MATHEMATICS TEACHERS' LEARNING THROUGH INQUIRY

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ABSTRACT

Inquiry-based teaching strives to engage students in learning mathematics with understanding in the classroom. Therefore, there is great interest in supporting teachers to meet this pedagogical challenge by developing practices that promote such an educational environment at different school levels. A powerful way for teachers to learn and transform their teaching is through teacher inquiry. This paper presents a model for inquiry into mathematics teaching based on the perspectives of theorists directly associated with teacher education. This model is described as an overarching inquiry cycle in which teachers begin with practice, pose a pedagogical problem, understand a key construct in the problem, hypothesize an inquiry-teaching model, test/apply it, and finally revise/apply this model. This approach is illustrated with a self-directed professional development process aimed at helping elementary teachers to develop understanding of inquiry-based teaching of mathematics.

KEY WORDS

Inquiry-based learning; Mathematics teacher learning;
Noticing; Dialogic inquiry; Inquiry stance.

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INTRODUCTION

Current learner-focused perspectives of mathematics education require teachers to use effective pedagogy that will actively engage students in learning mathematics with understanding. Inquiry-based teaching offers opportunities to achieve this in the mathematics classroom. This makes inquiry an important consideration in mathematics teachers' learning and practice. For teachers facing new pedagogical challenges, teacher inquiry can be a powerful vehicle for their learning and transformation of their practice. This paper discusses inquiry from the perspectives of theorists who deal directly with teacher education and the use of these perspectives to frame mathematics teachers' learning. It examines how inquiry has been interpreted and used in studies of practicing mathematics teacher education. Finally, it discusses a self-directed professional development process aimed at helping elementary teachers to develop an understanding of inquiry-based mathematics teaching, how it is related to the different perspectives of inquiry and the implications for the development of an inquiry stance.

PERSPECTIVES OF INQUIRY IN TEACHER LEARNING

Dewey's (1933/1971, 1938) work has provided a foundation for current perspectives of inquiry. For Dewey, what distinguishes inquiry from the trial and error that people are continually engaged in as they transact with their environment is that inquiry is «*controlled* or *directed* by means of reflection or thinking» (Biesta & Burbules, 2003, p. 58). Thus, *reflective thinking* is «central to all learning experiences enabling us to act in a deliberate and intentional fashion (...) [to] convert action that is merely (...) blind and impulsive into intelligent action» (Dewey, 1933/1971, p. 212). Dewey defines reflective thinking as an «active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends» (Dewey, 1933/1971, p. 9). He argued that encountering problems impels us to reflective thought, the essential characteristic of which is inquiry and that «We inquire when we question; and we inquire when we seek for whatever will provide an answer to a question asked» (Dewey, 1938, p. 105). Thus, for him, there is a direct relationship among questioning, reflective thinking, and inquiry. «Thinking is inquiry, investigation, turning over, probing or delving into, so as to find something new or to see what is already known in a different light. In short, it is questioning» (Dewey, 1933/1971, p. 265).

Dewey's (1933/1971) inquiry process begins when one encounters a puzzling situation, i.e., «a state of doubt, hesitation, perplexity, mental difficulty» (p. 12); «an entanglement to be straightened out, something obscure to be cleared up» by thinking (p. 6) and then entails the following phases or states of thinking:

1. Suggestions in which the mind leaps forward to a possible solution. If the solution seems feasible, it is applied, and full reflection does not occur. Otherwise, these phases take place:
2. Intellectualization of the difficulty or perplexity into a specific problem to be solved or question to be answered (i.e., placing the perplexity into a relevant context)
3. Development and use of a hypothesis to initiate and guide observation and other processes in the collection of empirical data (e.g., «searching, hunting, inquiring, to find material that will resolve the doubt, settle and dispose of the perplexity» [p. 12])

4. Elaboration of the hypothesis
5. Testing the hypothesis, either by overt action or thought experiment (imaginative action).

Dewey notes that «the sequence of five phases is not fixed» (p. 115). They also form a continuous process.

There are clear links to Dewey's view of reflective thinking in Schön's (1983) notion of *reflection-on-action* as the way practitioners focus on problem posing (questioning) to inquire into practice and meaningful situations. For Schön, reflection-on-action involves looking back at an event. It takes into consideration the context of the event by:

- analysing the circumstances of the event, including personal biases or misunderstandings
- planning actions based on careful consideration of all the information
- guiding future actions

This form of inquiry, according to Schön (1983, 1987), involves a process of posing and exploring problems or dilemmas identified by the practitioners themselves in order to examine their practice by analysing, adapting, and always challenging their assumptions in a self-sustaining cycle of reflecting on their theory and practice. This cycle allows them to learn from one problem to inform the next. This process of reflection (inquiry) enables practitioners to assess, understand and learn through their experiences. It is, therefore, a process that starts with their own experiences.

While Dewey's notion of inquiry is oriented towards a cognitive perspective, Wells' (1999) approach is oriented towards a socio-cultural perspective in which a «community of inquiry» is central. As Wells noted, «The construction of understanding is a collaborative enterprise» (p. 125). Wells (1999) defines *dialogical inquiry* as «a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them» (p. 122). He represents this as a «spiral of knowing» consisting of: experience, information, knowledge building, and understanding. He considers the relationship among experience, discourse, and the enhanced understanding to be the goal of all inquiry. He explains that each cycle of the spiral starts from past personal experience and new information is added from the current environment. The goal of each cycle is enhanced understanding that is reached through knowing in action a specific situation and almost always involves dialogic knowledge-building with others. This goal can be achieved through telling stories, developing explanations, making connections, and testing conjectures through action. The critical aspect of the spiral of knowing is interpersonal and collaborative and is always aimed at enhancing the understanding of both the group and participating individuals.

