{4.37}$$

$$\varphi_{oc} + \rho_{cms} \leq \alpha_{c1} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, m \in \mathcal{M}_c, s \in \mathcal{S}_m : a_o < \hat{w}_{cms} \tag{4.38}$$

$$\delta_{ci} + \rho_{cms} \leq 1 \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c, s \in \mathcal{S}_m, \hat{p}_{cm} < i \leq I_c \tag{4.39}$$

$$\begin{aligned} \sum_{1 \leq i \leq I_c} \sum_{o \in \mathcal{O}_c} \sum_{1 \leq j \leq \mathcal{J}_c} u_{ocji} + \sum_{1 \leq i \leq I_c} z_{ci} \leq \\ \sum_{m \in \mathcal{M}_c} \sum_{s \in \mathcal{S}_m} \bar{b}_m \rho_{cms} + B_c(1 - \lambda_c) \quad \forall c \in \mathcal{C} \end{aligned} \tag{4.40}$$

$$\sum_{1 \leq i \leq I_c} \hat{x}_{ci} \leq \sum_{m \in \mathcal{M}_c} \sum_{s \in \mathcal{S}_m} \bar{l}_{cm}\rho_{cms} + L_c(1 - \lambda_c) \quad \forall c \in \mathcal{C} \tag{4.41}$$

$$\sum_{s \in \mathcal{S}_m} \underline{l}_{cm}\rho_{cms} \leq \sum_{1 \leq i \leq I_c} \hat{x}_{ci} \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c \tag{4.42}$$

7. Process time of slitting lines: The right-hand-side of constraints (4.43) computes the time spent on cutting each coil, as the sum of the time needed to perform a crosscut, the time to slit the coil and the time to rewind the coil if it is partially used lengthwise. Constraints (4.44) assign this time to the slitting line on which the coil is cut. Constraints (4.45) compute the makespan.

$$\sum_{m \in \mathcal{M}_c} t_{cm} = \sum_{1 \leq i \leq I_c} \bar{t}(i-1)\delta_{ci} + \bar{t}\gamma_c^{\mathrm{P}} + \sum_{m \in \mathcal{M}_c} \sum_{s \in \mathcal{S}_m} \hat{t}_{cms}\rho_{cms} \quad \forall c \in \mathcal{C} \tag{4.43}$$

$$t_{cm} \leq \hat{q}_{cm} \sum_{s \in \mathcal{S}_m} \rho_{cms} \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c \tag{4.44}$$

$$\sum_{c \in \mathcal{C}_m} t_{cm} \leq C_{max} \quad \forall m \in \mathcal{M} \tag{4.45}$$

8. Accuracy: Constraints (4.46) compute the lack or excess weight served to each order.

$$\sum_{c \in \mathcal{C}: o \in \mathcal{O}_c} \sum_{1 \leq j \leq J_c} (D_c w_o) v_{ocj} - u_o^+ + u_o^- = b_o \quad \forall o \in \mathcal{O} \tag{4.46}$$

9. Strips of leftovers: The width of the strip of leftovers can be divided into retails and scrap (4.47). Analogously, the weight of the strip of leftovers can also be divided into retails and scrap (4.48). Constraints (4.49)-(4.53) ensure that the width and weight of retails are over the minimum established; otherwise, the strip of leftovers

is considered as scrap.

$$y_c = y_c^{\mathrm{r}} + y_c^{\mathrm{s}} \quad \forall c \in \mathcal{C} \tag{4.47}$$

$$z_{ci} = z_{ci}^{\mathrm{r}} + z_{ci}^{\mathrm{s}} \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.48}$$

$$\underline{a}\theta_c \leq y_c^{\mathrm{r}} \leq (W_c - 2r - \underline{w}_c)\theta_c \quad \forall c \in \mathcal{C} \tag{4.49}$$

$$\underline{b}\theta_c \leq \sum_{1 \leq i \leq I_c} z_{ci}^{\mathrm{r}} \quad \forall c \in \mathcal{C} \tag{4.50}$$

$$\sum_{1 \leq i \leq I_c} i z_{ci}^{\mathrm{r}} \leq B_c\theta_c \quad \forall c \in \mathcal{C} \tag{4.51}$$

$$y_c^{\mathrm{s}} \leq \overline{s}_c(1 - \theta_c) \quad \forall c \in \mathcal{C} \tag{4.52}$$

$$\sum_{1 \leq i \leq I_c} i z_{ci}^{\mathrm{s}} \leq B_c(1 - \theta_c) \quad \forall c \in \mathcal{C} \tag{4.53}$$

Constant $\overline{s}_c$ in constraints (4.52) is an upper bound of the width of the strip of leftovers from coil c to be considered as scrap. It can be computed as the maximum between $\underline{a}$ and the width corresponding to the leftovers of weight $\underline{b}$, $\overline{s}_c = \max\left\{\underline{a}, \frac{\underline{b}}{D_c \underline{L}_c}\right\}$

10. Variables' domain:

$$\gamma_c^{\mathrm{T}}, \gamma_c^{\mathrm{P}}, \beta_c^{\mathrm{T}}, \beta_c^{\mathrm{E}}, \lambda_c, \tau_c, \theta_c \in \{0,1\} \quad \forall c \in \mathcal{C} \tag{4.54}$$

$$\alpha_{cj} \in \{0,1\} \quad \forall c \in \mathcal{C}, 1 \leq j \leq J_c \tag{4.55}$$

$$\delta_{ci} \in \{0,1\} \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.56}$$

$$\hat{x}_{ci} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.57}$$

$$\rho_{cms} \in \{0,1\} \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c, s \in \mathcal{S}_c \tag{4.58}$$

$$x_c, y_c, y_c^{\mathrm{r}}, y_c^{\mathrm{s}}, e_c \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C} \tag{4.59}$$

$$z_{ci}, z_{ci}^{\mathrm{r}}, z_{ci}^{\mathrm{s}} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, 1 \leq i \leq I_c \tag{4.60}$$

$$u_o^+ \in [0, \overline{u}_o] \quad \forall o \in \mathcal{O} \tag{4.61}$$

$$u_o^- \in [0, \overline{u}_o] \quad \forall o \in \mathcal{O}^* \tag{4.62}$$

$$u_o^- \in \mathbb{R}_0^+ \quad \forall o \in \mathcal{O} \setminus \mathcal{O}^* \tag{4.63}$$

$$\varphi_{oc} \in \{0,1\} \quad c \in \mathcal{C}, o \in \mathcal{O}_c \tag{4.64}$$

$$\mu_{ocj} \in \{0,1\} \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.65}$$

$$v_{ocj} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c \tag{4.66}$$

$$u_{ocji} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, o \in \mathcal{O}_c, 1 \leq j \leq J_c, 1 \leq i \leq I_c \tag{4.67}$$

$$t_{cm} \in \mathbb{R}_0^+ \quad \forall c \in \mathcal{C}, m \in \mathcal{M}_c \tag{4.68}$$

