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DISCUSSION

Comments on: Static and dynamic source locations in undirected networks

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The paper under discussion deals with the *static* and *dynamic* source location problem in the form of covering problems. It is an integrated *facility location* and *maximum flow* problem where the aim is to determine the number and the positions of the uncapacitated facilities (referred to as *sources*) which minimize the overall location (installation) costs, guaranteeing that the maximum flow between the located facilities and any other vertex of the network is at least as large as the vertex demand. Hence this problem is referred to in literature as a *FlowLoc problem*.

The authors classify the literature about this problem with reference to several issues. In particular, they focus on:

- *Static* and *dynamic* networks;
- *Single* and *plural* vertex demand coverage;
- *Simultaneous* and *non-simultaneous* demand coverage;
- *Uniform* and *non-uniform* node parameters (vertex demands and location costs);
- *Single* and *multi*-commodity flows.

It is easy to understand that several problem variants, with different complexity, can be obtained by different combinations of them.

The authors focus on the single-commodity flow problems, discussing, for several variants of it, their complexity (in dependence also of the network topology), surveying

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the main contributions present in literature and proposing, in some cases, new theoretical and methodological findings (e.g. the structure of the single covering problem is re-interpreted as a matroid). In particular, the authors tackle:

– Static single covering problems with uniform and non-uniform setup costs;
– Static non-simultaneous and simultaneous plural covering problems on trees;
– Dynamic single covering problems.

They present for all these problems or variants of them, polynomial time algorithms based on the concept of minimal deficient sets, i.e. minimal node subsets whose demands cannot be supplied from outside.

The paper under discussion provides a wide and excellent review of the main literature about the problem, starting from the first published contributions. Moreover, it can be considered as a platform for further developments on the field, from both the methodological and practical point of view. From the methodological point of view, it opens new modeling perspectives and gives some hints about the further developments of more effective solution approaches. From the practical point of view, as also cited by the authors, the presented results and findings could be effectively used to optimize the exploitation of the communication and transportation networks in different real world problems. Indeed, most of the presented literature arises in the context of disaster management and evacuation planning. The authors are leading experts in the field and their works and achievements have significantly contributed to the development of the discipline.

For this reason, in this note, we will focus on another issue of the problem under investigation, that, in our opinion, has not received the appropriate attention in evacuation planning and should be taken into account in disaster management of a regional area.

To have a more effective and safe evacuation plan and on the basis of the local government needs that generally have to manage critical situations with limited human resources, conflicts between travelling flows at the junctions of the network have to be avoided. This means that, given an area to be evacuated, whose centroids are known, the evacuation of the people from each centroid towards the available shelters should occur using disjoint paths, where the disjunction has to be on the nodes (and so necessarily also on the links) and/or on the links (and so not necessarily on the nodes) of the network. Path disjunction problems (Andrews et al. 2010) have received more attention in communication network applications (Grötschel et al. 2014), but at the best of our knowledge, no contribution deals with them in evacuation planning.

This configures a multi-commodity flow design problem, where a commodity is associated with a centroid and it is represented by the people leaving a given area using disjoint paths (Melkote and Daskin 2001; Frangioni and Gendron 2009).

We propose here a simple *multi-commodity flow location ILP model* which allows to formulate the problem of maximizing the egress flow from the origins to the open shelters, using disjoint paths. It is easy to see that this problem, as the ones tackled in the paper under discussion, falls within the*FlowLoc* problems.

Let us consider a network $G(N, A)$, where N and A are the set of nodes and the set of directed links, respectively. The set of nodes N is partitioned into three subsets: E is the set of centroids, origins of flows, representing the areas (or sectors) to be evacuated;

S is the set of candidate positions for the shelters; V is the intermediate nodes. It is important to underline that, given the above described *one-to-one* correspondence, the commodity set corresponds to the centroid set.

Given this setting, we define the following parameters:

- m_{ij} capacity of each directed link (i, j), $(i, j) \in A$
- h_j fixed opening cost of the shelter j, $j \in S$
- B maximum available budget for shelter location

Moreover, the following decision variables are used:

- $f_{ij,e} \geq 0$ flow carried over the link (i, j), $(i, j) \in A$, coming from the sector e, $e \in E$
- $s_{j,e} \geq 0$ amount of flow coming from the sector e, $e \in E$, reaching shelter j, $j \in S$
- $y_{ij,e} = \{0, 1\}$ equal to 1 if the link (i, j), $(i, j) \in A$, is used by the sector e, $e \in E$, equal to 0 otherwise
- $y_{ij} = \{0, 1\}$ equal to 1 if a link (i, j), $(i, j) \in A$, is used by at least one sector, equal to 0 otherwise
- $y_j = \{0, 1\}$ equal to 1 if the shelter j, $j \in S$, is opened, equal to 0 otherwise