The importance of beginning with one's own experience and reflecting on it is also characteristic of the view of inquiry embodied in Mason's notion of noticing (Mason, 2002). Noticing, as a basis of teachers' learning, «is a collection of practices both for living in, and hence learning from, experience and for informing future practice» (p. 29). It is «a reference to lived experience through an invitation to check something out in your own experience» (p. xi). Mason defines it specifically as a collection of systematic practices consisting of four interconnected actions: preparing and noticing; systematic reflection; recognizing and labelling choices; and validating with others. This process is informed by research and shared practice through «*introspective observation* (in which an inner witness observes the self caught up in the action…); and *interspective observation* (in which people share observations as witness to each other, yielding objectivity from negotiated subjective information)» (Mason, 2002, p. 85). «The core of researching from the inside is attending to experience (…) so as to develop sensitivities to others and to be awake to possibilities» (Mason, 1994, p. 180).

Table 1 provides a summary of the *key components* of the preceding ways of viewing inquiry. Each column represents a complete inquiry cycle. However, it is not intended to represent a linear process with a definite end point. The relationship among components can be dynamic and cyclic. More importantly, the end point of each inquiry cycle (each column of Table 1) is an actual or potential beginning of a new cycle. In addition, each cycle begins with experience.

These approaches to constructing knowledge have been directly linked to the way teachers can learn and change. For example, Dewey (1933/1971) called for teachers to engage in inquiry or «reflective action» (action based upon thoughtful deliberation; intelligent action) that would transform them into inquiry-based, classroom practitioners. Inquiry provides teachers with a way to better understand their own practices, so that they can ultimately «transform actions into *intelligent* action» (Biesta & Burbules, 2003, p. 38) that result in growth. According to Biesta and Burbules (2003), the outcomes of this

REFLECTIVE THINKING DEWEY	A puzzling situation Generation of questions or specific problems Formulation of a solving suggestion (initial hypothesis) Transformation of hypothesis into possible lines of action Experimental testing of hypothesis in the concrete situation
REFLECTION-ON-ACTION SCHÖN	Problem or dilemma Analysing, adapting, and challenging assumptions Action plan Application to future actions
DIALOGIC INQUIRY WELLS	Personal experience Information Knowledge building Understanding
NOTICING MASON	Preparing and noticing Systematic reflection Recognizing and labelling choices Validating with others

TABLE I – SUMMARY OF KEY COMPONENTS OF PERSPECTIVES OF INQUIRY

process are changes in the way the teachers and students think and know and in the situation, which includes the way the curriculum gets enacted. Similar to Dewey, Schön (1983) argued that teachers could orchestrate their own change if they are helped to develop an inquiry (reflective) stance of looking at their own practice. This stance usually results in changes in their perspectives of a situation or new learning, which, if applied to practice, can result in improvement.

Wells (1999) made a case for teachers to engage in inquiry as a way to systematically investigate their own practice to find out what approaches, choice of activities and patterns of organization are most successful in their own particular situations. The outcome of this investigation is the improvement of both their pedagogical understanding and their practice. Mason (2002) directed his process of noticing specifically to mathematics teachers. He explained that a goal of this process is for teachers to examine their own experience of work on themselves, informed by research and shared practice, while addressing how to help their students to learn mathematics.

Table 1 also presents ways of interpreting inquiry that are consistent with «inquiry as stance» (Cochran-Smith & Lytle, 1999, 2009). This notion states that it is important for inquiry to be about teachers' learning as opposed to the tasks in which they engage. As Cochran-Smith and Lytle (1999) explained, «Inquiry as stance is distinct from the more common notion of inquiry as time-bounded project or discrete activity within a teacher education course or professional development workshop» (p. 289). Instead, it is about teachers

> working together in communities (…) [to] pose problems, identify discrepancies between theories and practices, challenge common routines, draw on the work of others for generative frameworks, and attempt to make visible much of that which is taken for granted about teaching and learning. From an inquiry stance, teachers search for significant questions as much as they engage in problem solving. They count on other teachers for alternative viewpoints on their work (pp. 292-293).

In addition, «from the perspective of inquiry as stance, teacher learning is associated more with uncertainty than certainty, more with posing problems and dilemmas than with solving them, and also with the recognition that inquiry both stems from and generates questions» (Cochran-Smith & Lytle, 1999, p. 294). In the context of inquiry as stance, Cochran-Smith and Lytle (2009) have broadened the scope of inquiry from a study of classroom practice to a lifelong habit of mind wherein teachers use an inquiry lens to question any aspect of the educational system. This added dimension of inquiry has a social justice goal of more equitable outcomes for students.

Table 1, then, provides a basis for a theoretical framework to guide and interpret an inquiry perspective of mathematics teachers' learning. The four ways of viewing inquiry have common features. However, they also have particular features that can be combined to produce a framework that recognizes the cognitive perspective of reflective thinking, the sociocultural perspective of dialogic inquiry and the importance of noticing in both of these perspectives. Such a framework is consistent with the view that knowledge is both an individual and a social construction and that individual and social dimensions of learning complement each other. This framework also represents a perspective of inquiry as a fundamental principle and a way of being in mathematics teacher education. Thus, it provides a basis for inquiry to be a norm of practice through teachers' development of an inquiry stance. An example of this framework is illustrated after discussing how inquiry has been addressed in research on practicing mathematics teachers' learning.

INQUIRY IN PRACTICING MATHEMATICS TEACHER EDUCATION

The reform movement in mathematics education and the focus on constructivism have provided support for inquiry as a mathematical process, as a way of teaching mathematics and as a way of developing mathematics teaching. However, several obstacles can arise for teachers when they try to teach from an inquiry perspective because it requires skills that are unfamiliar in traditional mathematics classrooms. In addition to holding deep understanding of mathematics for teaching, teachers must possess, for example, the ability to embrace uncertainty, foster student decision-making by balancing support and student independence, recognize opportunities for learning in unexpected outcomes, maintain flexible thinking, and tolerate periods of disorganization (National Research Council, 2000). Teachers are more likely to develop an understanding of such behaviours, and inquiry in general, if they learn through inquiry. But more importantly, as previously discussed, it is important for them to learn in a way that will help them to develop an inquiry stance as a central aspect of being a teacher of mathematics.