$$C_{max} \in \mathbb{R}_0^+ \tag{4.69}$$

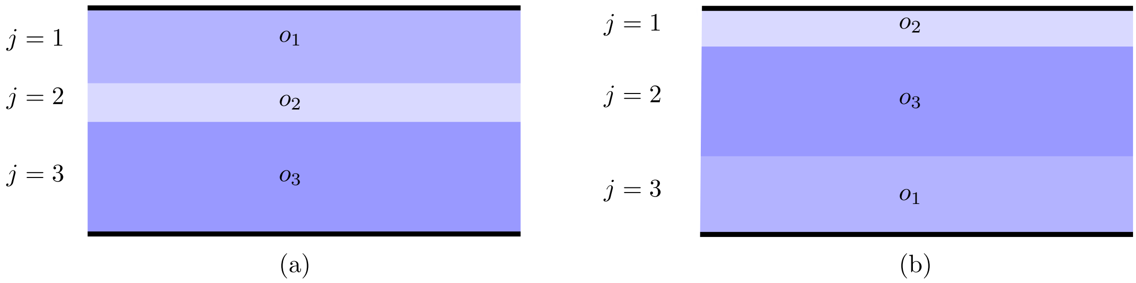


Figure 4.3: Two equivalent patterns for a coil.

4.4.3 Symmetry breaking constraints

Although the above described formulation is valid, it can be reinforced by eliminating symmetric solutions from the feasible region. The symmetry arises when assigning the widths of the orders to the strips of the coil. Fig. 4.3 shows two equivalent patterns that produce the same solution in terms of the operation. In pattern (a), the first strip is assigned to order o_1, the second to order o_2, and the third strip to order o_3, while in pattern (b) the first, second and third strips are assigned to orders o_2, o_3, and o_1, respectively. Both patterns and all other possible permutations produce the same solution (for a better performance, in practice, narrower strips are placed in the centre, and the wider strips at the edges). To avoid this situation, $\mathcal{O}$ can be considered as an ordered set and the following symmetry-breaking constraints can be added:

$$\mu_{ocj} + \sum_{o' \in \mathcal{O}_c : o \prec o'} \mu_{o'cj'} \leq 1 \quad \forall c \in \mathcal{C},\, o \in \mathcal{O}_c,\, j, j' : 1 \leq j' \leq j \leq J_c \tag{4.70}$$

These constraints ensure that if an order o is assigned to strip j, all subsequent orders $o' : o \prec o'$ cannot be assigned to a strip j' preceding j. The constraints improve the performance of the Branch and Cut procedure, reducing the set of feasible solutions by eliminating equivalent solutions.

4.5 Computational experience

In this section, we present the computational results of the experiments that have been carried out to validate the proposed model: a multi-criteria analysis to study the conflict among the different objectives, a performance analysis of the model in terms of both, model efficiency and quality of the solution, and an extended study of the impact of considering an integrated model for the cutting process and slitting line allocation.

The model has been implemented in AMPL (Fourer et al., 1990) and solved using the MIP Gurobi v.9.5.0 optimiser (Gurobi Optimization, 2020) with default settings, except

		Min.		**Max.**
Entry	Coil width (mm)	8/10	–	1,000/1,500
	Coil thickness (mm)	0.25/0.8	–	3/6
	Coil weight (kg)		–	10,000/25,000
	Coil outside diameter (mm)	600/700	–	1,900/2,100
Output	Strips weight (kg)		–	10,000/16,000
	Strips outside diameter (mm)	600/700	–	1,900/2,000

Table 4.1: Main characteristics of the slitting lines.

for the time limit, which is set to 30 minutes, unless otherwise stated. The experiments have been carried out in a virtual machine managed by OpenStack with 8 CPUs, 32GB RAM, Ubuntu 18.04 OS.

4.5.1 Data description

For the experiments, we have used real data provided by Cortichapa. The characteristics of the coils in stock and the customers' orders have been previously described in Section 3.5.1, therefore we will describe below the characteristics of the last element in our problem: the slitting lines.

The company owns four slitting lines with different technical characteristics and capabilities. In addition, each slitting line is able to work at two different speeds. Table 4.1 reports the valid range for the width, thickness, weight and external diameter of the coils that each slitting line can handle (which are used to determine the compatibility between the slitting line and the coil), and the maximum and minimum limits for the external diameter and weight of the set of strips of each pass (which are used to determine compatibility between the slitting line and the patterns).

The other two relevant characteristics of the slitting lines, which are needed to determine their compatibility with the coils and patterns, are the maximum number of knives that can be set in the slitter and the minimum cutting width of the strips (including the strip of leftovers). These parameters depend on the cutting speed, as well as, on the thickness and hardness of the coil. The company classifies the coils into different types according to their thickness and hardness. Note that, in contrast to the previous problem presented in Chapter 3, where the number of knives would only depend on the thickness of the coil, in this problem it also depends on the slitting machine, the cutting speed and the hardness of the coil.

Table 4.2 illustrates how the limits in the number of knives and cutting width vary among the different slitting lines and speeds for three selected types of coils. For instance,

Coil Type		Slitting lines 1 Slow	1 Fast	2 Slow	2 Fast	3 Slow	3 Fast	4 Slow	4 Fast
I	Max. number knives	15	10	20	15	18	-	20	20
	Min. cutting width	19	25	19	25	9	-	30	30
II	Max. number knives	15	10	30	25	19	19	30	30
	Min. cutting width	19	25	19	25	9	9	30	30
III	Max. number knives	-	-	-	-	4	-	5	-
	Min. cutting width	-	-	-	-	40	-	60	-

Table 4.2: Variability of the maximum number of knives and minimum cutting width.

for a Type I coil, up to 15 knives can be used if the coil is cut on Slitting line 1 at slow speed, whereas only up to 10 knives can be used if it is cut at fast speed. For the same type of coil, up to 18 knives can be used if the coil is cut on Slitting line 3 at slow speed (more than in the previous line) but it is not possible to cut it at fast speed. The same behaviour can be observed in the minimum cutting width. For a Type II coil, the width of strips has to be greater than 9 mm in the best scenario (Slitting line 3 at both speeds), but the width must be greater than 30 mm in the worst-case scenario (Slitting line 4). Finally, it can be observed that Slitting lines 3 and 4 are able to cut all three types of coils (Type III coil only at slow speed), whereas coils of Types I and II can only be cut on Slitting lines 1 and 2.

4.5.2 Impact of the symmetry breaking constraints

In order to study the impact of the symmetry breaking contraints, the model has been tested with the contraints and without them in a representative set of instances that will be introduced later in Section 4.5.4. Table 4.3 presents the results obtained. For the instances tested are shown: the characteristics of the instance (number of orders and coils), the model dimensions (number of constraints and variables) for the model without considering the symmetry breaking constraints, the computing time for the model with the symmetry breaking constraints (✓ SBC), without the symmetry breaking constraints (✗ SBC), its variation (Var.), the objective value for the model without the symmetry breaking constraints (Z_{IP}), and the optimality gap for both models.