Hence the problem can be formulated as follows:

$$Max\ z = \sum_{e \in E} \sum_{i \in V} f_{ei,e} \tag{1}$$

$$s.t.$$

$$\sum_{k \in V \cup S} f_{jk,e} - \sum_{i \in N} f_{ij,e} = \begin{cases} -s_{j,e} & \forall e \in E,\ \forall j \in S \\ 0 & \forall e \in E,\ \forall j \in V \end{cases} \tag{2}$$

$$f_{ij,e} \leq m_{ij} y_{ij,e} \quad \forall (i, j) \in A,\ \forall e \in E \tag{3}$$

$$y_{ij} = \sum_{e \in E} y_{ij,e} \quad \forall (i, j) \in A \tag{4}$$

$$\sum_{e \in E} s_{j,e} \leq M\, y_j \quad \forall j \in S \tag{5}$$

$$\sum_{j \in J} h_j y_j \leq B \tag{6}$$

$$y_j \in \{0, 1\},\ s_{j,e} \geq 0, \quad \forall j \in S,\ \forall e \in E \tag{7}$$

$$y_{ij,e} \in \{0, 1\},\ y_{ij} \in \{0, 1\},\ f_{ij,e} \geq 0 \quad \forall (i, j) \in A,\ \forall e \in E$$

The objective function (1) maximizes the flow in egress from the sectors to be evacuated. Constraints (2) represent the flow conservation at the intermediate nodes and at the shelter for each commodity e, $e \in E$. Constraints (3) are consistency constraints between the flow variables and the link usage variables and they impose that the maximum flow using a link cannot exceed its capacity. Constraints (4) impose that a link can be used to evacuate just a single commodity, i.e. it can be used just by the flow coming from a single sector. Constraints (5) are consistency constraints between the flow variables and the shelter location variables (the big M is used to have uncapacitated shelters). Constraint (6) is a budget constraint. Finally, constraints (7) are non-negativity and integrality constraints for the used variables.

It is easy to note that the presented model allows to determine paths which are just link disjoint. However, if we introduce a dummy link for each node of the network, it can be easily used to have totally disjoint paths.

Moreover, it is important to say that, obviously, if the amount of flow to be evacuated from an origin is higher than the maximum flow from this origin to the shelters, the concept of *temporally repeated flows* has to be applied (as explained in the paper under discussion) to have a solution guaranteeing the complete evacuation of the interested area.

Moreover, the proposed formulation leaves the possibility that each origin uses more not disjoint paths, different and disjoint from the paths used by the other origins. In other words, each path cannot be used by two different origins, but it could contain links which can belong also to other paths used by the same origin. Hence, in perspective, another important feature that should be taken into account is the possibility to have more disjoint paths for each origins. In other words, the flow starting from a given origin can evacuate using more paths, but, for an effective management of the process, its travel demand has to be split just at the origin node.

For the sake of the completeness, we have also to say that obviously, depending on the structure of the network of the area under investigation, on the volume of the travel demand, on the capacity of the links and on the available budget (i.e. on the number of available shelters), we could have situations where no solution using different disjoint paths for each origin exists.

In the following, we give few hints about the management of this situation. We could slightly modify the proposed formulation imposing that the variable y_{ij} is no more binary, but an integer variable counting the number of times each link is used by different origins. Then, we should add a second component to the objective function penalizing the multiple usage of each link. In this way, we push towards solutions maximizing the flow in egress from an origin and contemporaneously minimizing the number of times each link is used to evacuate different sectors.

Anyway the obtained result could allow the decision makers involved in the disaster management to tackle the strategic design problem of which links should be added to the available structure of the network and which (number and position) shelters have to be opened to plan an evacuation where no path is used by more than one origin (or, if not possible, each path is used by the least number of origins).

Hence research on the theme could take into account not only location decisions, but also design decisions to guarantee a secure and safe evacuation process.

As concluding remarks, on the basis of the above discussion, we recognize to this paper several merits. Firstly, it provides a significant, concise and clear review of the main theoretical and methodological contributions present in literature for the source location problems. Secondly, it sketches the guidelines for the advancement of the research in the field. Finally, it highlights and stimulates new work perspectives and challenges for the researchers and practitioners interested in the problem and its real applications to the transportation and communication networks.

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