Current professional standards for teaching and research in mathematics teacher education suggest approaches to teachers' learning that have potential to help teachers to develop an inquiry stance. For example, the National Council of Teachers of Mathematics [NCTM] (2000) Teaching Principle states:

> Opportunities [for teachers] to reflect on and refine instructional practice – during class and outside class, alone and with others – are crucial in the vision of school mathematics. (...) To improve their mathematics instruction, teachers must be able to analyze what they and their students are doing and consider how those actions are affecting students' learning. (...) Collaborating with colleagues regularly to observe, analyze, and discuss teaching and students' thinking or to do «lesson study» is a powerful (...) form of professional development (p. 19).

This perspective is reflected in practice-based learning communities, a current trend in mathematics teacher education. Practice-based learning communities are now viewed as a more desirable and meaningful way to facilitate mathematics teachers' learning and have been increasingly used in studies of teachers' professional development (e.g., Even & Ball, 2008; Krainer & Wood, 2008; Ponte *et al.*, 2008; Tirosh & Wood, 2008). A core feature of these learning communities is having teachers work collaboratively on a variety of activities linked to the context of their teaching. These activities are purposefully connected to their mathematics curriculum, their students' learning or work, and their classroom pedagogy. Thus, a common feature of this approach is to provide realistic or actual events and contexts of classroom situations that enable teachers to explore important mathematical and pedagogical ideas that relate to their own teaching.

The following examples of current studies on mathematics teacher education suggest some ways in which practicing teachers' learning has been facilitated through teachers working in groups over an extended period of time, investigating and discussing situations directly related to classroom teaching. Some studies engaged teachers in a collaborative process that included analyzing self-created videos of their teaching or researcher-created videos of teaching or students at work in the classroom. For example, Maher (2008) discussed a process of facilitating teachers' learning that included the use of researcher-created video recordings. This process involved:

> (1) teachers studying mathematics by working on a strand of tasks; (2) teachers collectively studying their own solutions; (3) teachers viewing and analyzing video recordings of children working on the same or similar tasks; and, (4) teachers implementing and analyzing together, the same or similar lessons in their own classrooms (p. 71).

van Es and Sherin (2010) also discussed a model of professional development called "video clubs" in which teachers watched and discussed excerpts of videos from their classrooms. In both studies, the approaches influenced teachers' thinking and teaching in positive ways.

Some studies involved the use of cases, as in Markovits and Smith (2008), who engaged teachers in a process that included:

> Solving and discussing the mathematics task on which the case is based, reading the case guided by a framing question, engaging in small and whole group discussions of the case centered on the framing question, and generalizing beyond the case to one's own teaching practice and to a larger set of ideas about mathematics teaching and learning (p. 47).

Other studies involved using students' work. For example, Kazemi and Franke (2004) initiated and organized a monthly work group of 10 teachers at an elementary school. Students' work from the teachers' classrooms guided the content and direction of discussions at each work-group meeting. Prior to the meetings, teachers used a common problem that they could adapt to their classes. For each meeting, teachers selected samples of students' work to share with the group. Work-group discussions centred on the students' work those problems generated. The mathematical domains the researchers chose to focus on during the work group reflected those that the teachers were working on in their classrooms. The approach helped the teachers to become more attentive to the details in the students' thinking.

In another example, the teachers' group work was based on observing students in an actual classroom. Francisco and Maher (2011) reported on the experiences of a group of elementary and middle school teachers who participated as interns in an after-school, classroom-based research project on the development of mathematical ideas for middle-grade students. For one year, the teachers observed the students working on well-defined mathematical investigations during research sessions in which the researchers taught the classes. In these classes, the researchers encouraged students to work collaboratively and justify their solutions, received their contributions positively, and gave them extra time to work on tasks and opportunities to refine and make connections between mathematical ideas. The teachers, in groups of two or three, observed a different group of four to six students in different sessions and occasionally followed the same group of students over several sessions. They received instructions about what to focus on in their observations and were told to refrain from interacting with students. This approach enabled the teachers to gain insights into the students' mathematical reasoning.

In these studies, inquiry is implied as consisting of situations or tasks for teachers to explore as they worked in groups. For the most part, this type of inquiry is influenced by the intentions and expectations of the researchers (the professional development leaders) and constrained by pre-set activities and goals. While such types of learning communities offer opportunities for teachers to construct knowledge about mathematics pedagogy, they are less likely to help them to develop an understanding of inquiry as a way of being and to adopt it as a way of framing their teaching. They do not offer the key aspects of the inquiry perspectives in Table 1 or the Cochran-Smith and Lytle

(1999) perspective of inquiry stance that are important for teachers to be able to develop an inquiry stance.

Lesson study, as practiced in Japan, also involves learning communities and has exerted an influence in other countries (e.g., Lewis, Perry & Hurd, 2009). In this approach, a small group of teachers works together to plan, teach, observe, and analyse the lessons. They start by identifying a goal or problem they want to explore. This is followed by a four-phased cycle: collaboratively developing a lesson plan, implementing the lesson with observation by colleagues and other experts, analytically reflecting on the teaching and learning that occurred, and revising the lesson for re-implementation (Curcio, 2002; Shimahara, 2002; Stigler & Hiebert, 1999). During each cycle of implementation, a different teacher teaches the lesson to his or her students in a normal classroom setting, while the other group members observe, taking notes on how it is being implemented. In the end, the teachers produce a report of what they learned, particularly with respect to their goal. This approach has the potential for teachers to engage in and develop an inquiry stance. However, the tendency is for it to be more theoretical than about personal experience, to have a specific purpose or outcome that is not about inquiry as a way of being, and to be based on a predetermined pre-learned process. As Yoshida (2008) pointed out, teachers who engage in lesson study need to learn how to investigate, plan a research lesson, observe it, and discuss it; and they need to receive strong support from other knowledgeable persons such as teacher educators.

In contrast to the preceding examples of the use of learning communities in practicing mathematics teacher education, Jaworski (2004) made a case for the use of «community of inquiry» based on Wells' perspective of inquiry, instead of «community of practice». As she stated, «In a community of inquiry, inquiry is more than the practice of a community of practice: teachers develop inquiry approaches to their practice and together use inquiry approaches to develop their practice» (p. 25). Jaworski (2004, 2006) discussed such an inquiry community in which teachers viewed themselves as researchers. In this community, teachers and didacticians/researchers worked together in a way that supported each other's learning through inquiry. The didacticians drew the teachers into inquiry in a variety of ways, such as workshops that created opportunities to do mathematics together in inquiry mode and exploration into what inquiry looks like in mathematics learning. The teachers formed an inquiry group to discuss what their teaching might look like from an inquiry perspective and to plan classroom activities that encouraged students to get involved in inquiry in mathematics.

