Monografías de la Real Academia de Ciencias de Zaragoza. **28:** 95–102, (2006).

Satellites around extrasolar planets?

Manuel Andrade and José Ángel Docobo

Astronomical Observatory R.M. Aller. Universidade de Santiago de Compostela

15782 Santiago de Compostela, Spain.

Abstract

We consider a star with two planets and analyse the chance of detecting a satellite in the outer planet by studying the observed perturbations in the inner orbit. The equations of motion are numerically integrated and applied to a pair of examples, one of them a real case (HD 37124).

1 Introduction

Since first extrasolar planet's discovery in 1995 (Mayor & Queloz) till now there are catalogued a number of extrasolar planets close to 200 (19 of them multiple planet systems). Certainly, this number will increase in the next years thanks to several ongoing programmes and future projects, both in ground and in space.

The first problem that one must face up to is the own definition of planet, especially taking into account that in the same scenarios we could find normal stars and brown dwarfs. Usually, it is assumed that the frontier between brown dwarfs and planets is given by the boundary of nuclear Deuterium burning at 13 $\mathcal{M}_{\text{Jupiter}}$, so that an object with a mass under that orbiting stars or stellar remnants will be considered a planet (in any other case, it will be considered a brown dwarf).

There are many types of extrasolar planets detection's methods: dynamical, astrometric, and photometric ones. Up to now, no doubt, the most important of all due to the amount of discovered planets has been the dynamic method known as method of the radial velocities. It consists in measuring variations in relative velocity of the star with regards to the Earth. Such variations, induced by the planet orbiting the star, analysed in the context of the two-body problem, allow us to state the existence of an object (in this case, a planet) and to determinate some of their orbital elements.

Nowadays, extrasolar planets with a few earth masses are being discovered, which implies to detect variations in radial velocities better than 1 m s^{-1}. In the near future, when this threshold decreases up to 0.09 m s^{-1}, it is expected that earth-like planets at 1 AU will be detected. While this happens, probabilities of satellite detection around extrasolar planets also increase.

Nevertheless, it is possible that earth-like size satellites orbit around the giant planets. Such satellites can not be detected by the radial velocity method, but their presence could be manifested on the perturbed orbits of the planets. Logically, we should expect to have the planetary orbits calculated with high precision to avoid that the perturbations are masked by the standard errors of the orbital elements.

From the point of view of the system stability it is notable that of the all known extrasolar planets only a few of them have orbital eccentricities nearly zero, those with smaller semi-major axes.

In the next section we consider some previous results for the 4(2,2)-model of the four-body problem. This model is applied to a pair of cases in Section 3. Finally, in Section 4 we draw up some conclusions and future work.

2 A model for the problem

The most simple model for a planetary system with a satellite is the three-body problem consisting in a star, a planet and a satellite around it. However, since we are interested in studying the perturbations produced by the satellite on other planet, we should consider a more complex system. Indeed, in this work, we will analyse a system comprising a star, an inner planet, and an outer planet with a satellite orbiting around it. Obviously, to detect perturbations on the inner orbit, satellite mass can not be negligible with regard to the remaining masses.

2.1 Double 4(2,2)-model

The aforementioned system must be studied in the context of the four-body problem. In this way we will consider a hierarchical pair of binaries revolving about their common mass center in disturbed Keplerian orbits. This is a very interesting model which can be also used to describe a binary-binary stellar system, a binary stellar system with one planet orbiting each star, etc.

We take the generalised Jacobi coordinated system (Walker 1983) showed in Figure 1, where m_1 is the star, m_2 and m_3 the planets, and m_4 the satellite orbiting m_3.

2.2 Equations of motion

The equations of motion, correct to the second order in the α_{kl}^{ij} (see definitions below), are:

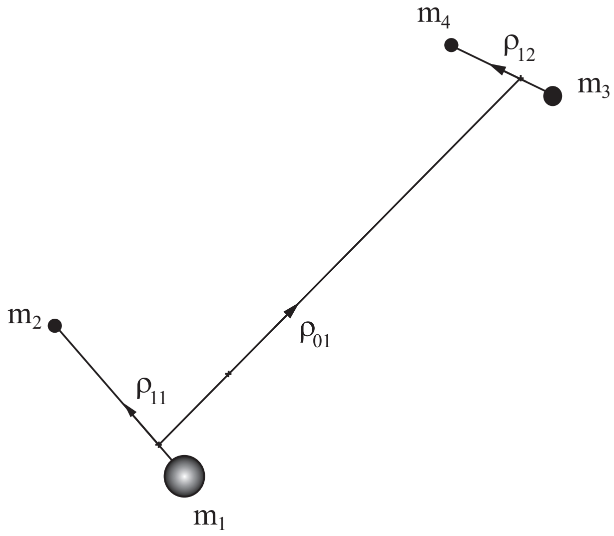

Figure 1.— Jacobian coordinate system.