The results obtained present a reduction of more than 25% on average in the computing time for instances solved up to optimality (see Var. column); an order of magnitude reduction in the optimality gap for instances where the optimiser cannot guarantee optimality after 1200 seconds; and, a good feasible solution is obtained for an instance where the optimiser cannot provide a feasible solution without these constraints. Taking into

Instance			Model dimensions			Computing time			Optimality gap (%)		
type	$\|\mathcal{O}\|$	$\|\mathcal{C}\|$	cons.	vars.	0-1 vars.	✓ SBC	✗ SBC	Var.	Z_{IP}	✓ SBC	✗ SBC
CO01	7	67	7,173	4,057	1,755	4	3	28%	209757.3	0.00	0.00
CO02	7	191	54,574	26,992	10,630	1200	1200	-	71851.2	98.61	90.31
CO03	7	65	9,780	5,078	2,193	44	39	13%	1974759.4	0.00	0.00
CO04[a]	8	65	18,582	9,927	3,476	269	1200	-78%	225002.4	0.00	0.92
CO05[a]	9	138	26,371	13,170	5,763	219	1200	-82%	124947.2	0.00	5.75
CO06[a]	11	213	26,563	14,503	6,250	61	225	-73%	1381237.8	0.00	0.00
CO07[a]	12	152	19,679	11,012	4,474	13	28	-55%	1846562.7	0.00	0.00
CO08[a]	14	179	45,169	28,345	7,660	392	1200	-67%	2326789.1	0.00	2.70
CO09[a]	14	160	19,654	10,895	4,632	9	14	-36%	1078144.3	0.00	0.00
CO10[a]	15	328	62,429	33,332	13,223	433	1200	-64%	420451.0	0.00	5.86
PS01[a]	25	114	29,267	15,078	5,638	39	99	-61%	791637.5	0.00	0.00
PS02[a]	25	117	23,511	12,099	5,000	38	59	-36%	1219565.1	0.00	0.00
PS03	25	119	22,901	11,787	4,770	1200	1200	-	1281949.7	1.57	1.37
PS04	25	129	24,626	13,454	5,145	49	43	15%	981700.3	0.00	0.00
PS05[b]	50	165	94,372	52,371	14,390	1201	1200	-	1970205.9	0.64	10.49
PS06[c]	50	175	65,432	33,660	11,373	1200	1201	-	-	21.24	-
PS07	50	189	66,146	37,339	11,018	1201	1200	-	1732986.2	3.83	6.35
PS08	50	196	115,139	74,158	14,983	1201	1201	-	1910983.0	7.15	12.61
PS09[b]	75	209	111,655	61,536	17,433	1201	1201	-	5280305.6	3.49	52.09
PS12[b]	75	222	72,031	39,362	12,644	1201	1200	-	2407410.7	1.67	18.22
XX01[a]	25	101	30,692	17,067	5,448	97	101	-3%	894415.2	0.00	0.00
XX02[a]	50	153	45,781	25,305	8,153	135	184	-27%	1914324.5	0.00	0.00
Avg.								**-37%**			

[a] The computing time is reduced.
[b] The optimality gap is reduced one order of magnitude.
[c] A good feasible solution is found thanks to SBC.

Table 4.3: Results obtained considering or not the symmetry breaking constraints.

	I1			I2		
Objective	f_1	f_2	f_3	f_1	f_2	f_3
f_1	<u>145,750</u>	1462	2352	<u>339,798</u>	1424	9250
f_2	370,080	<u>772</u>	3171	359,546	<u>1412</u>	7103
f_3	915,162	1493	<u>109</u>	1,120,400	2151	<u>16</u>

Table 4.4: Payoff matrix for selected instances.

account these results the symmetry breaking constraints are considered in the model.

4.5.3 Multi-criteria analysis

The first step of our analysis is to investigate the confrontation among the three different goals considered: the reduction of leftovers (measured in f_1), productivity maximisation (f_2) and customer satisfaction (f_3). We start by obtaining the payoff matrix: the optimal solution for each objective is obtained and evaluated for the other two. Taking into account the company's preferences, we have set the following values to the parameters in the different objectives. For the first objective we consider the following weights for the different types of leftovers: $s_1 = 1$ for strips that are for retails, $s_2 = 4$ for strips that are for scrap and $s_3 = 10$ for rolled leftovers. The second objective is penalised with $s_4 = 1$ for the total processing time and $s_5 = 10$ for the makespan.

Table 4.4 reports the resulting payoff matrix for two illustrative instances (with 10 orders and 50 coils). Each row represents the value of the three objectives in the solution obtained by solving the problem for f_1, f_2 and f_3 independently. The ideal value, underlined in the table, is a solution - most likely unfeasible - that attains the best possible outcome for each of the objectives. These instances reflect the two situations we have encountered. On the one hand, for instance I1 there is a major conflict among the three objectives, since the optimal solution for each objective provides very poor values for the other two. On the other hand, for instance I2, there is no great conflict between f_1 and f_2, since the optimal solution for each objective provides good values for both objectives (f_1 varies from $339,798$ to $359,546$ and f_2 varies from 1412 to 1424). However, these two objectives, related to process performance, are in conflict with f_3, which is customer satisfaction (note that the value of the objective for the optimal solution of f_1 is nowhere near its value for the optimal solution of the other two objectives).

Once the payoff matrix is obtained, we can approximate the Pareto region by using the augmented ε-constraint method presented in Mavrotas and Florios (2013). The Pareto region provides a broader picture of the confrontation among the different objectives. This allows the decision-maker to select a suitable solution, depending on their preferences, and based on the degree of conflict. Fig. 4.4 shows a graphical representation of the Pareto region for the two illustrative instances. The conflict between f_1 and f_2 can be observed in the graph on the left. In instance I1 there is no relationship between the two objectives, while in instance I2 there is a linear dependence. In the other two graphs, we can observe that the conflict between f_3 and the other two objectives is similar. The Pareto front can also help by selecting the multi-criteria method to be implemented, such as Goal Programming (firstly introduced by Charnes et al. (1955)), Compromise Programming (introduced by Cochrane and Zeleny (1973)) or another. Note that computing the Pareto front is often very time consuming, even for small instances. Therefore, in our approach, we have selected the set of weights $(\omega_1, \omega_2, \omega_3) = (30, 10, 1)$ for the different objectives according to the decision maker's preferences.