Jaworski (2004) also discussed a study that involved «learning study», which is different from lesson study in terms of its theoretical basis and its purposeful nature. She described learning study as

> a group of teachers designs innovative classroom activity, based on agreed theoretical principles, and explores the consequent teaching. *Design* and *innovation* offer purposeful directions. Teachers use inquiry as a tool to explore teaching, alongside didacticians who offer theoretical ideas and practical support and who research the processes of teaching development. Teachers develop their thinking and practice through successive cycles of inquiry. They each work in their own classroom, interpreting a design they have produced jointly. Observation of each other's teaching and group reflections lead to building of group and individual awareness through which *inquiry as a way of being* develops (p. 27).

Thus, Jaworski's work offers insights of a perspective of inquiry that can be related to key aspects of the perspectives in Table 1 and provide a basis to help teachers to develop an inquiry stance with regard to their teaching.

The preceding discussion provided a brief profile of the nature of inquiry in practicing mathematics teacher education based on examples of professional development situations involving community-of-learners. These examples suggest approaches that were effective in helping teachers learn specific aspects of pedagogical content knowledge. However, inquiry as an explicit focus was lacking, despite its importance to learning mathematics. More research attention is needed, as in Jaworski's case, where teachers' inquiry includes inquiry of inquiry as a basis of their learning and as a way of developing an inquiry stance in their teaching. The following section describes an example of such a study with practicing elementary teachers.

PRACTICING TEACHERS' SELF-DIRECTED INQUIRY-BASED LEARNING

This example draws on a study that focused on teacher learning *through* and *about* inquiry. In this study, the teachers engaged in a self-directed professional development process in which they decided what to do and how to do it. Chapman (2011) discusses the study from the self-directed aspect of the professional development experience. The focus here is to highlight key aspects of the process based on the theoretical perspectives in Table 1.

The participants were 14 practicing teachers with representation from grades 1 to 6 in the same elementary school. They had from 3 to 20 years of teaching experience; most had over 10. Teachers in Alberta are required to have a professional growth plan. Each school could choose its own way of implementing this. At the school of this study, the teachers were required to form disciplinary study groups of their choice. The teachers in this study chose the mathematics group because they thought mathematics was the area in which they needed the most help to bring their teaching more in line with curriculum expectations that fostered a constructivist or inquiry perspective. The curriculum was significantly influenced by NCTM (1989, 2000) standards. Although some teachers were beginning to make meaningful changes based on ideas in the textbooks linked to the curriculum, most were well behind in implementing the reform perspective of the NCTM standards in their classrooms. So, the participants' starting point was oriented towards a teacher-directed approach.

I was invited to join the group as an «expert-friend» and given consent to study the group's work. Since the teachers wanted to engage in a learning process based on their way of thinking, my role was to provide non-threatening, non-authoritarian support, by responding to their needs rather than imposing direction, and not deliberately influencing events by dictating what they should do or how they should do it. Therefore, the teachers' learning process was completely open-ended in that they controlled and made the decisions for every aspect of it.

Three of the teachers assumed the role of group leaders and were responsible for organizing the group's meetings and activities. The group met in the school once every three weeks for about one and a half to two hours after their last class. They were able to use one half day and one full day of their school's professional development days in each term for their group work. They also organized it so that they could take turns, in small groups, to observe their *research* lessons. They also sometimes met during lunch breaks to plan and reflect on the lessons. Although the study group continued beyond the first year, the focus here is only on year one because it consisted of the key activities in the self-directed approach that framed what occurred in subsequent years.

The actual process the teachers engaged in was too complex to describe here in detail because of its non-linear nature and multiple dimensions. It involved, for example, several layers of activities, multiple voices, negotiation of meaning and process, and inquiry within inquiry. Thus, only an overview of some key components of the process around which an inquiry stance unfolded over eight months during the school year is provided.

OVERVIEW OF THE TEACHERS' INQUIRY PROCESS

When I joined the teachers, they had already spent three of their group meetings sharing and reflecting on examples of what they were doing in their classrooms to engage students in learning mathematics. Based on this process, they had decided that they wanted to learn more about inquiry-based teaching and adopting it in their practice. Thus, their overarching *puzzling situation* was what it means to teach from an inquiry perspective and the best way for them to learn about it. Two parallel processes then emerged: learning about inquiry and pursuing an as-yet undefined path to achieve their aim. This allowed them to assume an inquiry stance as they embraced uncertainty in terms of the path they would take and what they would eventually learn. The following is an overview of key aspects of the resulting process based on the decisions they made beginning with when I joined the group:

Deciding on a Pedagogical Problem. The teachers began with the *puzzling situation* of what to do to get started. They discussed this by considering possibilities such as studying relevant theory, trying out and sharing ideas individually, and doing mathematics. They agreed that a process of trying out and sharing ideas made the most sense because it was practical. However, as they discussed how this process would work, they decided that being from different grades was an issue for it to be meaningful for all of them and if they divided up according to grades, they would lose the multi-grade community they wanted to maintain. One teacher suggested, «We should think of something we can all work with that cuts across the grades.» This resulted in a discussion of what topic of common interest would relate to everyone's teaching. Someone suggested working with the new curriculum, which they pursued, but were still unsure of what was common to all of them.