$$\left\{\begin{array}{l} \ddot{\vec{\rho}}_{01} = GM_{01}\vec{\nabla}_{01}\{\dfrac{1}{\rho_{01}}\left[1+\varepsilon_{01}^{11}P_2\left(C_{01}^{11}\right)+\varepsilon_{01}^{12}P_2\left(C_{01}^{12}\right)\right]\} \\ \ddot{\vec{\rho}}_{11} = GM_{11}\vec{\nabla}_{11}\{\dfrac{1}{\rho_{11}}\left[1+\varepsilon_{11}^{01}P_2\left(C_{01}^{11}\right)\right]\} \\ \ddot{\vec{\rho}}_{12} = GM_{12}\vec{\nabla}_{12}\{\dfrac{1}{\rho_{21}}\left[1+\varepsilon_{12}^{01}P_2\left(C_{01}^{12}\right)\right]\} \end{array}\right. \tag{1}$$

where

$$\left\{\begin{array}{l} \varepsilon_{01}^{11} = \dfrac{M_{21}M_{22}}{M_{11}^2}\left(\alpha_{01}^{11}\right)^2 \\ \varepsilon_{01}^{12} = \dfrac{M_{23}M_{24}}{M_{12}^2}\left(\alpha_{01}^{12}\right)^2 \\ \varepsilon_{11}^{01} = \dfrac{M_{12}}{M_{11}}\left(\alpha_{01}^{11}\right)^3 \\ \varepsilon_{12}^{01} = \dfrac{M_{11}}{M_{12}}\left(\alpha_{01}^{12}\right)^3 \end{array}\right. \tag{2}$$

with

$$\left\{\begin{array}{l} C_{kl}^{ij} = \dfrac{\vec{\rho}_{ij}\cdot\vec{\rho}_{kl}}{\vec{\rho}_{ij}\rho_{kl}}; \qquad \alpha_{kl}^{ij} = \dfrac{\rho_{ij}}{\rho_{kl}} \\ \rho_{ij} = \left|\overrightarrow{M_{i+1,2j-1}M_{i+1,2j}}\right| \\ M_{ij} = \sum_{k=a}^{b} m_k; \qquad \left(a=(j-1)2^{2-i}+1; \quad b=j2^{2-i}\right) \end{array}\right. \tag{3}$$

M_{ij} denotes the mass center of the masses m_k, which are the individual masses, $P_2\left(C_{kl}^{ij}\right)$ is the Legendre polynomial of order 2 in C_{kl}^{ij}, and $\vec{\nabla}_{ij}$ is the gradient operator with respect to $\vec{\rho}_{ij}$.

2.3 Empirical stability parameters

According to Walker (1983), each ε term gives a measure of the perturbation placed on each orbit of the system by the other members of the system. In this way, empirical stability parameters are obtained by considering the configuration for which the mutual perturbations are greatest. It corresponds to the case in that four bodies lie in the order $m_1\, m_2\, m_3\, m_4$ in a straight line, so that Legendre polynomial equals one.

$$\begin{cases} \Sigma_{01} = \dfrac{M_{21}M_{22}}{M_{12}^2}\left(\alpha_{01}^{11}\right)^2 + \dfrac{M_{23}M_{24}}{M_{12}^2}\left(\alpha_{01}^{12}\right)^2 \\ \Sigma_{11} = \dfrac{M_{12}}{M_{11}}\left(\alpha_{01}^{11}\right)^3 \\ \Sigma_{12} = \dfrac{M_{11}}{M_{12}}\left(\alpha_{01}^{12}\right)^3 \end{cases} \tag{4}$$

where Σ_{01} measures the perturbation on (M_{11}, M_{12}) subsystem, Σ_{11} the perturbation on (M_{21}, M_{22}) subsystem, and Σ_{12} the perturbation on (M_{23}, M_{24}) subsystem. For a given double four-body system to be stable for any time, each Σ_{ij} have to be much less than unity (and generally less than 10^{-2}).

3 Applications

3.1 Example A

This system comprises a star with 0.900 $\mathcal{M}_\odot$ and two big planets. In addition, we consider the existence of a satellite orbiting around the outer planet. With the aim to see clearly the influence of the satellite, we have taken a case where its mass is bigger than that of the inner planet.