Fig. 4.5 illustrates the weight of the coils used (consumption), the distribution of the coils among the slitting lines (time), and the satisfaction of customers' orders (accuracy) when each objective is optimised independently and when the proposed weighted sum is considered. When objective f_1 (reduction of leftovers) is optimised, in both instances the use of tons of coils is kept to a minimum and, above all, the rolled leftovers are reduced. However, the makespan and production time are large in instance I1, where the confrontation between these two objectives is higher. In instance I2, where the two objectives are not in high confrontation, the makespan and production time are similar to the ideal value. When objective f_2 (makespan and production time) is optimised, it can be observed that the coils are better distributed among the slitting lines, reducing the makespan and production time (which is greatly reduced in instance I1, where the conflict concerning the reduction of leftovers is higher). However, the cutting plan uses more tons of coils in instance I1 and produces a greater amount of rolled leftovers. Observe that, in instance I2, where the conflict between the reduction of leftovers and the makespan and production time minimisation are lower, the behaviour of both objectives is similar. When objective f_3 is optimised, the boxplot shows that almost all the orders are served with accuracy 1. However, this is achieved by increasing the number of tons used and the leftovers, especially rolled leftovers. This results in an increase in the makespan and in production time, due to the processing time needed to perform the crosscuts. Finally, when using a weighted sum of the three objectives, we obtain a solution whereby the used tons and leftovers are optimised, resulting in higher accuracy and shorter production times and makespan. Observe that in instance I2, the makespan is shorter than that obtained when optimising objective f_2, however, the production time is increased.

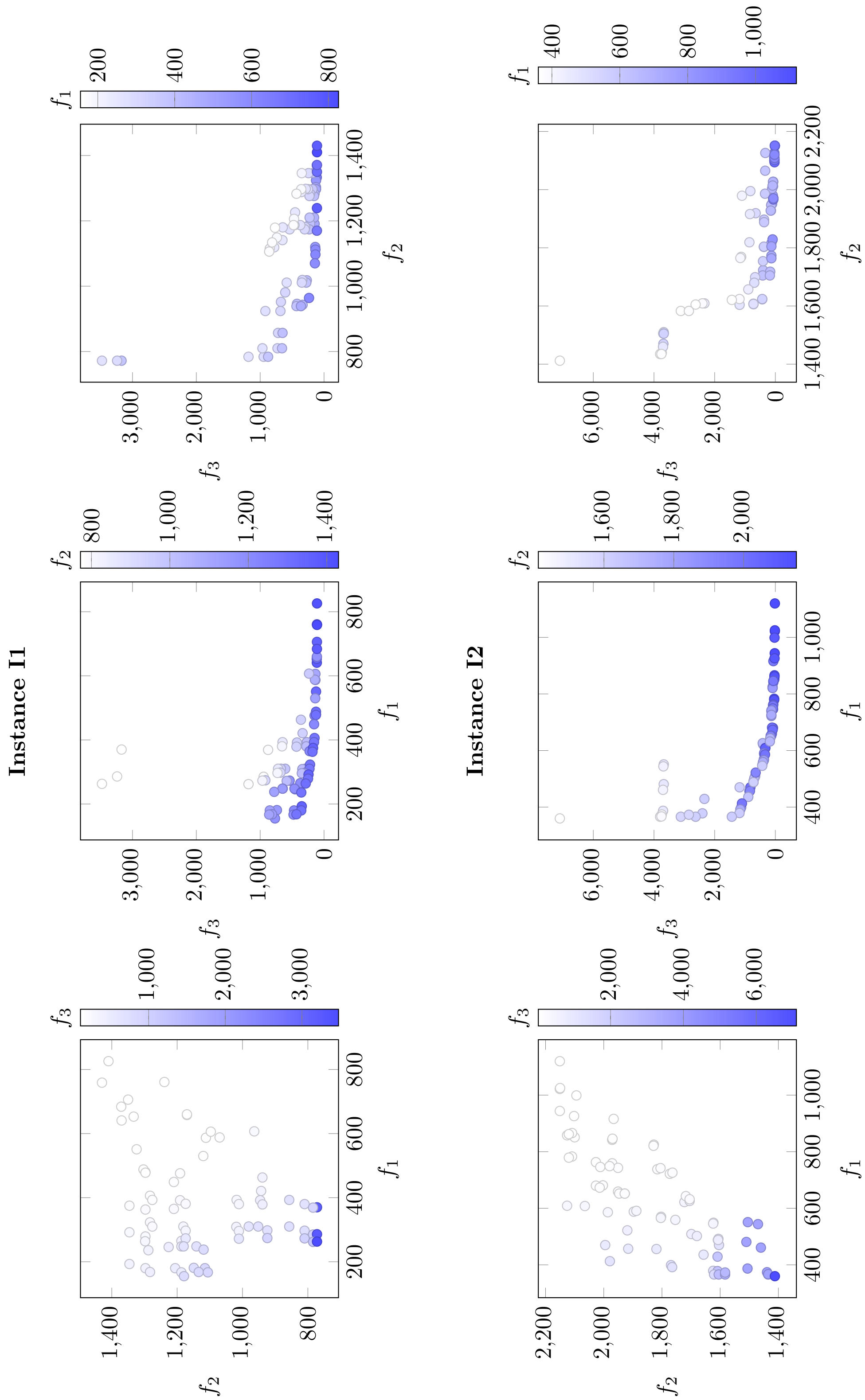


Figure 4.4: Representation of the Pareto region (projections) in the objectives space.

