On (m_X, m_Y)–approximately semi open maps between m_X–spaces[1]

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En este artículo se da la noción de mapeos semi abiertos aproximadamente (m_X, m_Y) entre m–espacios como una generalización del concepto de mapeos aproximadamente semi abiertos. También se dan algunas caracterizaciones de los mapeos aproximadamente semi abiertos (m_X, m_Y).

Palabras Claves: m–estructuras, conjuntos semi cerrados m_X, conjuntos cerrados m_X–sg.

In this article, the notion of (m_X, m_Y)–approximately semi open maps between m–spaces is given as a generalization of the concept of approximately semi open maps. Also some characterizations of (m_X, m_Y)–approximately semi open maps are given.

Keywords: m–structure, m_X–semi closed set, m_X–sg closed set.

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1 Introduction

The concept of minimal structure was introduced in 1999 by Maki *et al.* [11]. After this work, various mathematicians turned their attention in introducing and studying diverse classes of sets defined on the m–structure, because this notion is a natural generalization of many well known results related to generalized sets in topological spaces and several weaker forms of continuity. Each one of these classes of sets is, in turn, used in order to obtain different separation properties and new forms of continuity (see [1, 5, 6, 7, 8, 9, 12, 14, 15], for details). In this work, we use the notion of m–structure in order to define and characterize the (m_X, m_Y)–approximately semi open maps, (m_X, m_Y)–irresolute maps. Also, we find conditions under which the direct image of any m_X–sg open set in X is m_Y–sg open in Y and the inverse image of any m_Y–sg open set in Y is m_X–sg open in X. Finally we show that our results constitute a generalization of many of the results obtained by Caldas *et al.* [1].

2 Minimal structures

In this section, we introduce the m–structure and the m–operator notions. Also, we define some important subsets associated to these concepts.

Definition 2.1. [11]. Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the set of power of X. We say that m_X is an m–structure (or a minimal structure) on X, if $\emptyset$ and X belong to m_X.

The members of the minimal structure m_X are called m_X–open sets, and the pair (X, m_X) is called an m–space. The complement of an m_X–open set is said to be an m_X–closed set. Given $A \subseteq X$, we define m_X–interior of A abbreviate m_X-Int(A) as $\bigcup\{W/W \subseteq m_X, W \subseteq A\}$ and the m_X–closure of A abbreviate m_X-Cl(A) as $\bigcap\{F/A \subseteq F, X \setminus F \in m_X\}$. As an immediate consequence of the above definition is the following theorem.

Theorem 2.1. *Let (X, m_X) be an m–space and A a subset of X. Then $x \in m_X$-Cl(A) if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x. And satisfying the following properties:*

1. m_X-Cl$(m_X$-Cl$(A))$=m_X-Cl(A).

2. m_X-Int(m_X-Int(A))=m_X-Int(A).

3. m_X-Int($X \setminus A$)=$X \setminus$ (m_X-Cl(A).

4. m_X-Cl($X \setminus A$)=$X \setminus$ (m_X-Int(A).

5. If $A \subseteq B$ then m_X-Cl(A) $\subseteq$ m_X-Cl(B)

6. m_X-Cl(A) $\cup$ m_X-Cl(B) $\subseteq$ m_X-Cl($A \cup B$).

7. $A \subseteq m_X$-Cl(A) and m_X-Int(A) $\subseteq A$.

Proof. It is easy from m_X-Cl and m_X-Int definitions.

□

Rosas *et al.* [2] define the notion of α semi open sets as $A \subseteq X$ is an α–semi open set if there exists an open set U such that $U \subseteq A \subseteq \alpha(U)$. In the case that X is an m–space and using the notion of m_X–closure, we have the following definition.

Definition 2.2. Let (X, m_X) be an m–space. We say that $A \subseteq X$ is an m_X–semi open set if there exists $U \in m_X$ such that $U \subseteq A \subseteq m_X$-Cl($U$). Also we say that $A \subseteq X$ is an m_X–semi closed set if $X \setminus A$ is an m_X–semi open set.

We denote by $SO(X, m_X)$ the collection of all m_X–semi open sets of X and $SC(X, m_X)$ the collection of all m_X–semi closed sets of X. Observe that when m_X is a topology on X, the m_X-Cl(A) is exactly the Cl(A).

Definition 2.3. Let (X, m_X) be an m–space and $A \subseteq X$. The m_X–semi closure of A denoted by m_X-sCl(A) is defined as the intersection of all m_X–semi closed sets of X containing A and we define m_X–semi interior of A denoted by m_X-sInt(A) as the union of all m_X–semi open sets of X contained in A.

Similarly as in the Theorem 2.1, we have the following characterization.

Theorem 2.2. *Let m_X be an m–structure on X then $x \in m_X$-sCl(A) if and only if $U \cap A \neq \emptyset$ for every m_X–semi open set U such that $x \in U$. Also,*

1. m_X-sCl($\emptyset$)=$\emptyset$.

2. m_X-sCl(X)=X.

3. If $A \subseteq B$ then m_X-sCl(A) $\subseteq m_X$-sCl(B.)

4. m_X-sCl(m_X-sCl(A)) $= m_X$-sCl(A);

5. m_X-sCl($X \setminus A$) $= X \setminus$ (m_X-sInt(A)).

Proof. It is easy from m_X-sCl and m_X-$sInt$ definitions. □

From Theorem 2.1 and Theorem 2.2, for all $A \subseteq X$ we have the inclusion

$$m_X - sCl(A) \subseteq m_X - Cl(A)\,.$$

Observe that m_X-sCl(A)(resp., m_X-Cl(A)) is not necessarily an m_X–semi closed (resp., m_X–closed) set. At this point there are a natural question. There exist any condition on the set X or in the m–structure of X in order to guarantee that the m_X-sCl(A) (resp.,m_X-Cl(A))is an m_X–semi closed (resp., m_X–closed) set. At this point we introduce the following property.

Definition 2.4. [11]. Let (X, m_X) be an m–space. We say that m_X to have the property of Maki, if the union of any family of elements of m_X belongs to m_X.

