There is no evidence for order mattering; therefore, order does not matter: An appeal to ignorance

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Recibido el 23 de septiembre de 2016; aceptado el 16 de enero de 2017

No hay evidencia para la importancia del orden; por tanto el orden no importa: Una apelación a la ignorancia

Resumen.

Dentro del limitado campo de investigación sobre el conocimiento probabilístico del profesor, las respuestas incorrectas, inconsistentes o incluso inexplicables se explican frecuentemente usando marcos teóricos basados en las heurísticas y el razonamiento informal. Más recientemente ha emergido nueva investigación basada en falacias lógicas, que se ha mostrado efectiva para explicar ciertas respuestas normativamente incorrectas a tareas probabilísticas. Este artículo contribuye a esta área emergente, mostrando como una particular falacia lógica conocida como "apelación a la ignorancia" es útil para explicar un conjunto de respuestas normativamente incorrectas de profesores de matemática de enseñanza primaria y secundaria en formación a una nueva tarea probabilística. También se sugiere que el foco sobre el enfoque clásico en la enseñanza de la probabilidad teórica contribuye al uso de esta falacia en particular.

Palabras clave: Cognición; Falacias; Probabilidad; Conocimiento probabilístico; Profesores en formación.

Não há evidência de que a ordem seja importante; por isso, a ordem não importa: um apelo à ignorância

Resumo

No limitado campo de investigação sobre o conhecimento probabilístico dos professores, as respostas incorretas, inconsistentes e até mesmo inexplicáveis às tarefas probabilísticas são mais frequentemente explicadas pela utilização de teorias, estruturas e modelos baseados no raciocínio heurístico e informal. Mais recentemente, o surgimento de nova investigação baseada em falácias lógicas tem-se mostrado eficaz para explicar certas respostas normativamente incorretas a tarefas probabilísticas. Este artigo contribui para essa área emergente de investigação, demonstrando como uma falácia lógica particular, conhecida como "um apelo à ignorância", pode ser usada para explicar um conjunto específico de respostas normativamente incorretas dadas por futuros professores de

Para citar: Chernoff, E.J., Russell, G.L., Vashchyshyn, I., Neufeld, H. & Banting, N. (2017). There is no evidence for order mattering; therefore, order does not matter: An appeal to ignorance. *Avances de Investigación en Educación Matemática*, 11, 5 – 24.

matemática do ensino básico e secundário a uma nova tarefa de probabilidades. Sugere-se ainda que o foco na abordagem clássica para o ensino da probabilidade teórica contribui para o uso desta falácia lógica particular.

Palavras-chave: Cognição; Falácias; Probabilidades; Conhecimento probabilístico; Futuros professores.

There is no evidence for order mattering; therefore, order does not matter: An appeal to ignorance

Abstract.

Within the limited field of research on teachers' probabilistic knowledge, incorrect, inconsistent and even inexplicable responses to probabilistic tasks are most often accounted for by utilizing theories, frameworks and models that are based upon heuristic and informal reasoning. More recently, the emergence of new research based upon logical fallacies has been proving effective in explaining certain normatively incorrect responses to probabilistic tasks. This article contributes to this emerging area of research by demonstrating how a particular logical fallacy, known as "an appeal to ignorance," can be used to account for a specific set of normatively incorrect responses provided by prospective elementary and secondary mathematics teachers to a new probabilistic task. It is further suggested that a focus on the classical approach to teaching theoretical probability contributes to the use of this particular logical fallacy.

Keywords: Cognition; Fallacies; Probability; Probabilistic Knowledge; Prospective Teachers.

Il n'y a pas d'évidence que l'ordre importe ; alors, l'ordre n'importe pas : Un appel à l'ignorance

Résumé.

Dans le domaine limité de la recherche sur la connaissance des enseignants au sujet de la probabilité, des réponses incorrectes, inconsistantes et même inexplicables à des tâches probabilistes sont le plus souvent expliqués par le raisonnement heuristique et informel. Plus récemment, de nouvelles recherches basées sur les erreurs logiques se montrent efficaces pour expliquer certaines réponses incorrectes à des tâches probabilistes. Cet article contribue dans ce domaine de recherche émergente en démontrant comment une certaine erreur logique, connue sous le nom d'« appel à l'ignorance, » peut être utilisée pour expliquer un ensemble spécifique de réponses incorrectes fournies par de futurs enseignants du primaire et du secondaire à une nouvelle tâche probabiliste. On suggère aussi que l'accent mis sur l'approche classique d'enseigner la probabilité théorique contribuerait à l'emploi de cette erreur logique.

Mots-clés : Cognition ; Erreurs logiques ; Probabilité ; Connaissance probabiliste ; Futurs enseignants

1. Introduction

With some notable exceptions (e.g., Gómez, Batanero, & Contreras, 2013), there has been relatively little research to date on (prospective) teachers' probabilistic knowledge (Jones, Langrall, & Mooney, 2007; Stohl, 2005). The objective of this article, in general, is to contribute to this domain. More specifically, this article aims to contribute to the well-established domain of research that accounts for (through various theories, models and frameworks) incorrect, inconsistent, and sometimes inexplicable responses to a range of probabilistic tasks (Abrahamson, 2009; Chernoff, 2009; Kahneman & Tversky, 1972; Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; LeCoutre, 1992; Tversky & Kahneman, 1971, 1974).