At this point, they asked what I thought. I asked if they were familiar with the «front matter» of the curriculum. They were not but became curious and decided to read it for homework. The «front matter» outlined the perspectives of mathematics and learning and the mathematical processes that were required to enact the curriculum as intended. In the following group session, after three weeks to read and think about the «front matter», the teachers shared and discussed what might be meaningful to explore in relation to their practice. Their focus was on the mathematical processes emphasized throughout the curriculum (i.e., communication, connections, estimation and mental mathematics, problem solving, reasoning, and visualization). They became more interested in communication, connection and problem solving. After examining and evaluating these processes in relation to their teaching (e.g., what they did and did not do), they concluded that communication was the most meaningful for them to start with to make changes to their practice. A key reason for this conclusion was that inquiry-based communication would improve students' engagement and how they learned the mathematics. As one teacher explained, and the others agreed:

> Our students and their parents were used to doing math calculations but did not always have the experience or understand the importance of explaining and thinking through math. So it seems like a logical starting point for all levels of our learning community and our teaching.

Thus, at the end of the second group meeting, their pedagogical problem became what it meant to use communication to facilitate inquiry teaching.

Interpreting Key Construct in the Problem. Focusing on communication as the key construct to understand in their pedagogical problem and starting with their experiences, the teachers shared the types of questions they used in their teaching and how they engaged students. Some of the teachers shared ideas about questioning that they had read about. They eventually decided that it would be helpful to see what communication looked like in an inquiry lesson. They asked me for suggestions of how they could do this. I suggested a video study, which they liked, and decided to try. Some of them were aware of a Marilyn Burns' mathematics book, so they selected the Burns videos «Mathematics with Manipulatives» (Burns, 1988) from what I had access to for them to use.

The set of videos consisted of constructivist lessons that included inquiry-based learning approaches and communication in the elementary mathematics classroom. The teachers chose two of these videos, «Pattern Blocks» and «Cuisenaire Rods». Each video consisted of six lessons that covered the elementary grades. While the videos came with suggestions for use in professional development, the teachers were not interested in those guidelines. Instead, they discussed what they thought they should look for in the videos in relation to their practice. They decided to focus not only on communication but also on what they could learn about inquiry teaching. They asked if I had any advice before they looked at the first lesson. I suggested that they focus on what they could learn and use and not on being judgemental about the lesson for the sake of being critical. After clarifying what this meant, they then used the first lesson to orient their observation and record what stood out for them.

While there were many similarities in what the teachers observed, there were also differences that contributed to the variety of factors they found meaningful in the lesson. This outcome enabled them to decide on a common set of factors to focus their observations of the other video lessons. I helped them to organize these factors under broad categories that included students' role, teacher's role, questions posed by the teacher to stimulate/provoke and extend students' thinking, nature of tasks, and inquiry features of the lesson. After each lesson, they shared and built on each other's observations and used this to reflect on their own teaching in terms of what was lacking and what might be easy to begin to change. Two approaches they identified as applicable for all the grade levels were the use of groups and requiring students to share their thinking and not just give answers.

Creating an inquiry-teaching model. While the video study gave the teachers many ideas about inquiry-oriented practice and communication, they still had to decide on how to integrate these ideas into their teaching, not solely as individual techniques, but as a way of transforming their teaching. They decided they needed «a plan» – a systematic way to do this. Influenced by the structure they perceived in the video lessons, they decided to create a similar structure to guide their teaching, which they later called the inquiry-teaching model. Based on the video study and their discussions, they hypothesized that a model of inquiry teaching should include the following seven features: *free exploration, focused exploration, discussions, predictions, applications, evaluation, and extension* of the concept being taught. These are facilitated through communication, in particular, student-focused questioning by the teacher (e.g., What did you notice?) and students collaborating in small groups. Free exploration allowed students to see what they know on their own, while focused exploration involved the teacher providing a specific inquiry task.

Testing the inquiry-teaching model. In order to test their hypothesized inquiry-teaching model, the teachers planned an experimental lesson, then conducted, observed, analysed, and evaluated it. A Grade 1 teacher volunteered her classroom. The topic «explore and classify 3-D objects according to their properties» from the curriculum was selected to correspond with this teacher's schedule for the class. Based on their experience and new knowledge about inquiry, the teachers first brainstormed in small groups then shared different approaches to teaching the topic. Group 1 would: have students observe objects in the classroom; discuss why these objects have certain shapes; post pictures of objects in the real world around the classroom and use them to identify shapes; name geometric objects; make links to objects in class; refer to a chart with formal names; and have students investigate attributes and relate them to the real world (e.g., why things have certain shapes). Group 2 would: have students describe geometric objects in groups/pairs; list names of objects students suggest and descriptive words on a chart; have students build a model of one object and discuss and compare the model and an actual object; and introduce formal names. Group 3 would: pose a problem (e.g., build a house with this object); discuss attributes; have students explore attributes and classify attributes; and describe common features. Reflecting on these approaches and the seven features of the hypothesized inquiry-teaching model, the teachers sketched out the plan in Table 2 for grade 1 students' engagement in the lesson.

Brief introduction to set the tone
Free exploration of eleven 3-D geometric objects (Talk/experiment/observe in small groups)
Whole-class *discussion* of what they noticed
Individual *prediction*: Will shapes roll or slide? (using worksheet with pictures of the eleven 3-D objects and columns for rolls only, slides only and rolls and slides)
Discussion with a partner
Prediction if all will agree
Whole-class discussion of an *application* (think of self as a builder; Suppose I want to build a house on a mountain, what would I need to know about shapes?)
Focused exploration to test predictions (check with objects)
Discussion of findings with others in groups
Whole-class discussion of findings with justification and building of Venn diagram on white board with pictures
*Evaluation/*generalization (Venn diagram to sort pictures of shapes and make general statements about «What I know about 3D shapes!» 3-D vocabulary of objects)
An application *(extension)* task for homework (Look for things at home and around school that roll or slide.)

TABLE 2 — EXPERIMENTAL LESSON

Evaluating and revising the inquiry-teaching model. Following the evaluation of the lesson, the teachers discussed how well the model worked based on the level of students' engagement and learning. The Grade 1 students were «natural inquirers» and readily embraced the level of engagement of the lesson. The teachers were amazed and impressed with what the children were able to do, the richness of their thinking, and the depth of their learning of the concept. This provided evidence to support the meaningfulness and effectiveness of their inquiry-teaching model and understanding of student-focused communication. Follow up hypotheses and testing of the model involved questions that included: Does sequencing of the components matter? Are all components necessary in a lesson? Will the model work for different grades and topics? How can each of them implement the model successfully?