We integrate numerically the system of equations of motion given by (1) taking as initial values the Jacobi's coordinates obtained from the orbital elements shown in Table 1 (the remaining are set to zero). Integration is accomplished from $t = 0$ to $t = 50$ temporal units (tu), which corresponds to 135 initial periods of the inner orbit planet.

	Planet a	Planet b	Planet a	Planet b	Satellite
$\mathcal{M}$ ($\mathcal{M}_J$)	0.020	0.080	0.020	0.050	0.030
a (AU)	0.5	3.0	0.5	3.0	0.2
e	0.2	0.6	0.2	0.6	0.1
i (°)	5.0	10.0	5.0	10.0	3.0

Table 1.— Masses and orbital elements for the planets and the satellite.

Our aim is to compare the case where we have only two planets with that where the outer planet has a satellite companion. In this way, we will study the orbital elements evolution in both cases. In Figure 2 we can see this evolution, where planet+planet system is indicated by a solid line and planet+planet+satellite system is indicated by a dashed line.

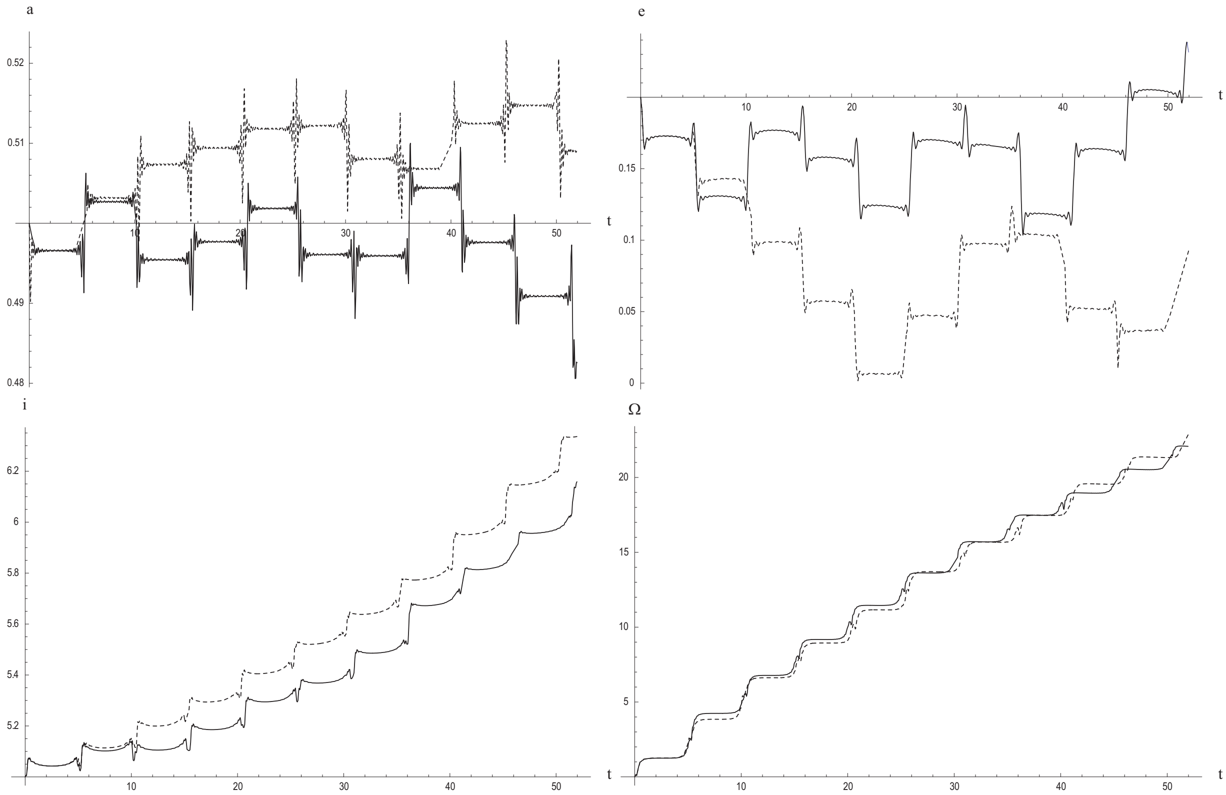

Figure 2.— Orbital elements evolution.

We should pay special attention to argument of periastron evolution because it is the orbital element whose evolution more easily allows us to discriminate between both cases (see Figure 3). We have seen that the existence of a satellite in the outer planet introduces a delay in the evolution of this orbital element. In this example, the value reached at the end of the integration interval ($t = 50$ tu) when there is a satellite, corresponds with $t = 48.66$ tu in the case there is not satellite, i.e., about 3.6 orbital periods. Therefore, measuring this angle, we could be able to determine whether there is or not a companion for the outer planet.