To meet the general and specific objectives stated, a group of prospective teachers were asked to determine and justify which of two student responses provided the correct answer and explanation to a question that involved determining the probability that a three-child family has two daughters and one son. In addition to contributing a twist to a task recently introduced to the research literature, we also utilize a unique lens to account for certain responses to that task. In our analysis, we demonstrate that logically fallacious reasoning—more specifically, an appeal to ignorance (there is no evidence for p; therefore, not-p)—accounts for certain prospective teachers' normatively incorrect responses to the task. By meeting the general and specific objectives presented, this article will contribute to the area of research (Chernoff, 2012a; Chernoff & Russell, 2011a, 2011b, 2012a, 2012b) that suggests that fallacious reasoning, taken here to mean the use of logical fallacies, can account for certain normatively incorrect responses to probabilistic tasks.

2. A brief summary of prior research

Research into probabilistic thinking and the teaching and learning of probability has largely focused on incorrect responses to probability tasks. (It must be noted that the focus on incorrect responses does not, in any way, suggest a negative view of the mind [see, for example, Kahneman, 2011]). Many theories, models, and frameworks have, traditionally, accounted for normatively incorrect responses to probabilistic tasks within the field of mathematics education. More recently, a new area of research suggests that the use of logical fallacies can account for certain normatively incorrect responses to probabilistic tasks. In the following section, we provide a brief overview of the research in this area to date; for an in-depth survey of research related to probabilistic intuition and difficulties associated with learning probability, see Batanero, Chernoff, Engel, Lee, and Sanchez (2016).

2.1. Heuristic and informal reasoning

In the early 1970s, psychologists Amos Tversky and Daniel Kahneman ran a series of studies that led them to conclude that "people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations" (1974, p. 1124). In particular, based on the results from multiple surveys that asked individuals to compare sequences of random events (like flipping a coin), they noted that "people expect that the essential characteristics of the process will be represented, not only globally in the entire sequence, but also locally in each of its parts" (1974, p. 1125). This type of reasoning was denoted the *representativeness heuristic*: whereby one "evaluates the probability of an uncertain event, or a sample, by the degree to which it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated" (Kahneman & Tversky, 1972, p. 431). For example, when considering a sequence of tosses of a coin, Tversky and Kahneman found that subjects deemed the sequence HTHTTH was more likely to occur than HHHHTH (where H stands for "heads" and T stands for "tails") because the ratio of the number of heads to the number of tails was not close enough to one.

As part of their research, Kahneman and Tversky also identified what they considered to be the root cause of the representativeness heuristic, heuristic reasoning, and individuals' intuitive notion of randomness: "people's intuitions about random sampling appear to satisfy the law of small numbers, which asserts that the law of large numbers applies to small numbers as well" (Tversky & Kahneman, 1971, p. 106) and, further, "this belief [in the law of small numbers] […] underlies the erroneous intuitions about randomness, which are manifest in a wide variety of contexts" (Kahneman & Tversky, p. 433). As Chernoff and Russell (2012a) explain, the representative heuristic, at the time, provided a new model for interpreting normatively incorrect responses to comparisons of relative probability, and as a result, heuristic reasoning became a dominant lens for research investigating comparisons of relative probability.

Since the seminal work of Tversky and Kahneman in the 1970s, a number of researchers have addressed issues associated with the inferential nature of Tversky and Kahneman's research. For instance, as Shaughnessy (1992) notes, "there was no attempt made [by Tversky and Kahneman] to probe the thinking of any of these subjects" (p. 473). Consequently, Shaughnessy (1977) brought two new elements to the task. First, "the subjects were asked to supply a reason for each of their responses. In this way it was possible to gain some insight into the thinking process of the subjects as they answered the questions" (p. 308). Second, in comparing the chances of occurrence of different sequences, Shaughnessy's iteration provided students with the option of "about the same chance" (p. 309) as one of the forced response items. The results of Shaughnessy's work, by and large, echoed the results of work by Tversky and Kahneman, even though the "about the same chance" option was unavailable to the participants Tversky and Kahneman's studies. However, with the new justification element of the task, Shaughnessy was able to determine that subjects did, in fact, find that a sequence such as BBBGGG (where B stands for "boy" and G for "girl," with the sequence representing birth order) was not representative of (the expected) randomness and that the sequence BBBBGB was not representative of the (expected) ratio of boys to girls.

The research of Lecoutre led to the recognition of another bias, the *equiprobability bias*, that Lecoutre suggested be "added to the list of biases observed in various situations of uncertainty" (1992, p. 558). This research was based on interpreting comparative probability task responses that involved a relationship between randomness and equiprobability. As Lecoutre (1992) found, some individuals seem to believe that random events are equiprobable "by nature" (p. 557), or, conversely, misconstrue equiprobability as the notion that "anything can happen" (Chernoff, 2012a, p. 80). For example, when a piece of candy is drawn from a box containing two orange flavored pieces and one lemon flavored piece, the events "draw a lemon-flavored piece" and "draw an orange-flavored piece" may be considered equally likely by some individuals because the process is random, and thus "the two results to compare are equiprobable because it is a matter of chance" (Lecoutre, 1992, p. 561). The equiprobability bias has accounted for certain normatively incorrect responses in more recent studies (e.g., Chernoff, 2012a).