Table 3 highlights the key components the teachers finalized for the model. They are situated in inquiry-oriented questions the teacher must pose to prompt or challenge students' thinking. Although the teachers described it as a teaching model, it focuses on learning and learners and not the teacher, representing a significant shift in their thinking.

Students: · reveal *prerequisite knowledge* · make *predictions* · engage in *free exploration* · engage in *focused exploration* · work on *applications* · engage in *discussion*, *comparison*, *evaluation* and *reflection* of their learning · work on *extension*

TABLE 3 – COMPONENTS OF THE INQUIRY-TEACHING MODEL

Applying the Inquiry-Teaching Model. The teachers determined that their inquiry-teaching model was flexible in terms of the components to be used and how they are to be sequenced in a lesson. This conclusion allowed them to personalize how they used the model in their teaching. Planning in teams according to grade levels, they started to adopt the model in their own ways to their teaching, reporting back to the whole group and reflecting on what worked and difficulties they encountered. The difficulties included lack of depth in understanding important aspects of the mathematics they were teaching, which along with problem solving, became the focus in the second year of their study group and the basis of ongoing inquiry cycles. By the end of the first year of the study group there were significant changes in the teachers' thinking and teaching. They did acknowledge, however, that this was just the beginning of an ongoing journey toward becoming an inquiry teacher. They summarized some of the key aspects of their learning at the end of the first year as a:

> deeper and more meaningful understanding of: inquiry teaching; questioning techniques for student thinking; open ended, thought provoking questions to motivate students to discuss and understand mathematics at a deeper level; student-centered strategies for listening to students and observing their problem-solving behaviours; and strategies that allow students to assume ownership of their knowledge and knowledge construction.

THEORETICAL FRAMEWORK UNDERLYING THE TEACHERS' DEVELOPMENT PROCESS

The preceding section described key activities in the first year of the teachers' self-directed professional development initiative. As these activities indicate, the process the teachers went through was an inquiry in itself because it was not predetermined. Within this process was a parallel process of inquiry into their practice and how to make it more inquiry-based. Both were dependent on the experience and knowledge they brought to these processes and the questions that emerged as the processes unfolded. Thus, both can be linked to the perspectives in Table 1, which provide a theoretical framework for interpreting the inquiry orientation of the teachers' self-directed learning approach. To illustrate this relationship, the approach is considered as being composed of an overarching inquiry cycle (Table 4) and a series of inquiry cycles (Table 5).

In Table 4, the column «teachers» represents the key components (overarching cycle) of the teachers' learning process as described in the preceding section. The cycle was initiated by a «puzzling situation» about inquiry teaching that grew out of the teachers' own experiences (practice). Each component is linked to a phase in Dewey's and Schön's processes. More importantly, each involved reflective thinking (Dewey) and reflection on action (Schön). For example, as previously described, the teachers reflected on their teaching, their elementary mathematics curriculum and the mathematical processes to decide on a pedagogical problem. They analysed their own teaching and the teaching in a video to understand communication (a key construct in the problem) to further understand the problem and generate an initial hypothesis of inquiry-based communication and teaching. In relation to Wells' perspective, each component of the teachers' process involved dialogic inquiry, i.e., beginning with personal experience and using it to obtain information to build knowledge and understanding through discussion (dialogic discourse). Similarly, in relation to Mason's perspective, *noticing* was important in each

TEACHERS	DEWEY	SCHÖN	WELLS	MASON
Beginning with practice	Becoming aware of puzzling situation in experience	Reflecting on experience	Reflecting on experience	Reflecting on experience
Posing a pedagogical problem	Generating a specific problem	Identifying a problem		
Understanding key construct in the problem	Formulating an initial hypothesis	Analyzing practice		
Hypothesizing an inquiry-teaching model	Elaborating on the hypothesis	Planning action	Dialogic inquiry	Noticing
Testing/applying the hypothesized inquiry-teaching model	Testing the hypothesis	Applying results to future action		
Revising/applying the inquiry-teaching model	Generating new problems	Generating new problems		

TABLE 4 – TEACHERS' OVERARCHING INQUIRY CYCLE

component to enable the teachers to bring to the surface issues and ideas and to recognize, label and validate choices. In addition, introspective and interspective processes were also involved as they thought about their own practice and that of the other teachers.

Table 4, then, illustrates how Table 1 can be used as a theoretical framework for the teachers' learning process. However, as is required for Table 1, the learning process was not linear in terms of moving from one component to the next in an unproblematic way. Instead, each component can be viewed as an inquiry cycle, as illustrated for three cycles in Table 5.

While Table 4 presents the macro-level of the teachers' learning process, Table 5 represents the micro-level of the first three cycles within the macro-level. The column «theoretical framework» represents a combination of the different perspectives in Table 1. Each of the «teachers' cycle» columns highlights the key components of the sub-cycles the teachers went through as they navigated their way through an undefined process. Cycle 1 corresponds to «deciding on a pedagogical problem,» cycle 2 to «interpreting key constructs,» and cycle 3 to «creating an inquiry-teaching model», wich are the components of the macro-cycle as discussed in the section on overview of teachers' inquiry process. In some cases, there were abbreviated cycles within the micro-cycles as the teachers' discussions and reflections diverged from their intended topic/problem. Such cycles were based solely on dialogic discourse and may or may not have led to a resolution.