Observe that any collection $\emptyset \neq \mathcal{J} \subseteq P(X)$, always is contained in an m–structure that have the property of Maki, as we know, $m_X(\mathcal{J}) = \{\emptyset, X\} \cup \{\bigcup_{M \in J} M : \emptyset \neq J \subseteq \mathcal{J}\}$. In particular, when $\mathcal{J} = m_X$, we denote by $m_X^{'} = m_X(\mathcal{J})$. Clearly $m_X = m_X^{'}$, if m_X have the property of Maki. Note that if m_X is an m–structure and $Y \subseteq X$, then $\{M \cap Y : M \in m_X\}$ is an m–structure on Y, and is denoted by $m_{X|Y}$, and the pair $(Y, m_{X|Y})$ is called an m–subspace of (X, m_X).

In general the m_X–open sets and the m_X–semi–open sets are not stable for the union. Nevertheless, for certain m_X–structure, the class of m_X–semi open sets are stable under union of sets, like it is demons treated in the following lemma.

Lemma 2.1. Let m_X be an m–structure which satisfy the property of Maki. If $\{A_i : i \in I\}$ is a collection of m_X–semi–open sets (resp., m_X–semi–closed sets), then $\bigcup_{i\in I} A_i$ (resp., $\bigcap_{i\in I} A_i$) is an m_X–semi–open set (resp., m_X–semi–closed set).

Proof. Suppose that m_X has the property of Maki and $\{A_i : i \in I\}$ is a collection of m_X–semi–open sets. For each $i \in I$, there exists a set $U_i \in m_X$ such that $U_i \subseteq A_i \subseteq m_X\mathrm{Cl}(U_i)$, in consequence, $\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} A_i \subseteq \bigcup_{i\in I} m_X\text{-}\mathrm{Cl}(U_i)$. Since m_X-Cl is a monotone operator, then $\bigcup_{i\in I} m_X\text{-}\mathrm{Cl}(U_i) \subseteq m_X\text{-}\mathrm{Cl}(\bigcup_{i\in I} U_i)$; and $\bigcup_{i\in I} U_i \in m_X$, because m_X has the property of Maki. It follows that $\bigcup_{i\in I} U_i \in m_X$ and $\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} A_i \subseteq m_X\text{-}\mathrm{Cl}(\bigcup_{i\in I} U_i)$, therefore $\bigcup_{i\in I} A_i$ is an m_X–semi–open sets. □

Theorem 2.3. *Let (X, m_X) be an m–space and m_X satisfying the property of Maki. For a subset A of X, the following properties hold:*

1. m_X-Int$(A) \in m_X$ and m_X-Cl(A) is m_X–closed.

2. $A \in m_X$ if and only if m_X-Int$(A) = A$.

3. A is m_X–closed if and only if m_X-Cl$(A) = A$.

Proof.

1. Obvious.

2. If $A \in m_X$. then

$$m_X\text{-}\mathrm{Int}(A) = \bigcup\{W/W \subseteq m_X, W \subseteq A\} = A$$

Conversely, since m_X satisfying the property of Maki, then m_X-Int$(A) \in m_X$. It follows that $A \in m_X$.

3. If A is an m_X–closed set, then $X - A \in m_X$. By definition of m_X–interior, m_X-Int$(X - A) = X - A$. Using Theorem 2.1(3), m_X-Int$(X - A) = X - (m_X$-Cl$(A))$. In consequence. m_X-Cl$(A) = A$.

Conversely, since m_X satisfying the property of Maki, then m_X-Int$(A) \in m_X$. It follows that A is m_X–closed.

□

In a similar form, we can prove the following theorem.

Theorem 2.4. *Let (X, m_X) be an m–space and m_X satisfying the property of Maki. For a subset A of X, the following properties hold:*

1. m_X-sInt(A) is m_X–semi open and m_X-Cl(A) is m_X–semi closed.

2. A is m_X–semi open if and only if m_X-sInt$(A) = A$.

3. A is m_X–semi closed if and only if m_X-sCl$(A) = A$.

From the last result, it follows that m_X–closed set $\Rightarrow m_X$–semi closed set, or equivalently, $m_X -$ open set $\Rightarrow m_X$-semi open set.

Corolary 2.3. Let (X, m_X) be an m–space and m_X satisfying the property of Maki. For a subset $A \subseteq X$, we have:

1. A is m_X–semi open if and only if $A \subseteq m_X$-Cl$(m_X$-Int$(A))$.

2. A is m_X–semi closed if and only if m_X-Int$(m_X$-Cl$(A)) \subseteq A$.

Proof. Obvious.

□

It is important to observe that the m–structure notion, uses in abstract form the properties of many important collections of generalized sets without the necessity of a topological structure, some of them are illustrated in the following situations:

1. Given a topological space (X, τ), the collection of: θ–open sets, (respectively semi open sets, pre open sets, β–open sets) denoted by τ_θ,(respectively $SO(X)$, $PO(X)$, $\beta(X)$) is an m–structures on X, and all satisfy the property of Maki. Also, the collection of closed sets in X is an m–structure and satisfy the property of Maki, if (X, τ) is an Alexandroff space.

2. If α is an operator associated with the topology τ on X in the sense of Carpintero *et al.* [2, 3], then the collections Γ_α and $\alpha - SO(X, \tau)$ are m–structures. Γ_α and $SO(X, \tau)$ also has the property of Maki and α–$SO(X, \tau)$ has the property of Maki, if α is a monotone operator.

3. If α, β are operators associated with a topology τ on X, the collection $(\alpha,\beta) - SO(X,\tau)$, introduced by Rosas *et al.* [14], also is an m–structure and satisfy the property of Maki.

4. If α, β and γ are operators associated with τ on X, the collection $\gamma - (\alpha,\beta)$-$SO(X,\tau)$, defined by Rosas *et al.* [15], is also an m–structure, and satisfy the property of Maki, when the operator γ is expansive on the class $(\alpha,\beta) - SO(X,\tau)$.

The following theorem improves the Lemma 1 in [7].