During the same period of time, Konold and colleagues (1993), building upon Shaughnessy's (1977) task modifications (i.e., including an equally likely response option and asking for a response justification), introduced a different version of the relative probability task (e.g., more than two sequences were provided to choose from) than had been seen in the past. Further, the researchers gave students a "Which is most likely?" version of the task followed by a "Which is least likely?" version of the same task. They found that, for the most likely version, certain participants answered using the outcome approach—"a model of informal reasoning under conditions of uncertainty" (Konold, 1989, p. 59)—according to which the participants interpreted the question as asking them to predict what will actually happen instead of taking probability into consideration. For the least likely version, researchers found the representativeness heuristic model to be applicable, as was the case with Kahneman and Tversky's (1972) research. The outcome approach became, as had the representativeness heuristic earlier, a new way to account for incorrect responses to comparisons of relative probability. As a result, Konold's notion of informal reasoning also became a dominant lens for research investigating comparisons of relative probability.

2.2. Logically fallacious reasoning

More recently, a slowly growing area of research suggests that fallacious reasoning, more specifically, the use of logical fallacies, can account for certain normatively incorrect responses to probabilistic tasks. For example, Chernoff and Russell (2011b, 2012a) demonstrated that certain prospective mathematics teachers, when asked to identify which event (a.k.a., outcome or subset of the sample space) from five flips of a fair coin was least likely to occur, instead of utilizing the representativeness heuristic (Kahneman & Tversky, 1972), the outcome approach (Konold, 1989) or the equiprobability bias (Lecoutre, 1992). Instead, they employed a particular logical fallacy, the *fallacy of composition*: when an individual infers something to be true about the whole based upon truths associated with parts of the whole. Specifically, Chernoff and Russel found that participants incorrectly deemed that the events were equally likely by following a particular line of reasoning: namely, the coins (i.e., the parts) are equiprobable; the events (i.e., the whole) are comprised of coins; therefore, the events are equiprobable (which is not necessarily true). It should be noted that use of the fallacy of composition accounted for both normatively correct and incorrect responses to the relative probability comparison task. In doing so, new light was shed on a subset of the answers that would have, traditionally, been accounted for with Lecoutre's equiprobability bias.

In subsequent research, Chernoff (2012a) and Chernoff and Russell (2011a) applied the fallacy of composition to a more traditional relative probability comparison. Prospective mathematics teachers were asked to determine which of five possible coin flip sequences, rather than events, were least likely to occur. As was the case in their prior research (e.g., Chernoff & Russell, 2011b), the fallacy of composition accounted for normatively incorrect responses to the task. More specifically, the researchers demonstrated that participants referenced the equiprobability of the coin, noted that the sequence is comprised of flips of a fair coin and, as such, fallaciously determined that the sequence of coin flips should also have a heads-to-tails ratio of one to one. In other words, the properties associated with the fair coin (the parts), which make up the sequence (the whole), are expected in the sequence. Once again, the fallacy of composition, instead of traditional theories, models, and frameworks associated with heuristic and informal reasoning, accounted for certain normatively incorrect responses to a probabilistic task.

A unique advantage of this new interpretation of certain normatively incorrect responses is that it avoids certain assumptions about individuals' understanding of the law of large numbers. While the work of Tversky and Kahneman suggested that the law of small numbers—an application to small numbers of the law of large numbers, which "guarantees that very large samples will indeed be highly representative of the population from which they are drawn" (Tversky & Kahneman, 1971, p. 106)—was related to the representativeness heuristic, subsequent research (e.g., Evans & Pollard, 1981; Kunda & Nisbett, 1986; Stohl, 2005) has demonstrated that individuals do *not* always have a proper understanding of the law of large numbers. Consequently, as Chernoff and Russell (2012a) recognized, the assumption that when individuals appear to be employing the law of small numbers, they have an appropriate understanding of the law of large numbers to make such an application, is worthy of note. On the other hand, the fallacy of composition framework, which was demonstrated to account for normatively incorrect responses to relative probability comparisons that, previously, may have been accounted for by the representativeness heuristic, avoids said note worthy issue.

Chernoff and Russell (2011a, 2011b, 2012a) contend, based on their research utilizing the fallacy of composition, that they have opened or reopened a new area of investigation for those researching probabilistic thinking and the teaching and learning of probability. They also suggest that more research will allow individuals to determine to what extent informal logical fallacies and fallacious reasoning can account for normatively incorrect responses to a variety of probabilistic tasks. The former and latter contentions provided the motivation (and research question) to determine whether or not any another logical fallacy, namely an appeal to ignorance, can account for normatively incorrect responses to a probabilistic task that has recently been introduced to the research literature.

3. Theoretical framework

A logical fallacy is, in simplest terms, an error in reasoning. In general, a logical fallacy occurs when the premise(s) of an argument are not sufficient to support the conclusion. Of the numerous fallacies that could potentially be utilized as a theoretical framework (e.g., equivocation, begging the question, the fallacy of composition, the fallacy of division, and others), the analysis of the results of the present study will rely on one particular logical fallacy: namely, an appeal to ignorance, which is "an argument for or against a proposition [*p*] on the basis of a lack of evidence against or for it" (Curtis, 2011, para. 3).

An appeal to ignorance can come in one of two forms: (1) there is no evidence against *p*, therefore, *p*, and (2) there is no evidence for *p*, therefore, *not-p*. Stated in more colloquial terms, the two cases of the fallacy can be described as follows: (1) the lack of evidence against a proposition is perceived as evidence that the proposition is true, and (2) the lack of evidence for a proposition is perceived as evidence that the proposition is false. As an example, consider the following question: Is there a lawn mower in my garage? If one does not look inside the garage, the lack of evidence *against* the existence of a lawn mower in my garage does not amount to evidence for its existence, because there may not be a lawn mower in my garage; this is the first form of an appeal to ignorance. On the other hand, the lack of evidence *for* the existence of a lawn mower in my garage does not amount to evidence of its absence, because there may, in fact, be a lawn mower in my garage; this is the second form of an appeal to ignorance.