From the point of view of inner orbit stability it appears that the case where there is a satellite as companion of the outer planet is slightly more stable. However, in this last case the whole system becomes more unstable (see Figure 4).

3.2 Example B (HD 37124)

This planetary system comprises a G4V star with 0.91 $\mathcal{M}_\odot$ and two Jupiter-type planets with orbital periods of 129 and 1942 days. In place of the last one we will suppose that there are a double planet-satellite with equal masses.

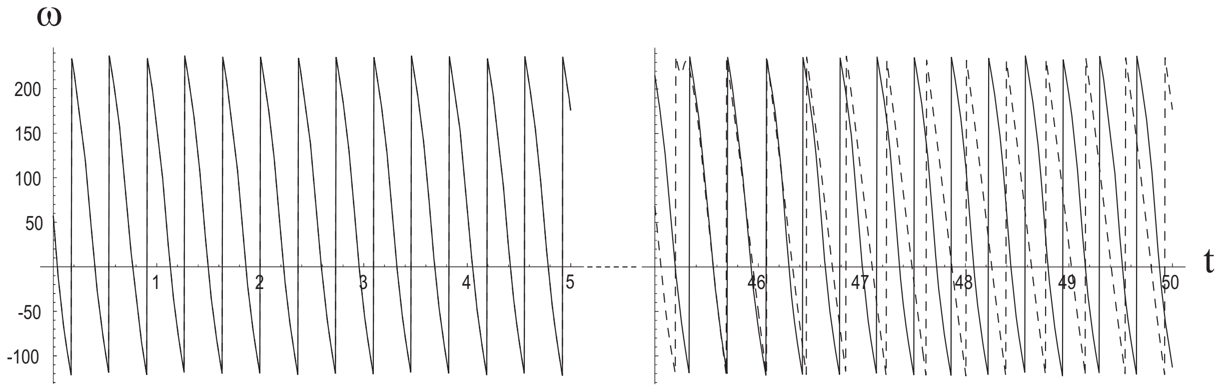

Figure 3.— Argument of periastron evolution.

	Planet a	Planet b	Planet a	Planet b	Satellite
$\mathcal{M}$ ($\mathcal{M}_J$)	0.86	1.01	0.86	0.59	0.59
a (AU)	0.54	2.95	0.54	2.95	0.05
e	0.10	0.4	0.10	0.4	0.4
i (°)	70.0	60.0	70.0	60.0	20.0

Table 2.— Masses and orbital elements for the planets and the satellite in HD 37124.

It is very important to emphasize that the easiest detectable systems with satellites will be those close to unstable configurations. In this way the empirical stability parameters allow us to know better the dynamical effect caused by the satellite. In fact, in Figure 5 we can see as the inner orbit remains stable while the whole system (star+planet plus planet+satellite) becomes, as in the previous example, more unstable.

4 Conclusions

We expect that by analysing in more detail the perturbations described here, we will be able to calculate how are the variations that they produce in radial velocities. Our aim is, given the radial velocity data, to determine whether the existence of one satellite around the outer planet would imply a better fit of data than in the case we are not considering any satellite.

Above all, a deeper study is necessary with regard to describe completely the long-time evolution of these systems. Besides this, we will have to take into account other models to explain different configurations, as for example those with the satellite in the inner planet or with satellites in more than one planet. Yet, it will be interesting to determine what configurations should be more suitable for the existence of satellites hosting earth-like life.