Theorem 2.5. *Let (X, m_X) be an m–space and $A \subseteq X$. If m_X satisfy the property of Maki. Then*

$$m_X\text{-sCl}(A) = A \cup m_X\text{-Int}(m_X\text{-Cl}(A))\,.$$

Proof. Since m_X satisfy the property of Maki, then m_X-sCl(A) is an m_X–semi closed set, using Corollary 2.1, we obtain that

$$m_X\text{-Int}(m_X\text{-Cl}(m_X\text{-sCl}A)) \subseteq m_X\text{-sCl}(A)\,.$$

Therefore

$$m_X\text{-Int}(m_X\text{-Cl}(A)) \subseteq m_X\text{-sCl}(A)\,.$$

It follows that $A \cup m_X$-Int$(m_X$-Cl$(A)) \subseteq m_X$-sCl(A).
The opposite inclusion, we observe that

$$\begin{aligned}
& m_X\text{-Int}(m_X\text{-Cl}(A \cup m_X\text{-Int}(m_X\text{-Cl}(A)))) \\
= \;& m_X\text{-Int}(m_X\text{-Cl}(A) \cup m_X\text{-Cl}(m_X\text{-Int}(m_X\text{-Cl}(m_X\text{-Cl}A)))) \\
\subseteq \;& (m_X\text{-Cl}(A)) \cup m_X\text{-Int}(m_X\text{-Cl}(m_X\text{-Int}(m_X\text{-Cl}(A)))) \\
= \;& m_X\text{-Cl}(A) \cup m_X\text{-Int}(m_X\text{-Cl}(A)) = m_X\text{-Cl}(A)\,.
\end{aligned}$$

Thus

$$\begin{aligned}
& m_X\text{-Int}(m_X\text{-Cl}(A \cup m_X\text{-Int}(m_X\text{-Cl}(A)))) \\
\subseteq \;& m_X\text{-Int}(m_X\text{-Cl}(A)) \subseteq A \cup m_X\text{-Int}(m_X\text{-Cl}(A))\,.
\end{aligned}$$

It follows that

$$\begin{aligned} & m_X\text{-Int}(m_X\text{-Cl}(A \cup m_X\text{-Int}(m_X\text{-Cl}(A)))) \\ \subseteq \ & A \cup m_X\text{-Int}(m_X\text{-Cl}(A))\,. \end{aligned}$$

In consequence, by Corollary 2.1, $A \cup m_X$-Int(m_X-Cl(A)) is an m_X–semi closed set and so m_X-sCl(A) $\subseteq A \cup m_X$-Int(m_X-Cl(A)).

□

The following example shows that the property of Maki in the Theorem 2.5 can not be removed.

Example 2.1. Let $X = \{a, b, c, d\}$. Define the m_X structure on X as

$$m_X = \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{a, b, d\},\, \{a, b, c\}\}\,.$$

Then

$$\begin{aligned} SO(X,\, m_X) \quad = \quad & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{a,\, c\},\, \{a,\, d\},\, \{b,\, c\},\, \{b,\, d\}, \\ & \{a,\, b,\, c\},\, \{a,\, b,\, d\},\, \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,, \end{aligned}$$

and

$$\begin{aligned} SC(X,\, m_X) \quad = \quad & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{c\},\, \{d\},\, \{a,\, b\},\, \{a,\, c\}, \\ & \{b,\, c\},\, \{b,\, d\},\, \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,. \end{aligned}$$

Observe that for $A = \{a, c\}$, we have that

$$m_X\text{-sCl}(A) \subset A \cup m_X\text{-Int}(m_X\text{-Cl}(A))\,.$$

But $A \cup m_X$-Int(m_X-Cl(A)) $\not\subset m_X$-sCl(A).

Corolaty 2.2. Let (X, m_X) be an m–space and $A \subseteq X$. If m_X satisfy the property of Maki. Then m_X-sInt(A) $= A \cap m_X$-Cl(m_X-Int(A)).

Proof. The proof follows from Theorem 2.1, Theorem 2.2 and Theorem 2.5.

□

Definition 2.5. Let (X, m_X) be an m–space and A be a subset of X:

1. The set $\bigcap\{U \in m_X : A \subseteq U\}$ is called the m_X–kernel of A, and is denoted by m_X-Ker(A).
2. The set $\bigcap\{U \in SO(X, m_X) : A \subseteq U\}$ is called the m_X–semi kernel of A and is denoted by m_X-sKer(A).

The following theorem, characterizes the m_X topological–kernel and the m_X–semi topological–kernel.

Theorem 2.6. *Let (X, m_X) be an m–space, A and B be a subsets of X, then:*

1. $x \in m_X$-Ker(A) if and only if $A \cap F \neq \emptyset$ for any m_X–closed set F that contain x.
2. $A \subseteq m_X$-Ker(A) and $A = m_X$-Ker(A) if A is m_X–open.
3. If $A \subseteq B$, then m_X-Ker$(A) \subseteq m_X$-Ker(B).
4. $x \in m_X$-sKer(A) if and only if $A \cap F \neq \emptyset$ for any m_X–semi closed set F that contain x.
5. $A \subseteq m_X$-sKer(A) and $A = m_X$-sKer(A) if A is m_X–semi open.
6. If $A \subseteq B$, then m_X-sKer$(A) \subseteq m_X$-sKer(B).

Proof. It is easy from Definition 2.5.

□

Definition 2.6. Let m_X be an m–structure on X. A subset $A \subseteq X$, is said to be an m_X–semi generalized closed set (abbreviated by m_X–sg–closed) if m_X-sCl$(A) \subseteq U$, whenever $A \subseteq U$ and U is an m_X–semi open set. A subset $A \subseteq X$, is said to be an m_X–semi generalized open set (abbreviated by m_X–sg–open) if, its complement is an m_X–sg–closed set.

The followings theorems, characterize the m_X–generalized closed sets and the m_X–semi generalized open sets.