Although the previous example may seem contrived, this fallacy is, in fact, quite common in everyday life. For instance, some people believe that a large and unusual type of animal (often called the Loch Ness monster) inhabits Loch Ness in the Scottish Highlands. While there is no evidence that the animal exists, there is also no conclusive evidence that it does *not* exist[1]. People may reason (fallaciously) about this absence of evidence in two different ways. In particular, for some, the absence of evidence against the existence of the Loch Ness monster acts as proof of its existence. This is the first form of the appeal to ignorance fallacy: there is no evidence against *p* (no one can prove unequivocally that Loch Ness does not exist), therefore *p* (Loch Ness exists, just beyond the grasp of our exploration). On the other hand, others interpret the lack of evidence for the existence of the Loch Ness Monster as proof of its *non*-existence, which is the second form of the fallacy: there is no evidence for *p* (no one has found unequivocal evidence that Loch Ness exists), therefore *not-p* (Loch Ness does not exist).

To give yet another example, if a drug has been subjected to numerous tests for harmful effects and none have been discovered, some may conclude that the lack of evidence that the drug is harmful (there is no evidence for *p*) is proof that the drug is not harmful (therefore *not-p*). In Western society, this second form of the fallacy is often rationalized and accepted, especially in scientific domains: as Curtis (2011, para. 14) writes, "when extensive investigation has been undertaken, it is often reasonable to infer that something is false based upon a lack of positive evidence for it." However, the acceptance of this fallacy does not negate the fact that it is indeed a fallacy—a point that, we acknowledge, is debated in certain cases, such as in the aforementioned case of drug testing. At best, when extensive investigation has been undertaken, a lack of evidence suggests that the *probability* is low that a given proposition is true; however, given that such reasoning is inductive rather than deductive, it is not conclusive, and therefore does not *prove* that the given proposition is true (Curtis, 2011).

Our analysis of the results of the present study will be concerned with this second form of the fallacy: there is no evidence for *p*, therefore *not-p*. In other words, our attention will focus on a set of individuals utilizing the absence of evidence as evidence of absence. Stated in terms of our first example, our analysis of the results will focus on those individuals who do not look inside the garage and use the lack of evidence of a lawn mower in the garage to fallaciously declare that there is no lawn mower in my garage. Of course, the participants in the study are not concerned with garages and lawn mowers—rather, they are concerned with whether or not order matters for a particular probabilistic task.

4. The Jane or Dianne task

The Jane or Dianne task, presented below in Figure 1, represents an alteration to the original "two boys and a girl task" (Chernoff & Zazkis, 2011, p. 21), which was utilized in previous research (ibid.) and introduced by Chernoff and Zazkis (2010).

What is the probability that a three-child family has two daughters and one son?

Jane's explanation: Out of the four possible outcomes (3 daughters, 0 sons; 2

[1] We do acknowledge the potential impact of a recent photo of the Loch Ness monster taken by Ian Bremner in late September, 2016.

daughters, 1 son; 1 daughter, 2 sons; and 0 daughters, 3 sons) only one outcome (2 daughters, 1 son) is favourable, so the probability is one-fourth.

Dianne's explanation: Out of the eight possible outcomes (daughter, daughter, daughter; daughter, daughter, son; daughter, son, daughter; son, daughter, daughter; daughter, son, son; son, daughter, son; son, son, daughter; son, son, son) only three outcomes (daughter, daughter, son; daughter, son, daughter; son, daughter, daughter) are favourable, so the probability is three-eighths

________________________ 's explanation is correct because.....

Figure 1: The Jane or Dianne Task

Fundamentally, the two boys and a girl task (Chernoff & Zazkis, 2010) is the same as the Jane or Dianne task. In other words, the core of the task—the probability question, which is what is the probability that a three-child family has two daughters and one son—is the same in both tasks. Previously, the task was utilized in order to elicit insight into prospective secondary school mathematics teachers' pedagogical approaches. Said preservice teachers (who had previously studied probability) were able to quickly give the normatively correct answer to the two boys and a girl task, and thus, were instead presented with the task of describing their pedagogical approach in response to a hypothetical student who gave a normatively incorrect answer of the probability being one-fourth. Chernoff and Zazkis (2011) suggest a "desirable pedagogical approach as one that uses the learner's ideas as a starting point" (p. 19), and their investigations focused on whether the prospective high school mathematics teachers' responses modeled this approach, how these preservice teachers would react to such an approach, and how personal experience influenced pedagogical choices.

In addition to the desirable pedagogical approach, Chernoff and Zazkis (2011) introduced the notion of the *sample set*, defined as "any set of all possible outcomes, where the elements of this set do not need to be equiprobable" (p. 18) in order to frame their investigation of prospective teachers' pedagogical responses. They argue that students who give the normatively incorrect response of one-fourth to the two boys and a girl task are in fact using a legitimate partition of the sample space which includes all possible outcomes, i.e. {three boys (BBB), two boys and one girl (BBG, BGB, GBB), one boy and two girls (BGG, GBG, GGB), three girls (GGG)}. This partition, they contend, is not a mistake, but in fact a useful step in probabilistic thinking. The error is made when students assign equal probability to said outcomes, rather than asking if, in fact, each outcome they have listed is actually equally likely. The study went on to present the preservice teachers with a more challenging probability question designed to have them experience what a theoretical tutee might experience, and then interact with both the desirable pedagogical approach, as well as their own suggested pedagogical approaches, which often included disregarding the student's response, and imposing the correct reasoning.