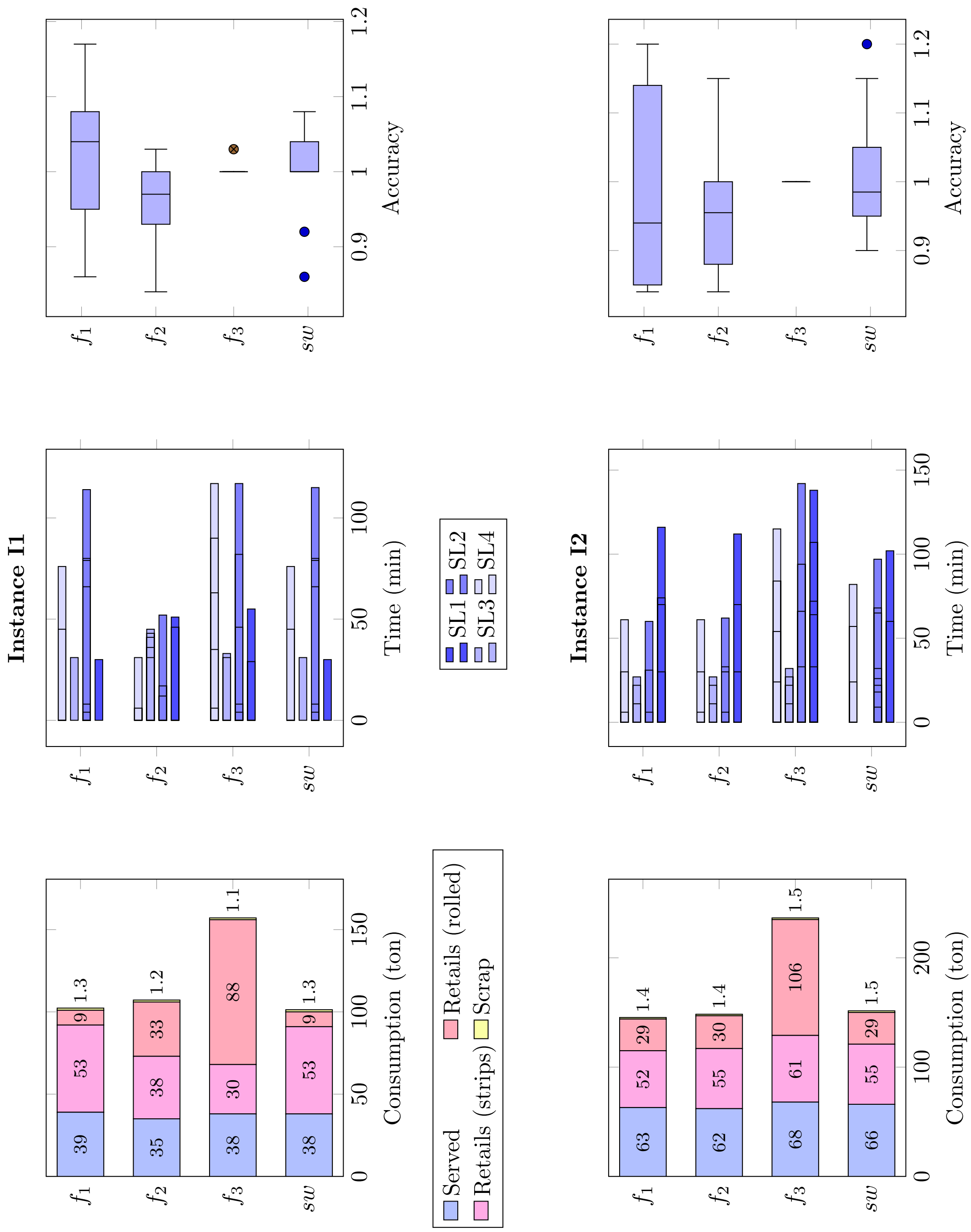


Figure 4.5: Main key performance indicators for the solutions in the payoff matrix.

4.5.4 Computational results

Using the aforementioned data provided by the company, three representative sets of instances have been created to test the model. The set of slitting lines and operational parameters are common for all instances, while orders and stock are different:

- Current Operation (CO): 10 instances that correspond to the operations of the company on ten different working days in 2019, for the orders that were planned as well as for the full compatible stock. The company has provided the cutting patterns used for these instances hence a comparison of the model performance can be established. Note that these instances might differ from the ones presented in the previous chapter since the information of the slitting lines has been added and the coils-slitting lines compatibility has been taking into account.
- Random generation of orders assuming All compatible Stock (AS): orders are randomly selected from a pool of around 300 historical orders, and then, all compatible coils in the current stock (that includes around 3000 coils) are considered. These instances reproduce a realistic situation.
- Random generation of orders and Partial Stock (PS): As in the previous instances, orders are randomly selected from a pool of around 300 historical orders, while the set of coils is merely a (random and representative) subset of compatible coils from the current stock.

For the second and third sets of instances, the number of orders is set to 25, 50, and 75, and for each size, four different instances are randomly generated. Then, each set includes 12 instances and, together with the CO instances, 34 instances have been tested in total. Instances with 100 orders have also been tested, nevertheless, no results have been obtained in less than 30 minutes.

Table 4.5 reports a number of statistics on the computational performance of the model for each instance: description (number of orders and coils in stock), model dimensions (number of constraints, variables, and binary variables), performance (computing time in secs., the value of the objective function for the best solution found, and the optimality gap for this solution). The key performance indicators of the solution proposed by the model are reported in Table 4.6: the number of coils used in the solution and their total weight in kg are indicated, together with the weight served, the weight of retails and the weight of the scrap. In addition, the percentage that each weight represents over the total weight used is also indicated. The following three columns report the minimum, average and maximum values of the order accuracy. We define the order accuracy as the ratio between the required weight and the served weight. In this way, the order accuracy values under 1 indicate that the required weight is more than the served weight. The Slit and

Instance			**Model dimensions**			**Performance**		
type	$\lvert\mathcal{O}\rvert$	$\lvert\mathcal{C}\rvert$	cons.	vars.	0-1 vars.	time	Z_{IP}	gap (%)
CO01	7	67	9,200	4,057	1,755	6	209757.3	0.00
CO02*	7	191	105,807	26,992	10,630	1800	58265.3	98.61
CO03	7	65	13,156	5,078	2,193	76	1974759.4	0.00
CO04	8	65	28,672	9,927	3,476	377	225002.4	0.00
CO05	9	138	40,581	13,170	5,763	367	124947.4	0.00
CO06	11	213	33,836	14,503	6,250	91	1381237.8	0.00
CO07	12	152	26,390	11,012	4,474	19	1846562.7	0.00
CO08	14	179	63,993	28,345	7,660	558	2326635.2	0.00
CO09	14	160	26,824	10,895	4,632	15	1078144.1	0.00
CO10	15	328	97,185	33,332	13,223	679	419654.8	0.00
AS01*	25	1032	670,971	162,354	61,053	1801	705443.3	13.01
AS02*	25	935	526,441	127,063	48,415	1801	1867384.7	0.09
AS03*	25	990	490,510	140,174	50,382	1801	955705.6	18.71
AS04*	25	798	433,551	120,868	45,749	1801	886606.9	41.53
AS05	50	1339	1,421,795	321,067	105,778	1801	–	–
AS07	50	1416	1,161,237	351,836	90,913	1801	–	–
AS06	50	1136	1,112,607	331,288	82,446	1801	–	–
AS08*	50	1579	889,651	227,370	81,118	1801	1412703.3	13.54
AS09	75	1523	2,005,134	443,430	134,438	1802	–	–
AS10	75	1724	2,234,881	572,637	142,991	1802	–	–
AS11	75	1690	1,247,056	330,168	98,853	1801	–	–
AS12	75	1783	1,553,106	387,274	114,482	1802	–	–
PS01	25	114	56,656	15,078	5,638	56	791637.5	0.00
PS02	25	117	43,664	12,099	5,000	51	1219565.1	0.00
PS03*	25	119	35,811	11,787	4,770	1800	1283426.6	1.61
PS04	25	129	44,667	13,454	5,145	77	981700.2	0.00
PS05*	50	165	233,475	52,371	14,390	1800	1953730.8	0.38
PS06*	50	175	114,901	33,660	11,373	1800	1584025.0	20.34
PS07*	50	189	138,676	37,339	11,018	1800	1723786.0	3.88
PS08*	50	196	225,528	74,158	14,983	1800	1870333.9	6.80
PS09*	75	209	261,427	61,536	17,433	1800	2789277.7	3.29
PS10	75	223	354,414	96,628	22,060	1801	–	–
PS11	75	214	358,676	82,544	21,680	1801	–	–
PS12*	75	222	140,582	39,362	12,644	1800	2346239.4	1.79

* Optimality has not been proved.

Table 4.5: Model dimensions and computational performance indicators.