Theorem 2.7. *Let* m_X *be an* m*–structure on* X *satisfying the property of Maki.* $A \subseteq X$ *is an* m_X*–sg-closed set if and only if there are not exist* m_X*–semi closed set* F *such that* $F \neq \emptyset$ *and* $F \subseteq m_X$*-sCl*$(A) \setminus A$*.*

Proof. Suppose that A is an m_X–sg–closed set and let $F \subseteq X$ be an m_X–semi closed set such that $F \subseteq m_X\text{-sCl}(A) \setminus A$. It follows that $A \subseteq X \setminus F$ and $X \setminus F$ is an m_X–semi open set, since A is an m_X–sg–closed, we have that $m_X\text{-sCl}(A) \subseteq X \setminus F$ and $F \subseteq X \setminus m_X\text{-sCl}(A)$. It follows that

$$F \subseteq m_X\text{-sCl}(A) \cap (X \setminus m_X\text{-sCl}(A)) = \emptyset,$$

implying that $F = \emptyset$. Conversely, if $A \subseteq U$ and U is an m_X–semi open set, then $m_X\text{-sCl}(A) \cap (X \setminus U) \subseteq m_X\text{-sCl}(A) \cap (X \setminus A) = m_X\text{-sCl}(A) \setminus A$. Since $m_X\text{-sCl}(A) \setminus A$ does not contain subsets m_X–semi closed different from the empty set, we obtain that $m_X\text{-sCl}(A) \cap (X \setminus U) = \emptyset$, and this implies that $m_X\text{-sCl}(A) \subseteq U$ in consequence A is an m_X–sg–closed.

□

As an immediate consequence of the above theorem, we have the next theorem.

Theorem 2.8. *Let* m_X *be an* m*–structure on* X *satisfying the property of Maki.* $A \subseteq X$ *is an* m_X*–sg–open if and only if there are not exist* m_X*–semi closed set* F *such that* $F \neq \emptyset$ *and* $F \subseteq A \setminus m_X - sInt(A)$*.*

3 Some continuous functions between minimal structure

In this section, we study the notions of approximately semi open maps and irresoluteness between m–structures and we looking for conditions under what the direct image of any m_X–sg open set in X is m_Y–sg open in Y and the inverse image of any m_Y–sg open set in Y is m_X–sg open in X.

Definition 3.1. A map $f : (X, m_X) \to (Y, m_Y)$ is called (m_X, m_Y)–irresolute (resp. (m_X, m_Y)–sg–irresolute) if, $f^{-1}(O)$ is m_X–semi open (resp. m_X–sg–open)in X for every $O \in SO(Y, m_Y)$ (resp. m_Y–sg–open in Y).

Remark 3.1. Observe that if in the above definition:

1. If m_X is a topology on X and m_Y is a topology on Y, then we obtain the definition of irresolute map [1].

2. If m_X is the collections of all sg–closed set in X and m_Y is the collections of all sg–closed set in Y, then we obtain the definition of sg–irresolute map [1].

Definition 3.2. A map $f : (X, m_X) \to (Y, m_Y)$ is called (m_X, m_Y)–contra irresolute if, $f^{-1}(O)$ is m_X–semi closed in X for every $O \in SO(Y, m_Y)$.

Remark 3.2. Observe that if in the above definition:

1. If m_X is the collections of all semi closed sets in X and m_Y is a topology on Y, then we obtain the definition of contra irresolute map [1].

2. If m_X is the collections of all sg–closed set in X and m_Y is the collections of all sg–closed set in Y, then we obtain the definition of sg–irresolute map [1].

Lemma 3.1. Let (X, m_X) and (Y, m_Y) be two m–spaces, where m_X satisfies the property of Maki. The following conditions are equivalent:

1. $f : (X, m_X) \to (Y, m_Y)$ is (m_X, m_Y)–irresolute function;

2. For each subset $A \subseteq X$, $f(m_X - sCl(A)) \subseteq m_Y - sCl(f(A))$;

3. For each m_Y–semi closed subset $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an m_X–semi closed in X;

4. For all $B \subseteq Y$, $m_X - sCl(f^{-1}(B)) \subseteq f^{-1}(m_Y - sCl(B))$.

Proof.

$(1 \Leftrightarrow 3)$. Follows from the definition of (m_X, m_Y)–irresolute function and the complement of set.

$(1 \Rightarrow 2)$. Let A be a subset of X and suppose that $y \notin m_Y$-sCl$(f(A))$, then there exists a m_Y–semi open set G in Y, such that $y \in G$ and $f(A) \cap G = \emptyset$, therefore, $f^{-1}(f(A) \cap G) = \emptyset$, it says that $A \cap f^{-1}(G) = \emptyset$. In consequence, m_X-sCl$(A) \subset X \setminus f^{-1}(G)$, it follows that $f(m_X$-sCl$(A)) \cap G = \emptyset$; and therefore, $y \notin f(m_X$-sCl$(A))$. But it is said that $f(m_X$-sCl$(A)) \subset m_Y$-sCl$(f(A))$ for all subset A of X.

$(2 \Rightarrow 3)$. Let V any m_Y–semi closed subset in Y, then $f^{-1}(V) \subseteq X$. By hypothesis $f(m_X$-sCl$(f^{-1}(V))) \subset m_Y$-sCl$(f(f^{-1}(V)))$, it follows that $f(m_X$-sCl$(f^{-1}(V))) \subset m_Y$-sCl$(V) = V$. In consequence, $f(m_X$-sCl$(f^{-1}(V))) \subset V$, it follows that m_X-sCl$(f^{-1}(V) \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is an m_X–semi closed set.

$(2 \Rightarrow 4)$. Let B be a subset of Y, then $f^{-1}(B) \subseteq X$. Using the hypothesis, that

$$f(m_X - sCl(f^{-1}(B))) \subseteq m_Y - sCl(f(f^{-1}(B))) \subseteq m_Y - sCl(B),$$

therefore, $m_X - sCl(f^{-1}(B)) \subseteq f^{-1}(m_Y - sCl(B))$.

