TOP (2015) 23:665–668
DOI 10.1007/s11750-015-0400-1

DISCUSSION

Rejoinder on: Static and dynamic source locations in undirected networks

Lara Turner[1] · Dwi Poetranto Groß[2] · Horst W. Hamacher[1] · Sven O. Krumke[1]

Published online: 6 October 2015

Thanks to all discussants for the positive reaction to our paper. We do appreciate the comments—most of which are very complimentary—and the stimulating suggestions for further research. Due to the strict time constraints for finishing this rejoinder we will not be able to give detailed answers to the questions raised by the discussants, but will formulate some thoughts on these questions, including conjectures on results and additional ideas, which may be followed up by the authors of this paper, the discussants or readers of TOP, or—even better—any combination of members of these groups.

We also want to comment on TOP's unique author–discussant–rejoinder scheme. After reading the comment papers, we obviously would love to rewrite our paper and include some of the ideas. In this scheme, we cannot do this, but will address these ideas in the rejoinder. For the reader of TOP, this is a beautiful example of transparency of a scientific development which we would like to see in many other scientific environments.

This rejoinder refers to the comments available at doi:10.1007/s11750-015-0393-9, doi:10.1007/s11750-015-0394-8, doi:10.1007/s11750-015-0396-6, doi: 10.1007/s11750-015-0397-5, doi:10.1007/s11750-015-0398-4.

✉ Horst W. Hamacher
hamacher@mathematik.uni-kl.de

Lara Turner
turner@mathematik.uni-kl.de

Dwi Poetranto Groß
dwi.Gross@inform-software.com

Sven O. Krumke
krumke@mathematik.uni-kl.de

1 Department of Mathematics, University of Kaiserslautern, P. O. Box 3049, 67653 Kaiserslautern, Germany

2 INFORM GmbH, Pascalstr. 23, 52076 Aachen, Germany

The comment of **Labbé** starts with a comparison of classical and network location and the source location in our paper. In the former, the all-pair shortest path matrix is crucial while in the latter the all-pair maximal flow value matrix is of prime interest. She then asks the interesting question what happens if we generalize the concept of all-pair shortest path and maximal flow value matrices to a general distance matrix. She points out that both, the classical network location problem and the source location problem, are special cases, but the former is known to be NP-hard while the latter is is solvable in polynomial time. Hence, the generalization suggested by Labbé lends itself nicely to a discussion of general complexity issues which are great "food for thought" for future research.

Another challenging research question by Labbé is to consider p-median versions of the source location problem. If we fix the cardinality p of the set of sources the problem of maximizing the overall flow or to maximize the number of nodes (or their cumulative fixed cost) whose demand can be satisfied is so interesting that we fully agree with her "... this field of research is wide open and promising" and hope that many readers of TOP will join us in this.

Carrizosa focuses on the relation between the source location problems and the general set covering problem. As in Labbé's comment, he finds the fact relevant that the optimal covering of flow demands can be computed by a polynomial algorithm while the general problem is known to be NP-hard.

We fully agree that Social Networks would be a nice example for real-world applications of the source location model—in a classical referee–author relationship we would certainly do so, and further work out this idea, as initiated by Carrizosa.

The idea to consider continuous versions of the source location problem where placement of sources is allowed somewhere on the edges (instead of just on the nodes in our paper) is intriguing. Our conjecture is that a node dominance result may hold as in the classical network location problem and we are very much looking forward to further consider this problem. The same is true for the second suggested continuous version in which the demand is discrete (on the nodes), but continuous over the edges.

Kara raises the interesting question of combining our results on dynamic source location problems with networks which change over time. This question is of decisive importance in any use of our model in the context of natural or manmade disasters, where transportation networks are very likely to change. The standard argument used in this situation is that one should work with the time expanded network. As we point out in Section 4.2 of our paper, this approach is, unfortunately, not helpful in our context. At this point of time, we cannot offer any idea or conjecture for this situation and consider it a very challenging future research question.

For her second suggestion of maximizing some utility function instead of minimizing cost, we claim that most of our algorithms can deal with this modification under the mild assumption of additivity of the utility function. Since the feasibility of (single and plural) node covers is not depending on the objective function, the choice of a feasible set with maximal overall utility is under this assumption easy to get. The reduction to finding a node in each minimal deficient set with maximal utility will solve the problem.