In this new version of the task, however, the focus is not on evaluating the prospective teachers' pedagogical responses. Rather, the focus is on which response prospective mathematics teachers deem mathematically correct and, relatedly, which explanation is deemed appropriate. Possible outcomes were included as justification for the responses in Jane and Dianne task, in part, because previous research has shown that normatively incorrect solutions do not necessarily point to an absence of correct probabilistic reasoning (Abrahamson, 2009; Chernoff, 2009; Chernoff & Mamolo, 2015; Konold et. al., 1993). In terms of the original context of the question from Chernoff and Zazkis (2011), the Jane and Dianne task gives insight into whether or not students are inclined to use a sample set without realizing that not all outcomes in the sample set are equally likely. As such, the task has been altered in order to contribute to the limited amount of research on what Jones, Langrall, and Mooney (2007) called "teachers' probabilistic knowledge" (p. 933).

5. Participants

The (n = 130) participants in our research were comprised of 52 (40%) prospective elementary school teachers (PESTs) and 78 (60%) prospective secondary school teachers (PSSTs). The PESTs were following a methodology course designed for teaching elementary school mathematics and the PSSTs were following a methodology course designed for teaching secondary school mathematics. The 52 PESTs were enrolled in two different classes (each including approximately 25 students); similarly, the 78 PSSTs were enrolled in three different classes (each including approximately 25 students). For both the PEST's and the PSSTs, the topic of probability had not yet been addressed in their course. Instead, content, strategies, and approaches garnered from research and practice related to the teaching and learning of probability were addressed after the data for this research was collected. To collect the data, participants were asked to determine, via written response, which of the two explanations (Jane's or Dianne's) was correct; and further, to justify their choice, also via written response. The participants were given as much time as required to complete the task.

6. Results

As seen in Table 1 below, there was, roughly, an even split between those individuals who declared and explained why Jane's response was correct and those who declared and explained why Dianne's response was correct. More specifically, about half (51%) of the participants declared that Dianne's explanation was correct: 40 of the 78 PSSTs (51%) and 26 of the 52 (50%) of the PESTs. Thus, as the data shows, there was little difference between the percentage of PESTs and PSSTs that chose Dianne and her explanation. Worthy of note, not all 66 of the 130 participants who chose the correct response provided an appropriate justification for why Dianne's explanation was correct. [Nine of the participants (7%) chose an option that was not presented to them. These individuals either indicated that neither choice and explanation was correct or that both of the choices and explanations were correct. These individuals have been placed in the 'Other' column in Table 1. The responses from those individuals who chose Dianne or who fell into the 'Other' category will not be part of the subsequent analysis of the results.]

Table 1: *Numerical results.*

Prospective teachers	n	Jane	Dianne	Other
Elementary school	52	23	26	3
Secondary school	78	32	40	6

Total	130	55 (42%)	66 (51%)	9 (7%)

Instead, our analysis of the results will focus on the 55 participants (42%) that chose Jane and her explanation. More specifically, 23 of the 52 PESTs (44%) and 32 of the 78 (41%) of the PSSTs chose Jane and her explanation, which, as was the case with Dianne, reveals little difference between the percentages associated with the PESTs and PSSTs. The 55 total participants who chose Jane and her explanation did not have similar justifications for why Jane's response and explanation was considered correct. As such, the 55 responses from those individuals who chose Jane have been further categorized in Table 2 below.

Jane responses were organized into two distinct categories: those responses that referenced order and those responses that did not reference order. Of the 55 participants that chose Jane's response, 45 (82%) referenced order in their justification and 10 (18%) did not reference order in their justification. More specifically, of the 23 PESTs that chose Jane's response, 15 (65%) referenced order and 8 (35%) did not make reference to order and, of the 32 PSSTs that chose Jane's response, 30 (94%) referenced order in the justifications and 2 (6%) did not reference order in their justifications, which represents a departure from the previous even split seen between PESTs and PSSTs.

Table 2*: Numerical results within Jane responses.*

Prospective teachers	n	No reference to order	Reference to order	
			No reference to question	Reference to question
Elementary school	23	8	7	8
Secondary school	32	2	17	13
Total	55	10	24	21

Refining "Jane" responses a step further, of the 45 people that referenced order in their response justifications, 21 individuals (47%) also made reference to the question (What is the probability that a three-child family has two daughters and one son?) in their response justifications. More specifically, 8 of the 15 PESTs (53%) and 13 of the 30 PSSTs (43%) referenced both order and the question. The 21 responses referencing both order and the question, which represent, concurrently, 16% (21/130) of all participants, 38% (21/55) of those who chose Jane's response and 47% (21/45) of those who chose Jane's response and referenced order in their justifications, are featured in the analysis of results.

6.1. Analysis of Results

Given the consistency associated with all 21 of the responses that referenced both order and the question, 5 of the 8 responses from the PESTs and 5 of the 13 responses from the PSSTs—10 in total—are presented below for analysis.

PESTs Response Justifications

In what follows, we analyse five exemplary responses from Sam, Rebecca, Carla, Ernie and Woody, which all evidence an appeal to ignorance. We begin by considering the responses of Carla and Woody.