$(4 \Rightarrow 3)$. Suppose that V is any m_Y–semi closed set in Y. Then $f^{-1}(V) \subseteq X$, by hypothesis, we obtain that

$$m_X - sCl(f^{-1}(V)) \subseteq f^{-1}(m_Y - sCl(V))\,.$$

But V is a m_Y–semi closed set, then $m_Y - sCl(V) = V$. In consequence,

$$m_X - sCl(f^{-1}(V)) \subseteq f^{-1}(V)\,.$$

But this says that $f^{-1}(V)$ is an m_X–semi closed set in X.

The following example shows that the property of Maki in the above lemma can not be removed.

Example 3.1. Let $X = \{a, b, c, d\}$. Define the m_X structure on X as $m_X = \{\emptyset, X, \{a\}, \{b\}\}$. Then

$$\begin{aligned} SO(X, m_X) \;=\; & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{a,\, c\},\, \{a,\, d\},\, \{b,\, c\},\, \{b,\, d\}, \\ & \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,, \end{aligned}$$

and

$$SC(X, m_X) = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

Let $Y = \{0, 1\}$, viewed as discrete space (*i.e.*, $m_Y = P(Y)$). Define $f : (X, m_X) \to (Y, m_Y)$ as

$$f(x) = \begin{cases} 1 & , \text{ if } x \in A \\ 0 & , \text{ if } x \notin A \end{cases}$$

where $A = \{a, b\}$. Observe that m_X-$sCl(A) = X$, then $f(m_X$-$sCl(A)) \neq m_Y$-$sCl(f(A))$. But f is (m_X, m_Y)–irresolute.

□

Lemma 3.2. Let (X, m_X) and (Y, m_Y) be m–spaces, where m_X satisfies the property of Maki. The following conditions are equivalent:

1. $f : (X, m_X) \to (Y, m_Y)$ is (m_X, m_Y)–contra irresolute function;

2. For each $A \subseteq X$, $f(m_X\text{-sCl}(A)) \subseteq m_Y\text{-sKer}(f(A))$;

3. For each m_Y–semi closed set $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an m_X–semi open set in X;

4. For each $x \in X$ and F be a m_Y–semi closed set in Y such that $f(x) \in F$, there exists an m_X–semi open set $U \subseteq X$ such that $x \in U$ and $f(U) \subseteq F$;

5. For each $B \subseteq Y$, $m_X - sCl(f^{-1}(B)) \subseteq f^{-1}(m_Y - sKer(B))$.

Proof.

$(1 \Rightarrow 3)$. Follows using complement.

$(3 \Rightarrow 2)$. Let A be a subset of X and suppose that $y \notin m_Y$-sKer$(f(A))$, then there exists a m_Y–semi closed set F in Y, such that $y \in F$ and $f(A) \cap F = \emptyset$, therefore $f^{-1}(f(A) \cap F) = \emptyset$, it said that $A \cap f^{-1}(F) = \emptyset$. In consequence, m_X-sCl$(A) \subset X \setminus f^{-1}(F)$, it follows that $f(m_X\text{-sCl}(A)) \cap F = \emptyset$, but, it said that $y \notin f(m_X$-sCl$(A))$. Therefore, $f(m_X\text{-sCl}(A)) \subset m_Y\text{-sKer}(f(A))$ for all subset A of X.

$(2 \Rightarrow 5)$. Let B be any subset in Y, then $f^{-1}(B) \subseteq X$. By hypothesis $f(m_X\text{-sCl}(f^{-1}(B))) \subset m_Y\text{-sKer}(f(f^{-1}(B)))$, it follows that $f(m_X$-sCl$(f^{-1}(B))) \subset m_Y$-sKer(B). Therefore, $m_X - sCl(f^{-1}(B) \subset f^{-1}(m_Y - sKer(B))$.

$(5 \Rightarrow 1)$. Let V be any m_Y–semi open set in Y, then $f^{-1}(V) \subseteq X$. By hypothesis, $m_X - sCl(f^{-1}(V))) \subset f^{-1}(m_Y - sKer(V))$, but $m_Y - sKer(V) = V$, it follows that $m_X - sCl(f^{-1}(V))) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is an m_X–semi closed set.

$(3 \Rightarrow 4)$ and $(4 \Rightarrow 3)$ are immediate.

□

The Example 3.1 shows that the property of Maki, in the Lemma 3.2, can not be removed too.

Definition 3.3. A map $f : (X, m_X) \to (Y, m_Y)$ is called (m_X, m_Y)–pre–semi closed (resp. (m_X, m_Y)–pre–semi open) if for every m_X–semi closed (resp. m_X–semi open) set B in X, $f(B)$ is m_Y–semi closed (resp. m_Y–semi open) in Y.

Remark 3.3. If in Definition 3.3, m_X is the collections of all semi closed (resp. semi open) sets in X and m_Y is a collection of semi closed (resp. semi open) sets in Y, then we obtain the definition of pre–semi closed (resp. pre–semi open) map given in [1].

Definition 3.4. Let (X, m_X) and (Y, m_Y) be m–spaces. A map $f : (X, m_X) \to (Y, m_Y)$ is said to be (m_X, m_Y)-approximately semi open (briefly, (m_X, m_Y)–ap–semi–open) if m_Y-sCl$(B) \subseteq f(A)$ whenever B is an m_Y–sg–closed set in Y, A is an m_X–semi closed set of X and $B \subseteq f(A)$.

Remark 3.4. If in Definition 3.4, m_X and m_Y are topologies on X and Y respectively, then we obtain the definition of ap–semi open given in [1].

Definition 3.5. Let $X, m_X)$ and (Y, m_Y) be m–spaces. A map $f : (X, m_X) \to (Y, m_Y)$ is said to be (m_X, m_Y)–approximately semi closed (briefly, (m_X, m_Y)–ap–semi–closed) if $f(B) \subseteq m_Y$-sInt(A) whenever A is an m_Y–sg–open set in Y, B is an m_X–semi closed set of X and $f(B) \subseteq A$.

Remark 3.5. If in Definition 3.5, m_X and m_Y are topologies on X and Y respectively, then we obtain the definition of ap–semi closed given in [1].