Sforza and Sterle suggest in their comments to organize the flow of evacuees along disjoint paths from the collection points (or sectors, as they call them) to enhance the control during the evacuation process. This is an interesting idea, both from a theoretical and practical point of view.

The proposed mixed integer program contains a constraint (4) (the numbering used here and in the subsequent paragraph refers to the one used in the discussant paper of Sforza and Sterle) which seems to contain on first sight a redundant variable y_{ij}, since (4) can be written as $\sum_{e \in E} y_{ij,e} \leq 1$ and because y_{ij} never appears as variable anywhere else. The reason why their formulation is, however, meaningful is explained later in their manuscript: Since a disjoint-path transportation of the evacuees may not exist, the binary variable in their formulation (1)–(7) may be replaced by an integer variable. Minimizing $max_{(i,j) \in A} y_{ij}$, i.e., the largest of these variables, would then be a measure of violating the desired disjointness of the transport. Sforza and Sterle suggest to add this objective to the objective (1) using a penalty for the new objective.

We like this approach and suggest an alternative way to look at their model: The two objective functions could be considered in a bi-criteria environment with a vector objective instead of a sum of the two objective functions. If one of them, the number indicating the violation of disjointness or the overall flow value, is more important than the other, one could use a lexicographical integer program approach. Else one could analyse the situation by identifying the efficient solutions.

This idea ties in with the suggestion given in the comments of **Ruzika and Trochiani**. They raise the question, whether it would be possible to replace the single objective function in the source location problems presented in the paper by multiple objectives in the framework of multi-criteria optimization. This proposal is intriguing not only because it relates to our previous remark, but since—as the authors state in their comment paper—most applications which use source location as solution model are likely to be of a multi-criteria type. The topics proposed in their paper are all very interesting.

The first one is on the number of non-dominated solutions. We conjecture that most of the multi-criteria versions of our problems are intractable. This may contrast the fact that the cost plays a secondary role to the capacities which define a partition of the node set into minimal deficient sets. Then the multi-criteria source location problem reduces to finding a minimal (multi-objective!) representative in each of these sets. We conjecture that the latter problem can be reduced to a shortest path problem which is known to be intractable. If this is true, the question of finding a suitable representative system of the non-dominated solution raised by the discussants is of particular importance. Their suggested ϵ-constraint scalarization approach can be directly implemented for all source location problem formulations as integer program. A future challenge is to identify cases, where the combinatorial algorithms of our paper, or new ones, can solve these ϵ-constraint problems. We conjecture that this will not be the case for general objective functions, but it would be interesting to identify classes of objectives where this is possible.

The idea to consider a weighted-sum scalarization of the different objectives is a generalization of the approach by Sforza and Sterle for the two specific objectives cost and disjointness discussed above. The question whether there is a bound on the number of the resulting non-dominated extreme cost vectors seems to be very challenging. The suggested approach to generalize the spanning tree result of Seipp (2013) to matroids is very promising.

As a final reaction to the comments of Ruzika and Trochiani, we claim that most algorithms presented in our paper can be carried over to the multi-criteria case, if we assume a total ordering in the objective space. This is, for instance, the case for the lexicographic ordering $x < y \Leftrightarrow x_i < y_i$ for $i := min\{j : x_j \neq y_j\}$, or the max ordering $x < y \Leftrightarrow max\{x_i\} < max\{y_i\}$, respectively. The justification for our claim is that the totality of the ordering implies that throughout any of our algorithms the scalar comparison can be replaced by the comparison in the objective space. Due to the time constraint in this rejoinder we cannot give the resulting detailed complexity bounds, but they will not be too difficult to get.

Acknowledgments This research has been partially supported by the European Union Seventh Framework Programme (FP7-PEOPLE-2009-IRSES) under Grant agreement no. 246647 and by the New Zealand Government as part of the OptALI project. Further, it is supported by the Federal Ministry of Education and Research (BMBF), Germany, Grants 13N12229, 13N12826, and 13N13198.

Reference

Seipp F (2013) On Adjacency, Cardinality, and Partial Dominance in Discrete Multiple Objective Optimization. Dr. Hut Verlag, München