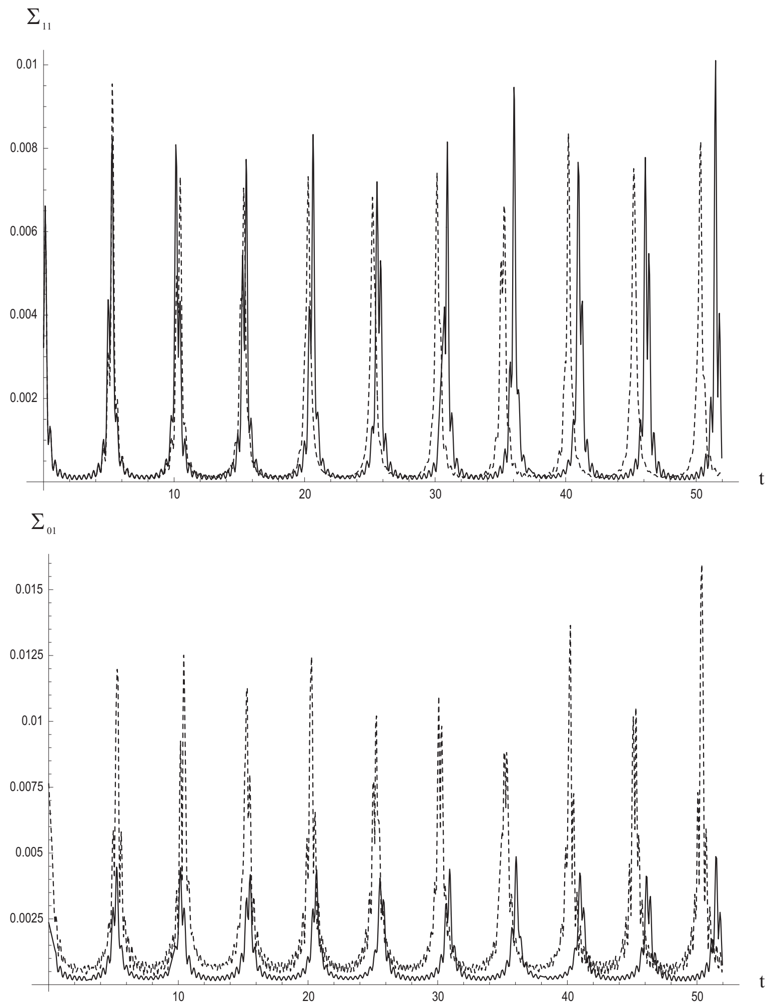

Figure 4.— Empirical stability parameters for star-inner planet subsystem and for star-inner planet and planet-satellite system, respectively (Example A).

References

[1] Mayor, M. & Queloz, D. A. 1995, *Nature*, **378**, 355-357.

[2] Walker, I. W. 1983, *Cel. Mech.*, **29**, 149-178.

[3] Walker, I. W. and Roy, A. E. 1983, *Cel. Mech.*, **29**, 267-294.

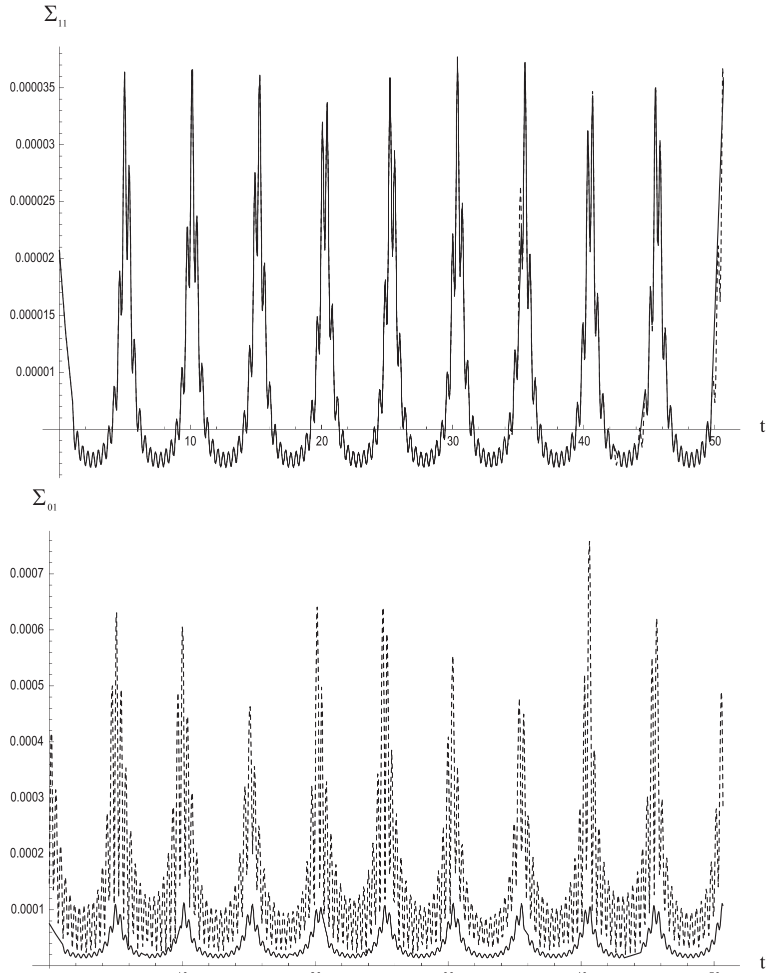

Figure 5.— Empirical stability parameters for star-inner planet subsystem and for star-inner planet and planet-satellite system, respectively (Example B).