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# λ-NUCLEAR SPACES WHICH ARE NUCLEAR

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It is well known [3] that every nuclear locally convex space $E$ is Hilbertian (that is $E$ has a basis $\mathfrak{U}$ of absolutely convex neighborhoods of $0$ such that each associated Banach space $\widetilde{E_U}$, $U \in \mathfrak{U}$ is a Hilbert space).

In general there is no converse of this result and several attempts have been made in order to find conditions under which Hilbertian locally convex spaces are nuclear. The framework of λ-nuclear spaces ( λ a normal scalar sequence space) is a suitable setting to investigate this property. Indeed, E. Dubinsky and M. S. Ramanujan, [1] , prove that if $\lambda$ is a perfect scalar sequence space such that $l_q \subset \lambda \subset l_p$ with $1 \leq q \leq p \leq \infty$ , $q \leq 2$ , $1/q - 1/p < 1/2$ , then every separable Hilbertian λ-nuclear space is nuclear.

In the main result of the present paper we exploit a former result of A. Persson and A. Pietsch [2] about Hilbert-Schmidt operators in Hilbert spaces to obtain the following

<u>Theorem</u>. Let $\lambda$ be any normal scalar sequence space and let $E$ be a λ-nuclear space. Then $E$ is nuclear iff $E$ is Hilbertian.

Sketch of the Proof. Let $\mathfrak{U}$ be a basis of Hilbertian neighborhoods of $0$ in $E$ . Since $E$ is λ-nuclear, for every $U \in \mathfrak{U}$ there exists $V \in \mathfrak{U}$ , $V \subset U$ , such that the canonical map $T_{UV} : \widetilde{E}_V \longrightarrow \widetilde{E}_U$ is λ-nuclear. We deduce that $T_{UV}$ is $l_\infty$-nuclear and then that $T_{UV}$ is a Hilbert-Schmidt map. This implies that $E$ is nuclear (cf. [3] , 4.4.2.).

The technical background involved in the proof of this Theorem is included in the following

<u>Proposition</u>. Let E and F be Hilbert spaces and $T : E \longrightarrow F$ a linear map. Then T is $l_\infty$-nuclear iff T is a Hilbert-Schmidt map.

## REFERENCES


[1] E. Dubinsky, M. S. Ramanujan, On $\lambda$-nuclearity. Mem. Amer. Math. Soc. 128, (1971)

[2] A. Persson, A. Pietsch, p-nukleare und p-integrale Abbildungen in Banachräumen. Studia Math. 33, (1969) 19-62

[3] A. Pietsch, Nuclear Locally Convex Spaces. Springer-Verlag, Berlin 1967