INQUIRY FRAMEWORK	TEACHERS' CYCLE 1	TEACHERS' CYCLE 2	TEACHERS' CYCLE 3
Encountering a puzzling situation based on interests, curiosities, thinking and experiences that initiates a process	Overarching puzzling situation about inquiry teaching; what inquiry means and how to transform practice	Puzzling situation resulting from cycle 1	Puzzling situation resulting from cycle 2
Identifying problem/ dilemma through reflective action, dialogic discourse and noticing	Problem of what topic to study to resolve puzzling situation	Problem of how to learn about communication in inquiry teaching	Problem of how to apply knowledge from video study to create an inquiry-teaching model to guide teaching
Creating and elaborating on hypothesis (plan of action) through reflective action, dialogic discourse and noticing	Hypothesis that the curriculum would be a good starting point in identifying a topic	Hypothesis that studying a video would be more meaningful as the basis for learning	Hypothesizing possible components for an inquiry-teaching model
Testing the hypothesis (applying the plan) and evaluating through reflective action, dialogic discourse and noticing	Studying «front matter» of the curriculum, focusing on mathematical processes	Studying videos of inquiry-based elementary school mathematics lessons	Planning experimental grade 1 lesson based on personal experience and hypothesized inquiry-teaching model
Drawing conclusions about the outcome (new knowledge created) through reflective action, dialogic discourse and noticing	Creating knowledge of mathematical processes that meant the most to the teachers' own teaching	Creating knowledge of inquiry tasks; student and teacher roles; inquiry questions/ prompts; inquiry lesson structure	Creating knowledge of possible ways to implement inquiry-teaching model to the teachers' practice
Generating a new problem/ dilemma through reflective action, dialogic discourse and noticing	New problem on how to learn about inquiry-based teaching focused on the communication process	New problem on how to apply knowledge from video study to the teachers' own teaching	New problem on how to test and observe experimental lessons in the classroom

TABLE 5 – TEACHERS' SERIES OF INQUIRY CYCLES

Tables 4 and 5 show how the perspectives in Table 1 can provide a theoretical framework for interpreting the teachers' learning process from an inquiry of inquiry perspective. They also demonstrate the complexity of the self-directed inquiry process in terms of the layers of inquiry that can emerge. These layers of inquiry were driven by the problems, challenges, and dilemmas the teachers encountered and their desire to pursue their interests and curiosities in ways that made sense to them as they tried to achieve their goal of engaging in a self-directed learning experience to understand inquiry teaching and transform their practice. The decisions they made in each cycle shaped the nature of their inquiry process, which in turn shaped the nature of the knowledge of inquiry teaching they constructed.

Although not elaborated on in describing the cycles, at both the macro and micro levels, reflection on experience and dialogic discourse played important roles in the teachers' learning. They often returned to experience to recall or detail salient events that resulted in new possibilities. In general, they began with self by examining what they knew, did not know, and wanted to know about a particular situation of interest. Their dialogical engagement opened possibilities for conducting the inquiry and creating a community of inquirers with shared goals. They shared stories of past and present experiences that formed a source and basis for their reflection. Their discourse took various forms, including telling stories of their classroom behaviours and the students' learning of mathematics, debating issues as they took sides, sharing and critiquing specific classroom experiences, sharing relevant experiences and knowledge from other subjects they taught, and sharing knowledge/thinking about mathematics and pedagogy.

An example of how the teachers shared and reflected on their experiences involves a session that was initiated by a puzzling situation some of them experienced while trying to get students to work in groups to solve a problem. They started with sharing situations/events involving the difficulties they experienced in getting students to share their group work (i.e., the puzzling situation). This evolved into the sharing of experiences about how their students' engaged with the problems. For example, Teacher L (a grade 3 teacher) shared:

> In the fractions [lesson], I did the ground work. (…) I knew they had some knowledge of fractions because when you brainstormed, they knew stuff. I asked them what they wanted to learn and they told me they wanted to add, subtract, do this. So I knew they knew what a fraction was. But it was interesting – out of all our talks, they did not know that the fraction needed equal parts. And those were my keen, keen ones [students]. So we actually cut things up into parts that were not equal for them to see how that would not represent a fraction. (…) Then we cut up into the equal. So that was really neat. So that was a good thing that came out of it. (…) Maybe I was wrong to expect that they would know that, I don't know. But I guess that's where my disappointment was. So maybe (…) that's not the best place to do it.

After Teacher L answered some questions about what she did, Teacher K (another grade 3 teacher who taught the same topic) then shared:

> Where I thought the fraction question was going to go, it didn't go there also. They all came up with pie charts and showed the 1/3. The question was: *If you put your hand into a bag of M & Ms and took out some M & Ms and 1/3 of them are red, what would that picture look like?* So I thought, «Oh, you can get some nice pictures here! Some of them might have 24 and some of them might have 12.» No! I got a pie chart divided into three equal pieces, (*laugh*), 1/3 red, and the other two coloured green or blue or whatever colours there were, right. (…) [One group said] «You should see the M & Ms. Let's draw a hand!» (…) So they drew the hand and they drew some M & Ms. [One student explained] «It's got to be three, and I don't know why, I don't know why exactly, but it's got to be three» because you are counting by threes, right? (…) This one group eventually came up with that. (…) But the others went to the pie chart.

Other teachers also shared related experiences. For example, Teacher B (grade 4) recounted the following:

> You know, I have to say that's what happened in the lesson that I did. They were to use equations that had their chosen number in it. So they chose like say, 25. And a lot of them had figured out the skip counting. So if it was 25 plus 37, they knew to jump down to 3 and then go over 7. But if it was a subtraction, they were okay minus-ing, but then they didn't know which direction to go, and so watching them struggle with that, you know, let me know where to go with the next lesson so that they knew where subtraction went on it. So the most valuable thing that I got out of it was not what they learned, but what they hadn't learned.

In this session, a key idea the teachers learned from their sharing, reflection and discussion was that, depending on how they listened to and observed their students, they could learn from the students' thinking and actions how or where to make changes in their teaching. For example,

> Teacher A: Listening to them you can find out 'Where do I need or how do I need to improve?' or 'Where do I need to go next?'
> (…)

Teacher C: That was the biggest part for me. (…) Like, when they all said (…) «we need this more» then you'll need to do this, so like you say –

Teacher L: It's a great indicator of that, what we need to do. Yeah. (…)

Teacher B: Yeah, listening to what they think and also look at what they do and not what we want them to do or to say. It can help us to help them more with how they are understanding the math.

They also became aware of how their thinking and expectations regarding their students' work could differ from the actual situation in significant ways and that they needed to be more open and flexible. This enabled them to make connections to the initial puzzling situation of how to get students to share their group work. The concern was that students did not know what to share and would share very little even with prompting. But as this excerpt of their discussion indicates, they became aware of a different way of viewing this.