Instance			**Used coils**		**Served**		**Retails**		**Scrap**		**Order accuracy**			**Cuts**		**Rolled**		**Productivity**	
Type	$\lvert\mathcal{O}\rvert$	$\lvert\mathcal{C}\rvert$	#	Weight	Weight	%	Weight	%	Weight	%	Min	Mean	Max	Slit	Cross	#	%	T	C_{max}
CO01	7	67	11	121038	103378	85.4	16025	13.2	1635	1.4	0.99	1.0	1.0	7	5	0	0.0	191	78
CO02	7	191	11	96758	92001	95.1	3522	3.6	1235	1.3	1.0	1.0	1.0	13	2	0	0.0	91	54
CO03	7	65	11	158706	123007	77.5	33675	21.2	2024	1.3	0.92	0.99	1.02	8	5	1	51.6	236	95
CO04	8	65	7	91605	75036	81.9	15354	16.8	1215	1.3	0.92	1.01	1.05	9	6	0	0.0	185	62
CO05	9	138	12	97401	89961	92.4	5721	5.9	1719	1.8	0.97	1.01	1.1	10	0	0	0.0	51	24
CO06	11	213	14	137914	94256	68.3	41764	30.3	1894	1.4	0.82	0.95	1.03	6	7	1	21.8	234	96
CO07	12	152	16	142417	90640	63.6	48646	34.2	3131	2.2	0.0	0.85	1.2	6	4	1	28.1	212	83
CO08	14	179	10	120140	79984	66.6	38617	32.1	1539	1.3	0.0	0.94	1.2	7	6	2	49.6	242	104
CO09	14	160	18	146624	111284	75.9	32830	22.4	2510	1.7	0.81	1.0	1.19	9	8	1	20.8	316	100
CO10	15	328	20	149634	116515	77.9	29971	20.0	3148	2.1	0.98	1.02	1.2	8	2	0	0.0	190	57
AS01	25	1032	50	225602	165906	73.5	57185	25.3	2511	1.1	0.0	0.96	1.12	4	7	0	0.0	292	98
AS02	25	935	35	295988	218752	73.9	74639	25.2	2597	0.9	0.85	1.0	1.09	6	6	1	14.6	326	123
AS03	25	990	35	355786	281281	79.1	70888	19.9	3617	1.0	0.86	1.0	1.15	7	12	0	0.0	450	197
AS04	25	798	43	322971	257550	79.7	61484	19.0	3937	1.2	0.0	0.92	1.18	7	7	0	0.0	402	140
AS08	50	1579	91	551135	449460	81.6	94562	17.2	7113	1.3	0.0	0.88	1.15	5	31	0	0.0	1152	426
PS01	25	114	22	249705	186286	74.6	60927	24.4	2492	1.0	0.0	0.91	1.17	6	14	0	0.0	444	168
PS02	25	117	25	316753	218563	69.0	94549	29.8	3641	1.1	0.0	0.92	1.15	6	13	0	0.0	436	193
PS03	25	119	26	336726	231364	68.7	102671	30.5	2691	0.8	0.0	0.94	1.14	7	9	0	0.0	348	130
PS04	25	129	25	241518	159815	66.2	78581	32.5	3122	1.3	0.0	0.79	1.13	8	15	0	0.0	487	160
PS05	50	165	41	472197	319225	67.6	147716	31.3	5256	1.1	0.0	0.87	1.2	8	23	0	0.0	782	242
PS06	50	175	52	548797	432804	78.9	110942	20.2	5051	0.9	0.0	0.91	1.15	7	19	0	0.0	757	269
PS07	50	189	42	450488	318052	70.6	128534	28.5	3902	0.9	0.0	0.83	1.18	6	23	0	0.0	712	246
PS08	50	196	42	493101	339727	68.9	148176	30.0	5198	1.1	0.0	0.79	1.18	8	24	0	0.0	805	261
PS09	75	209	54	693724	458752	66.1	228116	32.9	6856	1.0	0.0	0.71	1.19	8	25	0	0.0	969	330
PS12	75	222	53	597331	403353	67.5	188737	31.6	5241	0.9	0.0	0.71	1.17	7	27	0	0.0	906	326

Table 4.6: Solution performance indicators.

Cross columns provide the number of slits performed on average per coil, and the total number of guillotine crosscuts made in the solution. The Rewound column provides the number of coils that are rewound and the percentage of weight it represents over the weight of the retails. Finally, the last two columns report the total processing time of the coils used in the solution (T) and the makespan (C_{max}) of the slitting lines.

We observe that the utilisation of the coils is similar in all sets of instances, around 70%, including those instances where optimality has not been proved (marked with *). These values are similar to the values obtained in the model presented in Chapter 3, and outperformed the current solution implemented by the company, where the utilisation of the coils is around 50%.

Regarding the sets of instances AS and PS, we observe that as the size of the set of orders increases, it becomes more difficult to obtain a feasible solution within 30 minutes. For the same number of orders, it becomes more difficult when there are more coils in stock (only one instance with 50 orders is solved and none with 75 orders in set AS, whereas all instances with 50 orders and two with 75 orders are solved in set PS). These results suggest that, for larger problems, the model will need more time to obtain a feasible solution. Moreover, other strategies would have to be investigated in order to obtain good feasible solutions in a reasonable time.

4.5.5 Planning in two independent stages vs the integrated model

Currently, the company carries out the planning of the slitting process in two sequential stages. In the first stage, the company solves a 1.5D-CSP to select the coil and designs the cutting patterns, and then, in the second stage, each coil is assigned to a compatible slitting line.

In order to solve the problem at the first stage, several parameters must be set, such as the maximum number of knives. Although there are several criteria to choose this value, the company sets a maximum number of knives equal to the maximum value allowed by the slitting line, that allows the maximum value. For example, if we observe Table 4.2 which reports the slitting line restrictions, for Type I coils, the maximum number of knives is set at 20, which is what slitting line 2 allows. In practice, this criterion is the most logical since it provides more flexibility: if the company has to serve orders with a minimum width of 19 mm and the cutting line allows minimum widths of 19 mm, no additional restrictions appear to be imposed. However, as we will observe later, problems may arise, since this 19 mm limit must also be imposed on the leftovers which may lead to unfeasible situations.

The combined approach of both decisions regarding the design of the cutting patterns and cutting line allocations, allows overcoming these drawbacks. Consequently, it provides greater flexibility in the construction of the patterns, and achieves a better distribution of the workload among the different cutting lines.

We have tested the two strategies with part of the previous instances (instances with 25 and 50 orders, where a feasible solution was obtained within the time limit), by setting the time limit to one hour for all cases. Table 4.7 reports the main performance indicators for the solution obtained with the integrated model for each tested instance. The last row reports the average of these indicators. For each indicator, a column Dev. (%) has been added, showing the deviation of the integrated model from the two-stage model, except for order accuracy, since it is very similar for both strategies. Positive deviations indicate higher values for the integrated model than for the two-stage model. Indicators that improve when using the proposed model, are reported in bold. In general, it can be observed that the integrated model provides better values than the two-stage model, although the differences in the weight served, as well as in the weight of the retails and scrap, are not relevant. However, if we look at the time used by the slitting lines and the makespan, it can be observed that the integrated model provides much lower values, with an average reduction of more than 15%. There is also a strong reduction in crosscuts, which is one of the elements that most affects the processing time. Our proposal only provides worse times for instance PS02, although in terms of weight served and leftovers generated, it improves the two-stage model.