> *Carla:* Dianne's explanation gives possibilities of 2 girls and one boy *plus birth order possibilities – this is not what the question asked.*

> *Woody:* Dianne reuses some of the possibilities multiple times. *GGB is the same as GBG. The question is not asking anything about the order in which they were born.*

As the italicized portions of the responses from Carla and Woody show, both individuals reference that birth order should not be taken into consideration. On the one hand, Woody, declaring that "GGB is the same as GBG," is implicitly declaring that order does not matter. Carla, on the other hand, is more explicit in declaring that Dianne's explanation calculates the possibilities "plus" the birth order, which can be interpreted to mean that the order is in addition to what the question is asking. Both Woody and Carla, however, are quite clear in declaring why they have concluded that birth order does not matter. Essentially, both individuals make it clear that the question does not "ask" about the order. As seen in the responses from Sam, Rebecca, and Ernie, which are presented below and similarly italicized as above, they, too, reference that the question does not "say anything" or "mention" or "never asked" (respectively) about birth order.

> *Sam:* This explanation is better *because the question doesn't say anything about birth order*. It just wants to know the probability of 2 daughters and 1 son; *whether or not this occurs as DDS, DSD, or SDD does not matter.* I think they have a 1/4 chance of the 2 daughters and 1 son outcome.

> *Rebecca:* There are no repetitions of the number of each sex of child in Jane's. *The question doesn't mention birth order and therefore there is no need to consider that GGB and BGG is the same thing* in answering this particular question.

> *Ernie: The question never asked about order* so the only total is assumed and therefore needed ~ *disregard order patterns* so 4 possible outcomes.

Further, the responses from Sam, Rebecca, and Ernie indicate that there is no need to consider birth order or that one can, as stated by Ernie, "disregard the order patterns." Alternatively stated, for all three responses, given that the question does not make reference to order mattering, it is concluded that order does not matter, which, ultimately, leads to choosing Jane and her explanation as being correct.

Considered from within an appeal-to-ignorance framework, the responses from Sam, Rebecca, and Ernie and, further, from Carla and Woody (and the other 3 PESTs who referenced both order and the question in their responses), all note that the question does not provide evidence that order matters (i.e., there is no evidence for *p*) and, as a result, conclude that order does not matter (i.e., therefore *not-p*), which, ultimately, grounds a justification for why Jane's response and her explanation are correct. Similar results are found within the responses from the PSSTs.

PSSTs Response Justifications

In what follows, we analyse five exemplary responses from Frasier, Eddie, Robin, Paul and Glen, which also all evidence an appeal to ignorance. We first consider the responses of Paul and Eddie.

Paul: There are four different combinations that are possible *because it didn't specifically say that order mattered.* So generally, you can have 2 sons, 1 daughter; 2 daughters, 1 son; 3 sons; 3 daughters *in any order.*

Eddie: There is no specification as to what order the daughters & sons have to be born in. Therefore, there is only four possible outcomes causing a one in four chance.

As seen in the responses from Paul and Eddie, they make reference to the question and the fact that it does not specify that order matters. We note that in these particular responses, we are inferring that "it" for Paul and "there is no specifications" for Eddie are implicit references to the question. Working from this inference, for both Paul and Eddie, the reason that there are only four possible outcomes or that the different events can happen in any order are predicated on the question not providing evidence that order matters. The responses from Frasier, Robin, and Glen, presented below, are more explicit in their reference to the question.

Frasier: The question does not state the order of the siblings matters → they simply want a 2 girl + 1 boy family. *Dianne has multiples of the same outcome* such as DDS, and DSD.

Robin: The question did not ask what the probability is that the family has 2 daughters and one son *in that order, so there are only 4 possible outcomes*.

Glen: The question does not specify that the order of the children matters, they just want 2 daughters and a son. It shouldn't matter what order they come out in. *As far as the question is concerned, the DDS, DSD, SDD are all the same outcome.*

In fact, the above three responses are quite explicit in declaring that the question does not "state," "ask," or "specify" that the order of the children matters. Further, and working from the notion that the question does not specify that the order matters, all three participants conclude that the order does not matter, albeit in different ways (e.g., "multiples of the same outcome"; "there are only four possible outcomes"; "DDS, DSD, SDD are all the same outcome"). Alternatively stated, for the responses of Frasier, Robin, and Glen, given that the question does not make reference to order mattering, it is concluded that order does not matter, which, ultimately, grounds choosing Jane and her explanation as being correct.

Considered from within an appeal-to-ignorance framework, the responses from Paul, Eddie, Frasier, Robin, and Glen (and the other 8 PSSTs who referenced both order and the question in their responses) all note that the question does not provide evidence that order matters (i.e., there is no evidence for *p*) and, as a result, conclude that order does not matter (i.e., therefore *not-p*), which, ultimately, acts as a justification for why Jane's response and her explanation are correct.

6.2. Discussion

Research into the teaching and learning of probability and probabilistic thinking has focused on accounting for normatively incorrect, sometimes inexplicable responses to a variety of probabilistic tasks. Stemming from these investigations, a number of theories, models and frameworks have been developed to account for and to make sense of particular responses. Traditionally, this particular domain of research has focused on heuristic and informal reasoning. More recently, an emerging thread of research has (re)opened logical fallacies as a fresh perspective to account for certain response justifications. While it has been established that the fallacy of composition is able to account for incorrect responses to comparisons of relative probability (Chernoff

& Russell, 2011a, 2011b, 2012), it had not been determined to what extent logical fallacies can account for response justifications to other probabilistic tasks (other than relative probability comparisons) and which other fallacies could be utilized. Building upon this emerging thread of research, this article has demonstrated that a second logical fallacy, namely an appeal to ignorance, is able to account for particular responses to probabilistic tasks.