Definition 3.6. A map $f : (X, m_X) \to (Y, m_Y)$ is called (m_X, m_Y)–contra–pre–semi open if $f(O)$ is m_Y–semi closed set in Y for each set $O \in SO(X, m_X)$.

Definition 3.7. A map $f : (X, m_X) \to (Y, m_Y)$ is called (m_X, m_Y)–contra–pre–semi closed if $f(B) \in SO(Y, m_Y)$ for each m_X–semi closed set B of X.

Theorem 3.1. *Let $f : (X, m_X) \to (Y, m_Y)$ be a map. Then f is (m_X, m_Y)–ap–semi–open, if $f(O) \in SC(Y, m_Y)$ for every m_X–semi open subset $O \subseteq X$.*

Proof. Let $B \subseteq f(A)$, where A is an m_X–semi open set of X and B is an m_Y–sg–closed set in Y. Therefore the m_Y-sCl$(B) \subseteq m_Y$-sCl$(f(A))$. By hypothesis $f(A)$ is an m_Y–semi closed set, then m_Y-sCl$(f(A)) = f(A)$. Thus m_Y-sCl$(B) \subseteq f(A)$. In consequence, f is (m_X, m_Y)–ap–semi–open. □

Theorem 3.2. *Let $f : (X, m_X) \to (Y, m_Y)$ be a map. Then f is (m_X, m_Y)–ap–semi–closed, if $f(O) \in SO(Y, m_Y)$ for every m_X–semi closed subset O of (X, m_X).*

Proof. Let $f(B) \subseteq A$, where B is an m_X–semi closed set of X and A is an m_Y–sg–open set in Y. Therefore the m_Y-sInt$(f(B)) \subseteq m_Y$-sInt(A). By hypothesis $f(B)$ is an m_Y- semi open set, then m_Y-sInt$(f(B)) = f(B)$. Thus $f(B)) \subseteq m_Y$-sInt(A). In consequence, f is (m_X, m_Y)–ap–semi–open.

□

The following theorem improves the Theorem 2.7 given in [1], in the case that the m–structure on the set Y satisfies the property of Maki.

Therorem 3.3. *Let $f : (X, m_X) \to (Y, m_Y)$ be a map, where m_Y satisfies the property of Maki. If the m_Y–semi open and m_Y–semi closed sets of (Y, m_Y) coincide, then f is (m_X, m_Y)–ap–semi–open if and only if, $f(O) \in SC(Y, m_Y)$ for every m_X–semi open subset O of (X, m_X).*

Proof. Assume that f is (m_X, m_Y)–ap–semi–open. Let O be an arbitrary subset of (Y, m_Y) such that $O \subseteq B$ where $B \in SO(Y, m_Y)$, it follows that m_Y-sCl$(O) \subseteq m_Y$-sCl(B). But $B \in SO(Y, m_Y) = SC(Y, m_Y)$, then B is an m_Y–semi closed set, it follows that m_Y-sCl$(B) = B$. Therefore all subset of (Y, m_Y) are m_Y–sg–closed in Y, in consequence all are m_Y–sg–open. So for any $A \in SO(X, m_X)$, $f(A)$ is m_Y–sg–closed in (Y, m_Y). Since f is (m_X, m_Y)–ap–semi–open m_Y-sCl$(f(A)) \subseteq f(A)$. Therefore m_Y-sCl$(f(A)) = f(A)$. In consequence $f(A)$ is an m_Y–semi closed set in (Y, m_Y). The converse follows immediately from definition.

□

An immediate consequence of Theorem 3.3, is the following.

Theorem 3.4. *Let $f : (X, m_X) \to (Y, m_Y)$ be an (m_X, m_Y) be a map, where m_Y satisfies the property of Maki. If the m_Y–semi open and m_Y–semi closed sets of (Y, m_Y) coincide, then f is (m_X, m_Y)–ap–semi–open if and only if f is (m_X, m_Y)–pre–semi open.*

The following theorem, give to us conditions on the m–structure on the set Y in order to characterize the (m_X, m_Y)–ap–semi–closed.

Theorem 3.5. *Let $f : (X, m_X) \to (Y, m_Y)$ be a map, where m_Y satisfies the property of Maki. If the m_Y–semi open and m_Y–semi closed sets of (Y, m_Y) coincide, then f is (m_X, m_Y)–ap–semi–closed if and only if, $f(O) \in SO(Y, m_Y)$ for every m_X–semi closed subset O of (X, m_X).*

Proof. Assume that f is (m_X, m_Y)–ap–semi–closed. Let B be an arbitrary subset of (Y, m_Y) such that $B \subseteq O$ where $O \in SC(Y, m_Y)$ it follows that m_Y-sInt$(B) \subseteq m_Y$-sInt(O). But $O \in SC(Y, m_Y) = SO(Y, m_Y)$, then O is an m_Y–semi open set, it follows that m_Y-sInt$(O) = O$. Therefore all subset of (Y, m_Y) are m_Y–sg–open in Y, in consequence all are m_Y–sg–closed. So for any $A \in SC(X, m_X)$, $f(A)$ is m_Y–sg–open in

(Y, m_Y). Since f is (m_X, m_Y)–ap–semi–closed m_Y-sInt$(f(A)) \subseteq f(A)$. Therefore m_Y-sInt$(f(A)) = f(A)$. In consequence $f(A)$ is an m_Y–semi closed set in (Y, m_Y). The converse follows immediately from definition.

