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**J. M. Montesinos-Amilibia**

**Abstract.** It is announced that the Freudenthal compactification of an open, connected, oriented 3-manifold is a 3-fold branched covering of $S^3$.The branching set is as nice as can be expected. Some applications are given.

La compactificación de Freudenthal de una 3-variedad abierta conexa y orientable es una cubierta de 3 hojas ramificada sobre $S^3$. La ramificacion es tan simple como podría esperarse.Se ofrecen algunos corolarios.

## Open 3-manifolds as 3-fold branched coverings

**Resumen.** La compactificación de Freudenthal de una 3-variedad abierta conexa y orientable es una cubierta de 3 hojas ramificada sobre $S^3$,y en ciertos casos, de dos hojas. La ramificacion es tan simple como podría esperarse.Se ofrecen algunos corolarios.

Manifolds of dimension 2 and 3 in this paper will be separable metric spaces so that they are locally finite simplicial complexes [10](see also[11]). Let $M$ be an open, connected, oriented 3-manifold (an open 3-manifold for brevity). Denote by $\widehat{M}$ its Freudenthal compactification (see [5],[3],see also[2]). Denote by $E(M)$ the *end space* $\widehat{M} - M$ . Some years ago, Bobby Winters asked me about the possibility of representing an open 3-manifolds as a branched covering of a dense subset of $S^3$.The purpose of this paper is to announce the following results.

**Theorem 1** *Let $M$ be an open, connected, oriented 3-manifold. Let $\widehat{M}$ denote its Freudenthal compactification.Then, there exist a 3-fold branched covering $p : \widehat{M} \to S^3$ such that p maps the end space $E(M)$ of $M$ homeomorphically onto a tame subset $T$ of $S^3$.The 3-fold branched covering $p \mid M : M \to S^3 - T$ is simple, and the branching set is a locally finite disjoint union of strings (properly embedded arcs).*

This Theorem generalizes the Theorem of Hilden ([6] and[7]) and the author ([12]and[13]).

**Corollary 1** *Let $M$ be an open, connected, oriented 3-manifold with just one end. Then there exist a 3-fold covering onto Euclidean 3-space $p : M \to R^3$, branched upon a locally finite disjoint union of strings.*

This is the case of the uncountably many open, contractible 3-manifolds.

**Corollary 2** *Every closed, oriented 3-manifold is a 3-fold covering of $S^3$branched over a wild knot.*

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**Corollary 3** *The open contractible Whitehead manifold is a 2-fold covering of $R^3$ branched over a string.*

Smith proved in [15] that for any orientation-preserving involution of $S^3$, the fixed point set is either null, the 1-sphere, or $S^3$ itself. Montgomery and Zippin [14], following pioneering work by Bing [1], showed that there is an involution in $S^3$ whose fixed point knot is not the unknot, since it contains a Cantor set wose complement is not simply connected. The following Corollary qualifies that result.

**Corollary 4** *There exist a 2-fold branched covering $p : S^3 \to S^3$ defined by an orientation- preserving involution $u$ of $S^3$ whose fixed point knot contains a non tamely embedded Cantor set. Therefore $u$ is not equivalent to the standard involution. Moreover $p$ sends that Cantor set homeomorphically upon a tamely embedded Cantor set.*

# References

[1] Bing, R. H. (1952). A homeomorphism between the 3-sphere and the sum of two solid horned spheres. *Ann. of Math.* (2) **56**. 354–362.

[2] Brown, E. M., Tucker, T. W. (1974). On proper homotopy theory for noncompact 3-manifolds. *Trans. Amer. Math. Soc.* **188**, 105–126.

[3] Fox, R. H. (1957). *Covering spaces with singularities. A symposium in honor of S. Lefschetz*, Princeton University Press, Princeton, N.J., 243–257.

[4] Fox, R. H. (1972). A note on branched cyclic covering of spheres. *Rev. Mat. Hisp.-Amer.* (4) **32**, 158–166.

[5] Freudenthal, H. (1945). Über die Enden diskreter Räume und Gruppen. *Comment. Math. Helv.* **17**, 1–38.

[6] Hilden, H. M. (1974). Every closed orientable 3-manifold is a 2-fold branched covering space of $S^3$. *Bull. Amer. Math. Soc.* **80**, 1243–1244.

[7] Hilden, H. M. (1976). Three-fold branched coverings of $S^3$. *Amer. J. Math.* **98**, no. 4, 989–997.

[8] Hoste, J. (1957). Framed link diagrams of open 3-manifolds. In *KNOTS '96 (Tokyo)*, World Sci. Publishing, River Edge, NJ, 515–537.

[9] Kuperberg, G. (1996). A volume-preserving counterexample to the Seifert conjecture. *Comment. Math. Helv.* **71**, no. 1, 70–97.

[10] Moise, E. E. (1952). Affine structures in 3-manifolds. V. The triangulation theorem and Hauptvermutung. *Ann. of Math.* (2) **56**, 96–114.

[11] Moise, E. E. (1977). *Geometric topology in dimensions 2 and 3.* Graduate Texts in Mathematics, Vol. 47. Springer-Verlag, New York.

[12] Montesinos, J. M. (1974). A representation of closed orientable 3-manifolds as 3-fold branched coverings of $S^3$. *Bull. Amer. Math. Soc.* **80**, 845–846.

[13] Montesinos, J. M. (1976). Three-manifolds as 3-fold branched covers of $S^3$. *Quart. J. Math. Oxford Ser.* (2) **27**, no. 105, 85–94.

[14] Montgomery, D., Zippin, L. (1954). Examples of transformation groups. *Proc. Amer. Math. Soc.* **5**, 460–465.

[15] Smith, A. (1939). Transformations of finite period: II, *Ann. of Math.* (2) **40**, 690–711.

J. M. Montesinos-Amilibia
Facultad de Matematicas
Universidad Complutense
28040 Madrid
Spain