Teacher K: We usually want for the sharing to be about what they did to get the answer, what they are able to do to get the answer. So the point is not that or whether you get an answer, but when it gets to sharing, could you talk – well –

Teacher A: Explain your thinking –

Teacher K: Yeah, talk about your thinking, and it could be about what they can't do or don't understand.

Based on this new understanding, two new «problems» emerged from their *reflection on experience* for further inquiry: (i) What does it mean to observe and listen to students in an inquiry classroom? Initially, prior to this session, they considered it to be about what they wanted to know; now they hypothesized that it should be about learning from the children. (ii) What does it mean for students to share their work? Initially, they wanted students to get to a correct answer and share how they got it; now they hypothesized it should be about explaining their thinking, regardless of how or whether they completed the task.

In relation to the framework in Table 4, in this session, the teachers engaged in Wells' dialogic process (Table 1) by sharing experiences of practice and obtaining information from it that led to their development of new knowledge and a different understanding of their teaching. They engaged in Mason's noticing (Table 1) by reflecting on and attending to significant actions and moments in their individual and collective experiences and validating their understandings (choices) with each other. They also engaged in Dewey and Schön-like reflective thinking by reflecting in and on action to analyze their teaching. This enabled them to identify specific «problems,» formulate hypotheses, and plan actions during the session, to test and apply the hypotheses in their teaching, and to follow up with discussions/reflection, thus completing an inquiry cycle. The excerpts of their sharing presented above also show how they were beginning to develop an inquiry stance, discussed in the next section, by reflecting, noticing, then acting.

THE TEACHERS' INQUIRY STANCE

By embarking on self-directed professional development to learn about teaching through inquiry, the teachers engaged in inquiry in a way that is consistent with developing an inquiry stance. It was a journey that challenged them to confront their practice and thinking in order to make changes. When the journey began, while they all participated in the discussions and decision-making, only a few seemed reflective and open to confronting their own teaching. This changed as they started to see themselves in each other's experiences in ways that resonated or conflicted with their own thinking and practices. This prompted them to share their own stories and open up their practice for examination by themselves and others. Comparing their practice to the «front matter» of their curriculum and to the teaching/lessons in the videos they studied, and planning and testing the experimental lessons were also instrumental in helping them to learn to reflect more deeply and notice aspects of their thinking and practice that they had taken for granted. The experiences prompted them to start making changes to their teaching throughout the journey prior to the completion of their inquiry-teaching model. For example, they started trying to get students to share and justify their thinking, to work in groups, and to explore with manipulatives.

In addition to the teachers' learning process being consistent with the inquiry perspectives presented in Table 1, it was consistent with Cochran-Smith and Lytle's (1999) perspective of the inquiry stance. For example, the teachers worked together to pose significant problems relevant to their teaching and their learning, challenge the status quo in their practice and engage in ways to bring about change. They reflected on each other's work and counted on each other for alternative viewpoints. They envisioned and theorized their practice, and interpreted and questioned the theory and research of others (e.g., the curriculum, the videos, and later readings from professional journals for mathematics teachers). They embarked on a process that involved uncertainty, posing problems and dilemmas, and recognized that inquiry both stems from and generates questions. This enabled them to learn to embrace uncertainty, to become flexible, and to notice. They embraced and learned from the process in a way that placed them on a path toward ongoing development of an inquiry habit of mind to question their practice and achieve the goals of becoming «life-long learners» as mathematics teachers and meaningfully engaging their students in learning mathematics.

Cochran-Smith and Lytle (1999) also noted that «teachers (...) who take an inquiry stance work within inquiry communities to generate local knowledge» (p. 289), which they considered «knowledge that may also be useful to a more public educational community» (p. 290), i.e., local knowledge with broad implications. The teachers' «local knowledge» included meaningful ways of engaging students in communication and an inquiry-based model for teaching mathematics. In addition, with encouragement from me, by the end of the study group's second year, some of the teachers presented at teachers' conferences and were invited to conduct workshops within and outside their school system. A couple of years later, a few of them accepted appointments to be «teacher leaders» in schools that were receiving professional development funds to start study groups. Cochran-Smith and Lytle (1999) also noted that «The most significant questions about the purposes and consequences of teacher learning are connected to teacher agency and ownership» (p. 293). Since the teachers engaged in a self-directed process, teacher agency and ownership were central to the process. The teachers also talked about how much they valued the collegiality of the learning community and learning an approach that they could use for ongoing learning and growth in their teaching. In general, the teacher's learning process was effective in helping them to develop an inquiry stance in relation to their practice.

CONCLUSION

If we accept that one aspect of being a teacher of mathematics is to develop an inquiry stance, then we need to think of inquiry as more of an ongoing, recursive process of learning than is generally reflected in studies of mathematics teacher education. Developing an inquiry stance requires an attitude of openness and acceptance of the idea that learning from inquiry is not only a path with no end, but one that is also a continual source of professional growth. Being able to accept this requires that teachers develop a willingness to participate in ongoing reflection and learning as part of their everyday practice. In sum, learning from inquiry requires an attitude of openness towards one's own teaching.

This paper illustrated one way in which mathematics teachers can engage in inquiry. It is based on a process of learning *through* and *about* inquiry, which in turn leads to the development of an inquiry stance. More importantly, the paper has illustrated how four interrelated inquiry perspectives can form a theoretical framework for mathematics teachers' learning. Such a framework requires that teachers engage in an open-ended process in which they determine – or play a key role in determining – the initial topic and questions to pursue. In this process, their personal experience and practice are crucial to their learning to «interpret and theorize what they are doing» (Cochran-Smith & Lytle, 1999, p. 291). The intent here is not to imply that teachers should embark on a self-directed learning process, but that whatever the approach, if it is from the perspective of the inquiry stance or the proposed theoretical framework, the teachers' perspectives and experiences are central. Research to further explore this framework should consider both self-directed situations with an «expert-friend» and situations supported by others as in Jaworski (2006) to shed more light on the roles of the teachers and «mentors» in creating an effective process involving the inquiry stance.

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