□

Example 3.3. Let $X = Y = \{a, b, c, d\}$. Define the m_X structure on X as

$$m_X = \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{a,\, b,\, d\},\, \{a,\, b,\, c\}\}\,.$$

Then

$$\begin{aligned} SO(X,\, m_X) \;\;=\;\; & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{a,\, c\},\, \{a,\, d\},\, \{b,\, c\},\, \{b,\, d\}, \\ & \{a,\, b,\, c\},\, \{a,\, b,\, d\},\, \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,, \end{aligned}$$

and

$$\begin{aligned} SC(X,\, m_X) \;\;=\;\; & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{c\},\, \{d\},\, \{a,\, b\},\, \{a,\, c\}, \\ & \{b,\, c\},\, \{b, d\},\, \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,. \end{aligned}$$

The set of all m_X–sg–closed is

$$\begin{aligned} & \{\emptyset,\, X,\, \{a\},\, \{b\},\, \{c\},\, \{d\},\, \{a,\, b\},\, \{a,\, c\},\, \{a,\, d\},\, \{b,\, c\}, \\ & \{b,\, d\},\, \{c,\, d\},\, \{a,\, b,\, c\},\, \{a,\, b,\, d\},\, \{a,\, c,\, d\},\, \{b,\, c,\, d\}\}\,, \end{aligned}$$

Define $f : (X, m_X) \to (Y, m_Y)$ as $f(a) = b, f(b) = a$ and $f(c) = f(d) = c$. Observe that f is (m_X, m_Y)–pre–semi–open, (m_X, m_Y)–pre–semi–closed and (m_X, m_Y)–ap–semi–open. But f is not (m_X, m_Y)–irresolute.

The following theorem improves the comments after Remark 2.3 given in [1] .

Theorem 3.6. *Let $f : (X, m_X) \to (Y, m_Y)$ be a map. If f is (m_X, m_Y)–pre–semi open, then f is (m_X, m_Y)–ap–semi–open.*

Proof. Let A any m_X–semi open set in X and let B any m_Y–sg closed set in Y such that $B \subseteq f(A)$ since f is (m_X, m_Y)–pre–semi open, then $f(A)$ is an m_Y–semi open set in consequence the $m_X -$ scl$(B) \subseteq f(A)$ and therefore f is (m_X, m_Y)–ap–semi–open.

□

The following example give to us a map $f : (X, m_X) \to (Y, m_Y)$, where m_Y satisfies the property of Maki,f is (m_X, m_Y)–ap–semi–open but not (m_X, m_Y)–pre–semi–open

Example 3.3. Let $X = Y = \{a, b\}$. Define the m_X structure on X as $m_X = \{\emptyset, X, \{a\}\}$. Observe that m_X is a topology. Then $SO(X, m_X) = \{\emptyset, X, \{a\}\}$ and $SC(X, m_X) = \{\emptyset, X, \{b\}\}$. The set of all m_X–sg–closed is $\{\emptyset, X, \{a\}, \{b\}\}$. Define $f : (X, m_X) \to (Y, m_Y)$ as $f(a) = b$ and $f(b) = a$. f is (m_X, m_Y)–ap–semi–open but not (m_X, m_Y)–pre–semi–open.

Definition 3.8. Let m_X be an m–structure on X. (X, m_X) is said to be an $m_X - sT_{1/2}$ if each m_X–sg–closed is an m_X–semi closed set.

The following theorem, characterizes the m_X-$sT_{1/2}$ when m_X satisfy the property of Maki.

Theorem 3.7. *Let m_X be an m–structure on X that satisfies the property of Maki. Then (X, m_X) is an $m_X - sT_{1/2}$ if and only if each unitary set $\{x\}$ in X is an m_X–semi open set or an m_X–semi closed set.*

Proof. (Sufficiency). Suppose that (X, m_X) is an $m_X - sT_{1/2}$. Then for any $x \in X$, the unitary set $\{x\}$ can be m_X–semi closed set or not. In the case that $\{x\}$ is an m_X–semi closed set, the result follows. In the other case, $X \setminus \{x\}$ is an m_X–sg–closed in m_X. Now using hypothesis, we obtain that $X \setminus \{x\}$ is an m_X–semi closed set and therefore, $\{x\}$ is an m_X–semi open.

(Necessity). Let A be an m_X–sg–closed in m_X and $x \in m_X - sCl(A)$. If $\{x\}$ is an m_X–semi open set, then $\{x\} \cap A \neq \emptyset$ and therefore, $x \in A$. In the case that $\{x\}$ is an m_X–semi closed set, then we have that $x \in A$, because if $x \notin A$, then $\{x\} \subseteq m_X - sCl(A) \setminus A$. but this is impossible.

□

The following example shows that the property of Maki in Theorem 3.7, can not be removed.

Example 3.4. Let $X = Y = \{a, b, c\}$. Define the m_X structure on X as $m_X = \{\emptyset, X, \{a\}, \{b\}, \{c\}\}$. Then $SO(X, m_X) = \{\emptyset, X, \{a\}.\{b\}, \{c\}\}$ and $SC(X, m_X) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{a, b\}\}$. The set of all m_X–sg–closed is $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Observe that the set $\{a\}$ is an m_X–sg–closed set that does not is m_X–semi–closed neither m_X–semi–open.

Theorem 3.8. *Let $f : (X, m_X) \to (Y, m_Y)$ be an (m_X, m_Y)–irresolute map and (m_X, m_Y)–pre–semi closed, where m_X satisfies the property of Maki, then:*

a. For all m_X–sg–closed set A in X, $f(A)$ is an m_Y–sg–closed set in Y.

b. $f^{-1}(B)$ is an m_X–sg–closed set in X for all m_Y–semi closed set B in Y.

Proof.

a. Let V be a m_Y–semi open set in Y such that $f(A) \subseteq V$, then $f^{-1}(V)$ is an m_X–semi open set and $A \subseteq f^{-1}(V)$, it follows that $m_X - sCl(A) \subseteq f^{-1}(V)$, since f is an (m_X, m_Y)–pre semi closed, then $f(m_X - sCl(A))$ is a m_X–semi closed set. In consequence,

$$\begin{aligned} m_X - sCl(f(A)) &\subseteq m_Y - sCl(f(m_X - sCl(A))) \\ &= f(m_X - sCl(A)) \\ &\subseteq V\,. \end{aligned}$$

Therefore, $f(A)$ is a m_X–semi generalized closed set.

b. Let B be a m_Y–semi closed set in Y, and suppose that U is an m_X–semi open set in X such that $f^{-1}(B) \subseteq U$. Consider $F = m_X - sCl(f^{-1}(B)) \cap U^c$. Then we may conclude that F is an m_X–semi closed set, therefore $f(F)$ is a m_Y–semi closed set and

$$\begin{aligned} f(F) &= f(m_X - sCl(f^{-1}(B)) \cap U^c) \\ &\subseteq f(m_X - sCl(f^{-1}(B))) \cap f(U^c) \\ &\subseteq m_Y - sClf^{-1}(f(B)) \cap f(U^c) \\ &\subseteq m_Y - sCl(B) \cap Y \backslash = \emptyset. \end{aligned}$$

In consequence, $f(F) = \emptyset$, and $F = \emptyset$, therefore, $f^{-1}(B)$ is an m_X semi generalized closed set.