Speaking more generally, a major purpose of studying teachers' probabilistic knowledge is to better understand how teacher knowledge affects the teaching and learning of probability in classrooms. Consider Stohl's work with probability in teacher education, which demonstrates that there are two very important facets to the success of a probability curriculum: namely, it relies on a "teachers' understanding of probability as well as a much deeper understanding of issues such as students' misconceptions" (2005, p. 351). This deeper understanding has been referred to as knowledge of content and students (KCS), which is built on the teacher's previous experiences with students and informs their future instructional design (Hill, Ball & Schilling, 2008). Thus far, the focus of the article has been on providing strong evidence for the existence of an appeal to ignorance in pre-service teacher responses to certain tasks in probability, but no attempt has yet been made to develop a deeper understanding as to why such appeals occur. We now consider two instances.

Framed using the notion of a sample set (Chernoff & Zazkis, 2011), participants who chose Jane's response began with a legitimate partition of the sample space (as the set did indeed list all possible outcomes), which was perhaps naturally suggested by the language of the prompt, "…two daughters and one son." Up until this point, it could be argued, then, that they were employing "correct" probabilistic reasoning. The participants' faltered when, rather than asking if each of the outcomes in the sample set were equally likely, they used an appeal to ignorance to justify their sample set rather than expanding it to the sample space. Their belief that order could be disregarded based on a lack of evidence for order mattering ended their trajectory of correct probabilistic thinking and resulted in choosing Jane's response.

Further, given that Stohl (2005) also claims that teachers lacking knowledge in probability content and/or pedagogical content knowledge will likely make mistakes similar to those of their students based on beliefs and intuitions, it is interesting speculate a re-casting of roles in order to begin to consider why an appeal to ignorance appears as a consistent response. This re-casting (very) loosely posits the researcher as the classroom teacher (albeit in a very limited role) and the preservice teacher participants as students of probability in a typical classroom setting (albeit in a very limited role) in which the data was collected. By doing so, speculation based on this study, which contained a researcher-preservice teacher dynamic, could then perhaps inform a classroom teacher-student dynamic.

Based on the above comparison, perhaps this article can inform teachers' (researchers') knowledge of probability by drawing attention to the use of logical fallacies (as a new area of investigation for future research on probabilistic thinking, the teaching and learning of probability, and teachers' probabilistic knowledge). However, Stohl's second facet also deserves some attention: How can this study contribute to a deeper understanding of how the classroom teacher (researcher) understands the cause of students' (pre-service teachers) misconception of an appeal to ignorance? In other words, beyond claiming its existence, it is worthwhile to consider the role of a classroom teacher to better inform possible factors supporting the use of an appeal to ignorance as a logical fallacy.

One possible explanation is based on the work of Konold and colleagues (1993). In their work of studying the causes for inconsistent, and sometimes contradictory, lines of probabilistic reasoning, they highlight the existence of maxim-like beliefs in reasoning around coin-flipping. These maxims often influence decisions in probabilistic environments, and can sometimes be found to be held in contradiction to one another. The power of these beliefs is so influential that students may hold two maxims to be unequivocally true when stated in isolation (e.g. "One cannot predict for certain the results of coin flipping," and "Heads and tails occur about equally often in a sample of flips"), and still cling to these 'truths,' albeit sometimes begrudgingly, when the result of their combination is shown to result in contradictory reasoning (Konold et al., 1993, p. 407-408).

Initially, it may be argued that the participants who chose Jane's response in favour of Dianne's were following the maxim "*All outcomes of an experiment are equally likely.*" They then generated a list of possible family compositions (with regards to gender) and proceeded to treat them all as equally probable because (by the students' reasoning), as with all random events, randomness can only be handled by assigning every outcome an equal probability (this line of reasoning was termed the "equiprobability bias" by Lecoutre [1992]). The issue with this interpretation is that both Jane and Dianne's solutions present lists of outcomes—one in the form of the normative sample space and one as a sample set (Chernoff & Zazkis, 2011)—that are presented as being equiprobable. Therefore, if "*all outcomes of an experiment are equally likely*" were the only guiding maxim, it would not lead to choosing one response over the other. In other words, the equiprobability bias cannot fully explain why some participants chose Jane's response over Dianne's.

Instead of relying on a maxim related the nature of probability, it seems that some of the participants who chose Jane's justification did so because of the way the question was constructed. Each of the 21 participant responses that were the impetus for this article explicitly referenced the information, or lack thereof, in the question. The language of their responses (e.g. "*it didn't specifically say that order mattered,*" and "*the question doesn't mention birth order and therefore there is no need to consider that*") signals that they may be acting according to a social maxim (Yackel & Cobb, 1996; Yackel, Rasmussen, & King, 2000) regarding the nature of all questions in probability, which can be stated as "*A question must contain all pertinent information to arrive at a correct solution.*" In other words, their mathematical action is justified by the ultimate authority of the question. We term this the *explicitness maxim*. It falls outside the realm of maxims describing the mathematics of the task, but rather speaks to the sociomathematical sphere in which tasks are encountered.