Note that instance AS07 is unfeasible with the two-stage strategy, while the integrated model provides a solution. In order to illustrate what is happening, let us suppose that the company holds a 500mm Type I coil and has orders of different widths compatible with this coil. The patterns for the coil may have up to 19 strips (including the strip of leftovers, if applicable) if the coils are cut on slitting line 2, but only up to 17 if they are cut on slitting line 3, since these slitting lines allow a maximum of 20 and 18 knives, respectively. In the two-stage model, patterns are designed without taking into account the slitting line allocation and, therefore, the company must set a maximum number of knives (20 in this case), forcing the minimum width of the leftovers to be set at 19 mm. Let us assume that one of the orders is for 120 mm width strips, which must be fulfilled with four strips; then, from the 500 mm width of the coil, 480 mm are dedicated to these four strips and 5+5 mm to the edge trimming, resulting in a 10 mm leftover strip. This pattern cannot be obtained as a possible solution in the first stage model, since a minimum width of 19 mm was imposed for all the strips. Then, the first stage model discovers that the instance is unfeasible. Nevertheless, this pattern could be cut on slitting line 3, since its minimum cutting width is 9 mm.

Given that the different slitting lines are not considered in the first stage, it is not

Instance			Used coils				Served		Retails		Scrap		Productivity					
name	$\lvert\mathcal{O}\rvert$	$\lvert\mathcal{C}\rvert$	#	Dev.	Weight	Dev. (%)	Weight	Dev. (%)	Weight	Dev. (%)	Weight	Dev. (%)	Cross cuts	Dev.	T	Dev. (%)	C_{max}	Dev. (%)
PS02	25	117	25	0	316753	0.00	218563	0.00	94549	0.00	3641	0.00	**13**	**-1**	**436**	**-5.01**	193	+2.66
PS03	25	119	26	0	336726	0.00	231401	0.00	102634	0.00	2691	0.00	**9**	**-2**	**348**	**-11.68**	**130**	**-11.56**
PS04	25	129	25	+2	241518	-1.80	159815	-0.01	**78581**	**-6.26**	3122	+36.39	15	+3	487	+26.49	160	+42.86
PS05	50	165	41	0	472198	-0.42	**319225**	**+0.46**	**147716**	**-2.42**	5256	+3.96	**23**	**-10**	**780**	**-21.21**	**242**	**-15.09**
PS06	50	175	54	+2	550650	-0.72	**436072**	**+0.28**	**109755**	**-4.36**	**4823**	**-4.21**	**17**	**-13**	**703**	**-27.23**	**256**	**-30.81**
PS07	50	189	42	0	450488	0.00	**318052**	**+0.26**	**128534**	**-0.63**	3902	0.00	**23**	**-13**	**712**	**-27.27**	**246**	**-28.70**
PS08	50	196	42	-1	493101	+0.24	**339731**	**+0.14**	148172	+0.41	5198	+1.72	**24**	**-12**	**803**	**-23.60**	**261**	**-25.00**
AS01	25	1032	46	-1	224625	-0.15	**165925**	**+0.05**	**56340**	**-0.91**	2359	+3.87	7	0	**283**	**-1.74**	107	**+1.90**
AS02	25	935	35	0	295988	-0.02	218752	0.00	74639	0.00	**2597**	**-2.70**	**6**	**-2**	**326**	**-12.83**	**123**	**-13.99**
AS03	25	990	36	+4	358580	-0.20	**281657**	**+0.14**	**73954**	**-1.17**	**2969**	**-6.66**	**11**	**-4**	**438**	**-19.04**	**202**	**-4.27**
AS04	25	798	40	+1	318951	-0.66	257839	-0.65	**57470**	**-1.52**	3644	+15.06	**6**	**-4**	**381**	**-13.80**	**141**	**-11.88**
AS07*	50	1416	65	-	585393	-	454980	-	124172	-	6242	-	21	-	829	-	284	-
AS08	50	1579	80	+2	547508	-0.10	449376	-0.11	**91182**	**-0.10**	6951	+0.42	**27**	**-8**	**982**	**-18.57**	**322**	**-24.24**
Avg.			39.5	+0.7	373599	-0.27	**275592**	**+0.06**	**94189**	**-1.33**	3819	+2.52	**15**	**-5**	**548**	**-16.39**	**196**	**-15.16**

Table 4.7: Main performance indicators comparison.

possible to make the lower limit of the width of the strip of the leftovers dependent on the characteristics of the line. However, this is achieved with the integrated model presented in this chapter.

4.6 Conclusions and future research

A mixed integer linear optimisation model has been presented to solve a complex 1.5-dimensional cutting stock problem in the steel industry. The model integrates both, the coil selection and the definition of the cutting patterns, as well as the coil-slitting line allocation. This integration allows the specific constraints for the slitting lines to be assumed when selecting the coils from the stock and, when defining the cutting patterns. The steel company owns a heterogeneous set of slitting lines, each of which has particular specifications. Subsequently, unless integration is considered, the solution proposed could overload the most versatile slitting line, or may even obtain cutting patterns that cannot be processed in any of the slitting lines.

The computational results show how the integrated model improves the company's current operations, without reducing the service quality of the orders. Consequently, the leftovers are reduced and the workload distribution for the slitting lines is improved: the makespan and the total processing time are also reduced.

Furthermore, a multi-criteria analysis has been presented in order to study the existing confrontation among the different objectives considered. It has been observed that there exists a conflict between the accuracy of the order (deviation from the ordered weight) and the process productivity (including both, the leftovers and the cutting times). However, it has been studied how a selection of weights for the objectives is a useful approach in order to find solutions with a good trade-off among these objectives.

As far as future research is concerned, two main lines are identified: on the one hand, incorporating coil scheduling in the slitting lines, which will allow considering customer deadlines and weekly planning. While, on the other hand, as the model is too complex (in computational terms) and unable to solve large instances, metaheuristics or matheuristics could help to solve the problem or, at least reduce the feasible solutions region.

Chapter 5

Conclusions, original contributions and future research

Contents

The final conclusions of this PhD are presented in this chapter, together with the original contributions and some future lines of research.

5.1 Conclusions

The main objective of this PhD, as stated in Section 1.2, was to develop decision support systems to help companies to tackle their complex business problems, and make optimal decisions that maximize their operational efficiency. This has been achieved through the development of three mathematical optimisation models to tackle the planning problems of two international companies, one in the retail sector and the other in the steel industry.