□

Corolary 3.1. Let $f : (X, m_X) \to (Y, m_Y)$ be an (m_X, m_Y)–irresolute map and (m_X, m_Y)–pre semi closed, where m_X, m_Y are minimal structures satisfying the property of Maki, then:

1. If f is an injective function and (Y, m_Y) is a $m_Y - sT_{1/2}$ space, then (X, m_X) is an $m_X - sT_{1/2}$ space.

2. If f is a bijective function and (X, m_X) is an $m_X - sT_{1/2}$ space, then (Y, m_Y) is a $m_Y - sT_{1/2}$ space.

Proof.

1. Suppose that A is an m_X–sg–closed set in X. Using Theorem 3.8, and the hypothesis, $f(A)$ is a m_Y–sg–closed set in Y, but Y is a $m_Y - sT_{1/2}$ space, then $f(A)$ is a m_Y–semi closed set in Y. Now, using the fact that f is an injective function and (m_X, m_Y)–irresolute, it follows that $A = f^{-1}(f(A))$ is an m_X–semi closed set in X. In consequence, (X, m_X) is an $-m_X - sT_{1/2}$ space.

2. Given $y \in Y$, there exists a unique point $x \in X$ such that $y = f(x)$, it follows that each unitary set in Y is a m_Y–semi open set or m_Y–semi closed set and therefore Y is a $m_Y - sT_{1/2}$ space from Theorem 3.6.

□

The following theorem improves the Theorem 2.10 in [1]

Theorem 3.9. *Let $f : (X, m_X) \to (Y, m_Y)$ be an (m_X, m_Y)–irresolute, bijective and (m_X, m_Y)–ap semi open, where m_X, m_Y are minimal structures satisfying the property of Maki, then the inverse image of any m_Y–sg open is m_X–sg open.*

Proof. Let A be a m_Y–sg open set. Suppose that $F \subseteq f^{-1}(A)$ where $F \in$ SC(Y, m_Y). Taking complements we obtain $f^{-1}(A^c) \subseteq F^c$, it follows that $A^c \subseteq f(F^c)$. Using Theorem 2.4 and corollary 2.2, m_X-sCl$(A) =$

$A \cup m_X$-Int(m_X-Cl(A)) and m_X-sInt(A) $= A \cap m_X$-Cl(m_X-Int(A)). Since $(m_Y-\ \text{sInt}(A))^c = m_Y-\ \text{sCl}(A^c)$ and m_Y satisfies the property of Maki, we obtain $m_Y-\ \text{sCl}(A^c) = A^c$. In consequence, $(m_Y-\ \text{sInt}(A))^c \subseteq f(F^c)$, because f is (m_X, m_Y)–ap semi open, it follows that $f^{-1}(m_Y-\ \text{sInt}(A))^c \subseteq f^{-1}(f(F^c)) = F^c$ and therefore $F \subseteq f^{-1}(m_Y-\ \text{sInt}(A)$. Since f is an (m_X, m_Y)–irresolute, it follows that $f^{-1}(m_Y-\ \text{sInt}(A)$ is an m_X–semi open set and therefore $F \subseteq f^{-1}(m_Y-\ \text{sInt}(A) = m_X-\ \text{sInt}(f^{-1}(m_Y-\ \text{sInt}(A)) \subseteq (m_X-\ \text{sInt}(f^{-1}(A))$. This implies that $f^{-1}(A)$ is an m_X–sg open set.

□

The following example shows a surjective map $f : (X, m_X) \to (Y, m_Y)$ that is (m_X, m_Y)–irresolute and (m_X, m_Y)–pre semi open, where m_X, m_Y are minimal structures satisfying the property of Maki and does not satisfying that the inverse image of any m_Y–sg open in Y is an m_X–sg open set in X. In consequence in Theorem 2.10 in [1] the condition of bijectively on f is necessary.

Example 3.5. Let $X = \{a, b, c, d\}$. Define the m_X structure on X as $m_X = \{\emptyset, X, \{a, b, c\}, \{b, c, d\}\}$.Then

$$SO(X, m_X) = \{\emptyset, X, \{a, b, c\}, \{b, c, d\}\}$$

and $SC(X, m_X) = \{\emptyset, X, \{a\}, \{d\}\}$. The set of all m_X–sg–closed is $\{\emptyset, X, \{a\}, \{d\}, \{a, b, d\}, \{a, c, d\}\}$. The set of all m_X–sg–open is

$$\{\emptyset, X, \{b\}, \{c\}, \{a, b, c\}, \{b, c, d\}\}.$$

Let $Y = \{a, b, c\}$. Define the m_Y structure on Y as

$$m_Y = \{\emptyset, Y, \{a, b\}, \{b, c\}\}.$$

Then $SO(Y, m_Y) = \{\emptyset, Y, \{a, b\}, \{b, c\}\}$ and $SC(Y, m_Y) = \{\emptyset, Y, \{a\}, \{c\}\}$. The set of all m_Y–sg–closed is $\{\emptyset, Y, \{a\}, \{c\}, \{a, c\}\}$. The set of all m_Y–sg–open is $\{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$. Define $f : (X, m_X) \to (Y, m_Y)$ as $f(a) = a, f(b) = f(c) = b$ and $f(d) = c$. Observe that f is a surjective (m_X, m_Y)–irresolute and (m_X, m_Y)–pre semi open map but $f^{-1}(\{b\}) = \{b, c\}$ does not is an m_X–sg–open set.

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