It is likely that this neat and tidy perception of the probability task comes from the PSST and PEST's experiences with probability in classrooms. While the study of probability contains ample opportunity for the experimental, this branch is often avoided in favour of the classical, normative approach based on counting principles—theoretical probability. It is not a surprise that teachers favour this classical approach. It is accessible through counting techniques and algorithms, leads to a single answer, and somehow "avoids a realistic interpretation of that value" (Stohl, 2005, p. 347). (Certainly, the value attributed to various methods also depends on the aim of teaching—conceptual progress or numerical progress [Borovcnik, 2012]—, and it may be argued that the classical approach favors the latter.) This is the social paradigm from which probability questions emerge. Classical probability treats problems in a theoretical space where necessary information such as sample space, favourable outcomes, and existence of order are either systematically calculated or explicitly stated. An explicitness maxim easily gains influence in this culture. Without an explicit reference to attending to order, the social norm is maintained by assuming that no mention of order (no evidence for *p*) means that order doesn't matter (therefore *not-p*). It is so powerful that it is offered as the only discernable difference between the responses of Jane and Dianne. Worthy of note, the computational and procedural (i.e., classical) approach of teaching probability has been found to be more prevalent among secondary teachers as compared to primary teachers (Stohl, 2005), and the notion of order is a key concept underlying the counting techniques of this classical probability. In particular, the route to a correct solution often lies in the answer to the question, "Does order matter?" It is interesting to note that these findings are echoed in the results of this study, where of the 23 PESTs that chose Jane's response, 15 (65%) referenced order in the justifications and, of the 32 PSSTs that chose Jane's response, 30 (94%) referenced order in the justifications while just 2 (6%) did not reference order in their justifications.

In an attempt to better understand the misconceptions of students rather than to simply categorize their existence, the *explicitness maxim* is offered as an explanatory classroom edict developed through experience with probability questions. It seems that given the overwhelming prevalence of the normative or classical notion of probability that dominates mathematics classrooms, and thus students' encounters with probabilistic reasoning, an appeal to ignorance may be "neither a reflection of basic deficits in logical reasoning not a result of simple carelessness" (Konold et al., 1993, p. 411). In other words, it may, in some sense, be logical. In short, it seems that the structure of school mathematics class may perpetrate the fallacy, but more research will determine to what extent logical fallacies—such as the fallacy of composition and an appeal to ignorance—play a part in teachers' and students' knowledge of probability, and what factors (both mathematical and social) contribute to their use.

7. Concluding Remarks

As demonstrated in the analysis of results, all 10 responses that were analyzed can be framed within the logical fallacy know as an appeal to ignorance. More specifically, all 10 responses made reference, whether implicitly or explicitly, to the question not "stating," "asking," "indicating," or "declaring" that order matters (i.e., there is no evidence for *p*) and, as such, the responses concluded that the order (of the outcomes) does not matter (i.e., therefore *not-p*), which was represented differently by different individuals and which led, ultimately, to their decision to choose Jane's response and explanation.

Although only 10 responses were presented in the analysis of results, we note, based upon the striking similarities between the 10 responses presented and the 11 responses not presented, that the logical fallacy, known as an appeal to ignorance, accounts for 100% (21/21) of the participants whose responses referenced both order and the question, which also represents, concurrently, 47% (21/45) of the responses who chose Jane's response and referenced order in their justifications, 38% (21/55) of those responses who chose Jane's response, and 16% (21/130) of all the participants involved in the current research.

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There is no evidence for order mattering; therefore, order does not matter: An appeal to ignorance

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Within the limited field of research on teachers' probabilistic knowledge, incorrect, inconsistent and inexplicable responses to probabilistic tasks are most often accounted for by utilizing theories, frameworks and models that are based upon heuristic and informal reasoning, such as the representativeness heuristic, the equiprobability bias, the outcome approach, and others. More recently, new research based upon logical fallacies has been proving effective in explaining certain normatively incorrect responses to probabilistic tasks. However, while it has been established that the fallacy of composition is able to account for certain incorrect responses to comparisons of relative probability, it had not been determined to what extent logical fallacies can account for responses to other probabilistic tasks and which other fallacies could be utilized. This article contributes to this emerging area of research by demonstrating how a second logical fallacy, known as "an appeal to ignorance," can be used to account for a specific set of normatively incorrect responses provided by prospective elementary and secondary mathematics teachers to a new probabilistic task.

In particular, a group (n = 130) of prospective elementary and secondary school teachers were asked to determine and justify, via written response, which of two student responses provided the correct answer and explanation to a question that involved determining the probability that a three-child family has two daughters and one son, an alteration to a task recently introduced to the research literature. In this new version of the task, altered with the goal of contributing to the research on teachers' probabilistic knowledge, the focus is not on evaluating the prospective teachers' pedagogical responses, but rather on which response they deem to be mathematically correct.

It was found that 49% of the prospective teachers chose the incorrect student response, with little difference between the percentage of prospective secondary and prospective elementary school teachers that chose this response. The analysis, however, focuses on 21 responses referencing both order and the given question in their justifications. In particular, these participants reason that, given that the question does not explicitly "state," "indicate," or "declare" that birth order matters in this task, birth order is not relevant to the situation. It is demonstrated that logically fallacious reasoning—more specifically, an appeal to ignorance (there is no evidence for proposition p; therefore, not-p)—accounts for these normatively incorrect responses to the task.

In addition to demonstrating the use of the appeal to ignorance fallacy by prospective mathematics teachers, the article also explores possible factors supporting and perpetuating its use. In particular, and in line with previous research on the existence of maxim-like beliefs related to probabilistic situations, it is suggested that individuals employing this fallacy are following a belief that is termed the *explicitness maxim*,

whereby it is assumed that a question must contain all pertinent information to arrive at a correct solution. It is further proposed that this maxim reflects the sociomathematical sphere in which probabilistic tasks are encountered, and in particular, that a focus on the classical approach to teaching theoretical probability may contribute to the use of this particular logical fallacy.