In the first case, the model has allowed the company to gain a better knowledge of its network of stores, while in the second case there has been a concrete transference of the results through the implementation of the proposed models in the planning systems of the company. The main conclusions are detailed below:

- **Facilities delocation in the retail sector**: The problem of redesigning the network of stores of a retail chain has been studied, considering possible closures or changes in the management policy of the stores. In order to tackle this problem, a mathematical optimisation model has been proposed to decide whether the stores should change their management policy or should be delocated, with the aim of improving the operation of the network. The type of decisions extracted from this model do not apply directly to the structure of the network, since the network is affected by other criteria (such as the company policy or the competition, among others) that cannot always be taken into account. However, the model allows the company to know what is the best structure in terms of profit maximisation, and to make more informed decisions.

- **The slitting problem in the steel industry**: An integer linear optimisation model has been developed to obtain a cutting plan for the slitting process in a steel manufacturing company. With the aim to meet the demand, the cutting plan defines which coils will be selected from the stock and the cutting patterns for each of them. The model developed has provided the company with a planning solution that improves its current operations by reducing the time invested in planning, and by improving several operational aspects: the utilisation of the coils has been significantly improved, from using 50% of the material to serve customer orders in the current operation, to using 80% in the solutions proposed by the model. This means a considerable reduction in the leftovers, both reusable and non-reusable. In addition, the model proposes solutions where smaller coils are selected from the stock that correspond to leftovers from previous cutting processes. This fact, together with the reduction of leftovers, will lead to a reduction in the stock processed in the medium term, with the consequent improvement in the stock management. Furthermore, the model is able to better adjust the weight served to the weight ordered, improving the company's profit by reducing penalties or discounts due to these deviations.

- **The slitting problem with slitting lines allocation**: The model proposed considers the integration of the coil selection, the cutting pattern design and the slitting line allocation. Although it is not yet in production, this model allows a better planning of the slitting process, since it improves the distribution of the workload among the different slitting lines and solves some unfeasibility issues that appeared in the previous model, without deteriorating the objectives previously considered. Furthermore, it allows us to include orders that can be served in several days, forcing only orders that are required to be completed within the current cutting plan, and allowing to plan non required orders as long as the utilisation of the coils is improved.

5.2 Contributions

This section is devoted to present the main contributions obtained during the development of this PhD. Two papers have been published in journals indexed in the *Journal Citation Report* (JCR), an extended abstract has been published in the proceedings of a conference, and another paper has been submitted to a journal indexed in the JCR. They can be found in Section 5.2.1.

Additionally, the main results of this thesis have been presented in several national and international conferences. They are listed in Section 5.2.2.

5.2.1 List of publications

Published papers

- **Sierra-Paradinas, M.**, Alonso-Ayuso, A., Martín-Campo, F.J., Rodríguez-Calo, F. & Lasso, E. (2020), 'Facilities Delocation in the Retail Sector: A Mixed 0-1 Nonlinear Optimization Model and Its Linear Reformulation', *Mathematics* **8**(11):1986. doi:10.3390/math8111986.
- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (2021), 'An exact model for a slitting problem in the steel industry', *European Journal of Operational Research* **295**(1), 336-347. doi:10.1016/j.ejor.2021.02.048.

Extended abstract published in proceedings

- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (2021), 'A mathematical methodology for planning the slitting process in the steel industry.', Galán, J. M., Díaz-de la Fuente, S., Alonso de Armiño Pérez, C., Alcalde Delgado, R., Lavios Villahoz, J. J., Herrero Cosío, Á., Manzanedo del Campo, M. Á., del Olmo Martínez, R. *Proceedings of the 15th International Conference on Industrial Engineering and Industrial Management and XXV Congreso de Ingeniería de Organización*

Submitted paper

- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. (submitted March 2022), 'An exact model for the 1.5-dimensional cutting stock problem in the steel industry with heterogeneous parallel slitting lines allocation', *European Journal of Operational Research*.

5.2.2 Conferences

- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. A mathematical methodology for planning the slitting process in the steel industry. 15th International Conference on Industrial Engineering and Industrial Management (ICIEIM), Burgos. 2021.
- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. An exact model for a slitting problem in the steel industry. 34th Conference of the European Chapter on Combinatorial Optimization, Madrid. 2021.
- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. The slitting problem in the Steel industry: A case study. Mini symposium Math-In: Success Stories between Academia and Industry. XXVI Congreso de Ecuaciones Diferenciales y Aplicaciones XVI Congreso de Matemática Aplicada, Gijón. 2021.
- **Sierra-Paradinas, M.**, Soto-Sánchez, Ó., Alonso-Ayuso, A., Martín-Campo, F.J., & Gallego, M. A mathematical model for the slitting problem in a Spanish steel manufacturing company. New Bridges between Mathematics and Data Science, Valladolid. 2021.

5.3 Future work

This section is aimed to present some lines of future research for the problems tackled in this PhD thesis.

- **Facilities delocation in the retail sector**: We are assuming that all the parameters are known, however, this is not always the case. The amount of goods consumed that corresponds to the demand of the network may change over time. To incorporate the uncertainty in the demand we could consider Stochastic Optimisation via Scenario Analysis. In this approach, the uncertainty in the parameters is represented by their probability distributions. Then, a scenario tree is created from a representative set of the possible values of the parameters. The models obtained when using this approach are large and complex, but by properly exploiting the structure of the formulations, it is possible to use decomposition methods to solve problems with even millions of variables.

  From the point of view of resolution, we have observed that when considering larger networks, the complexity increases and it is not possible to solve large instances in a reasonable computational time. For these cases, it is necessary to investigate other optimisation techniques, such as metaheuristics or matheuristics. Although they are not exact methods, they provide good solutions in reasonable computing times.

- **The slitting problem in the steel industry**: As far as the future investigation is concerned, the company is interested in carrying out the planning process over a week including more orders with their corresponding deadlines and sequencing the workload over different days, indicating which coils should be cut each day to meet the due dates. During the past years, interest in the combination of cutting stock problems and sequencing has grown. Recently Pitombeira-Neto and Prata (2019) Pitombeira-Neto and de A. Prata (2019) presented a matheuristic for the 1D-CSP combined with the scheduling process that appears in a multi-period problem. Arbib and Marinelli (2014) Arbib and Marinelli (2014) proposed an exact integer linear optimisation formulation that assigns patterns to periods and developed primal heuristics for a cutting stock problem with due dates. Reinertsen and Vossen (2010) Reinertsen and Vossen (2010) addressed the 1D-CSP when orders must be completed before the due date and presented novel optimisation models that were solved with column generation procedures. Set up costs due to the adjustments of the knives will need to be considered to obtain the changing times between coils in the different slitting lines.

  This new model will be far more complex and it will require more computational effort to solve full-size instances. Therefore, we need to consider more time-efficient resolution techniques. To solve the problem presented in Chapter 3 we are working on a metaheuristic that is composed of a constructive phase with a random component and a variable local search. The results, although they are good and obtained in a few minutes, they are yet not comparable to the solutions obtained with the model presented in this work. For the problem presented in Chapter 4 we are working on applying a matheuristic approach considering that the problem can be easily separated into two phases: the definition of the cutting patterns and the allocation of the slitting lines.

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